

Research on the Influence of Financial Mathematics on Modern Financial Market

Zhixuan Bao

School of Sdu-anu Joint Science College, Shan Dong University, Weihai, China

202100700001@mail.sdu.edu.cn

Abstract. In the process of continuous reform and development of financial markets, financial theories are constantly updated and the research and development of financial mathematics becomes more and more important. This paper focuses on the impact of financial mathematics on modern financial markets. Starting from the emergence and development prospect of modern financial mathematics, the impact of financial mathematics on the development of modern financial market is comprehensively elaborated by analyzing financial mathematical theories, such as harness theory, stochastic optimal control theory and optimal stopping theory, and analyzing the application of mathematical models in financial theory. Financial mathematics not only effectively drives the innovation and iteration of financial instruments, but also has a direct role in the development of financial markets. The article focuses on the application of mathematical knowledge in several financial problems, aiming to provide reference and reference for advancing the application of mathematical knowledge in the field of finance.

Keywords: Financial Mathematics, theory of harnessing, stochastic optimal control theory, optimal stopping theory, financial mathematical models, option pricing.

1. Introduction

With the rapid development of the world economy and the advent of the era of big data, mathematics, as a fundamental discipline in the natural sciences, plays a key role in the development of social economy, and thus the hybrid discipline of financial mathematics has emerged. To be precise, financial mathematics belongs to the economic discipline, which is of great value in the modern financial market and will make accurate judgments on market development to ensure that correct decisions can be made. At the same time, it can also rely on the construction of mathematical models to develop ideas for the development of the financial market, which will lead to the gradual expansion of the market scale, towards diversification and diversification, and continuously drive the market economy. This paper focuses on the impact of financial mathematics on modern financial markets, and solves common financial-type problems by constructing a variety of mathematical models. Through this study, the contribution of financial mathematics to the social economy and its shortcomings are summarized.

2. Organization of the Text

2.1. Financial Mathematics

2.1.1 Basic concept

Unlike the disciplines of financial economics and mathematics, the ideas involved in financial mathematics are more abstract; it is the application of mathematical thought models and theoretical methods to financial economics. Financial mathematics is concerned with problems in financial scenarios such as selecting multiple portfolio securities and pricing portfolio investment assets under multiple uncertainties. The basic ideas and three fundamental concepts of financial mathematics are arbitrage, optimality and equilibrium, and its core content is the study of portfolio, optimal choice and asset pricing theory in stochastic environments and under multiple conditions.

2.1.2 The emergence and prospects of modern financial mathematics

The year of the introduction of financial mathematics as a modern scientific term is only about 30 years old, but the earliest history of its activity dates back to the fundamental valuation relationship first identified and explained by Irving Fisher in 1896. This relationship is one of the core ideas of finance, stating that the value of an asset is equal to the sum of the present values of the future cash flows it generates [1]. A few years later, the French mathematician Bachelier's dissertation, "Speculative Theory", talked about the use of Brownian motion to study the stock market, a theory that in turn gave impetus to the development of financial mathematics. With the rise of computers, scholars who used mathematical tools to study financial problems were frequently awarded the Nobel Prize in Economics. In the context of the two Wall Street revolutions, modern financial mathematics arose and has flourished since the early 1980s, with the following three broad categories of people currently engaged in its study: probabilistic and stochastic analysis scholars, stochastic cybernetics scholars and mathematical statisticians [1]. It can be seen that the emergence of financial mathematics is an inevitable development of financial economy and has important practical significance.

2.2. The Analysis of the Impact of Financial Mathematical Theory on Modern Financial Markets

2.2.1 Harnessing theory

In most cases, financial market investments are in pursuit of minimizing opportunity costs. It is based on this condition that the harness theory was born, the core of which lies in the study and analysis of investment options and returns are both increasing functions of time, and the investment option curve is tangent to the investment return curve, and the investment opportunity cost is minimized at the position of the tangent point, which can also be said to be the optimal investment time, stage. The theory is able to tap into the laws in the process of financial market development, and conceptual definition for stochastic, equivalence harness. The main role at present is to solve the problem of product derivation in the financial market, including the price positioning of certain products, etc., to ensure that it is more scientific and reasonable. The main role of harness theory is to solve the problem of product derivation in financial markets, including the price positioning of certain products, to ensure that it is more scientific and reasonable. The theory of harness has an extremely important position in the financial market today, and will affect more investment return relationships in the market, or have a direct connection, so the theory is also being fully promoted and applied [2].

2.2.2 Stochastic optimal control theory

Stochastic control, as an important branch of control theory, is a theory that combines stochastic control theory and optimal control theory to study stochastic control systems, and is now widely used in the field of finance. The application of stochastic control theory to solve optimal investment strategy problems in finance mainly relies on Belinan's dynamic programming principle to obtain the dynamic programming equation, namely the Hamilton-Jacobi-Bellman (HJB) equation, and to solve it [3]. Stochastic optimal theory emerged in the late 1960s as an important theoretical credential in the theory of financial economics to deal with stochastic problems. Stochastic optimal control approaches are widely used in the current financial industry and will continue to be developed and used as well.

2.2.3 Optimal stopping theory

Optimal stopping theory is an area of probability theory with a strong application background, and it is a theory that is widely used in the pricing of financial derivatives. Its main applications are in investment decisions, evaluating option values, and pricing problems of cash flows. Optimal stopping theory is based on the irreversibility and irrevocability of time, where a finite amount of time is available to make the best possible choice.

2.3. Mathematical Models in Financial Theory

2.3.1 Present value formula model

In order to understand specifically the intrinsic value of a financial investment, it is necessary to calculate the present value and assess the value for the sake of money. The process is based on the link between the discount rate and the present value, specifying the rate of payoff in the investor's expectations and applying the present value formula model. The present value formula needs to be derived from the compound interest formula, $F = P(1+r)^n$ (P : principal; r : annual interest rate; F : final value), get $P = F/(1+r)^n$, The discount factor $1/(1+r)^n$ is denoted as $(P/F, r, n)$, which in turn leads to the derivation of the present value formula as $P = F (P/F, r, n)$ [4].

In real life, the bank mortgage to buy commodity houses, customers pay in installments; online shopping, customers can choose installment services. The economic meaning of present value is to discount the cash flow received in the future to the present value, and these services can provide convenient services for social production and life while promoting the flow of funds. In the process of installment repayment, applying the present value formula model, the total discounted sum of n installment repayments can be derived, the $P = \frac{S}{i} [1 - (\frac{1}{1+i})^n]$ (S : repayment amount per period; i : monthly interest; n : number of periods) leads to $S = P \cdot \frac{i}{1 - (1+i)^{-n}}$, as the number of repayment periods increases, the interest value increases, so we can reduce the interest by decreasing the number of periods.

2.3.2 Linear programming model

In economic activities, the application of linear programming models is needed to achieve the maximum creation of economic benefits through the rational allocation of resources [4]. The linear programming method is often used in problems of return and risk of financial investments, such as choosing a suitable portfolio of investments and rational allocation of resources to obtain the maximum return. Based on the objective function, the calculation of the minimum and maximum values of the problem is performed. Applying linear equations, setting up constraints, specifying the range of values of decision variables, and then finding feasible solutions. The decision variables can be set according to the decision of the financial activity. Based on this, a linear programming model is constructed using the optimization objective and constraints, according to which the optimal solution can be found. This way to rationalize the allocation of resources and reduce the risk of investment.

2.3.3 Portfolio model

Markowitz synthesized probability theory and optimization theory to pioneer quantitative portfolio research, which uses the variance of returns to measure investment risk and the expectation of returns to measure the return on investment, proposing the mean-variance model in finance [5].

Suppose an investor invests in n securities to construct a portfolio investment and the return of the i th asset is r_i ($i=1, 2, \dots, T$), then the mean $E[r_i]$ and variance $V[r_i]$ of r_i are

$$\bar{r}_i = E[r_i] = \frac{1}{T} \sum_{t=1}^T r_{it},$$

$$\sigma_i^2 = V[r_i] = E[r_i - E(r_i)]^2 = \frac{1}{T} \sum_{t=1}^T (r_{it} - \bar{r}_i)^2,$$

Covariance of r_i and r_j :

$$\sigma_{ij} = E[(r_i - E(r_i))(r_j - E(r_j))] = \frac{1}{T} \sum_{t=1}^T (r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)$$

Then, the expectation and variance of the portfolio:

$$E(\sum_{i=1}^n r_i x_i) = \sum_{i=1}^n \bar{r}_i x_i,$$

$$V(\sum_{i=1}^n r_i x_i) = \sum_{i=1}^n \sigma_i^2 x_i^2 + \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_{ij} x_i x_j$$

Where risk is expressed in terms of variance and return is expressed in terms of expectation, then minimizing risk is the optimal investment strategy for a given return R [5].

2.4. Financial Mathematics for the Development of Modern Financial Markets

2.4.1 Strengthen the degree of precision of financial services

The application of financial mathematics to the modern financial markets enables the optimization and improvement of the business in a comprehensive manner, leading to the efficient development of financial services. At the same time, describing the randomly fluctuating markets in precise terms allows people to obtain real and reliable information, achieve accurate modeling of large databases, etc., and clear the obstacles in the way of modern financial development, allowing academic research to proceed smoothly. Improving the accuracy of the data, financial mathematics makes it possible to solve financial problems on the banking side [6]. In the process of applying financial mathematics, it enables researchers to scientifically use modern computer technology and enterprise data management methods to improve the quality of all aspects of work, ensure access to accurate data information, and improve the level of financial services.

2.4.2 Promote the construction of new ecology of financial market

The application of theories and models of financial mathematics promotes the development of scientific modernization of financial theory research, effectively builds a new ecological structure and model of financial theory research under new technologies, and its core idea is to use new ideas and technologies of technological development to promote the sustainable and healthy development of financial theory research, to promote the continuous innovation of technological development of China's financial industry, so that the contemporary financial market can take a sustainable development Road, scientific construction of new technology ecological new financial market theory research environment [7]. The application of financial mathematics can also promote the transformation and upgrading of modern financial enterprises, and therefore, financial mathematics can also promote the overall development of the financial industry. At the same time, relevant technical personnel should enhance their own awareness of development and progress to promote the continuous implementation and progress of financial mathematics, and then come to complete the construction and development of the new ecology of the financial market.

2.4.3 Enriching the theoretical foundation of financial market

The current theoretical basis in financial figures is relatively large, which can provide rich theoretical support for the financial market and lead to the improvement of the practical operation level under the guidance of theory [8].

First, the theoretical basis of the stochastic optimal control type. This type of theory is mainly formed in the mid-1960s, emphasizing that the stochastic optimal control theory is in a continuous time environment, in-depth analysis of the optimal consumption theory, after the study can highlight the financial industry transactions have boundaries, with relative regularity, and the reality of the situation there are certain differences; second, harness theory. This theory belongs to the part of the modern financial market with certain frontier characteristics of financial theory, a comprehensive integration of all financial theories, and proposes a highly feasible and effective solution to the problem of derivatives of financial products, and makes reasonable price positioning for imperfect financial products in the market field; third, optimal stopping time theory. This kind of theory is one of the branches of probability, and currently accounts for a small proportion of mathematical theory, mainly because experts at home and abroad have not yet studied this kind of theory in depth, but from the future development trend has a greater prospect, which can provide accurate guidance for investment in the financial market; fourth, differential response theory. Considering that the financial market is not unchanging and has dynamic characteristics, it is because of the dynamic changes that directly affect the existence of many uncertainties within the financial market, once the phenomenon of market volatility will lead to securities, futures and stocks affected, the current stochastic dynamic model theory or pricing theory can not accurately predict market changes, while differential

countermeasure theory can be based on the differential response theory can make assumptions based on the macro market perspective, accurately study the various uncertainties that adversely affect the market order, and comprehensively integrate advantageous resources to come up with a financial data theory that can solve the problem of market uncertainty; fifth, other intelligent and empirical theories. Currently the world is in the stage of rapid development of information technology and digitalization, the financial mathematics and modern information technology are integrated with each other, and various mathematical computing information systems such as genetic algorithm technology and wavelet analysis technology and traditional financial mathematics can be integrated with each other to create modern data models, summarize the regular characteristics of the financial market, verify the regular state, and enhance the credibility of the theoretical content [8].

2.4.4 Analysis of financial mathematics in option pricing and investment decision making

Option pricing as well as investment decisions are important aspects of mathematical knowledge applications that can be solved using mathematical knowledge of calculus. Regarding the option pricing aspect, in the specific financial market development, if once there are large fluctuations, this not only violates the law of motion to a large extent, but also causes certain adverse effects on the pricing of options in the market [7]. The integration of calculus in financial theory enables a reasonable analysis and targeted study of the actual market situation, and when the market is volatile, the corresponding mathematical model early warning system will react and thus show the volatility in a more scientific way. In the investment decision, through the application of mathematical knowledge of calculus, it can scientifically formulate the investment plan, determine the investment direction and control the investment risk. Modern financial mathematics covers a wide range of contents, and based on these theories, it is possible to make specific scientific applications to gain more space for development.

2.5. Problems and Improvements in the Application of Financial Mathematics in Modern Financial Markets

2.5.1 Uncertainties

In the operation of financial markets, it is often difficult to find out specific laws, which are stochastic and nonlinear, especially in the case of large fluctuations in financial markets, the stochasticity will be more obvious, which will lead to the difficulty of summarizing information in financial markets and increase the difficulty of financial mathematics applications. Therefore, it is necessary to strengthen theoretical applications in the analysis of financial markets and promote the sustainable development of financial mathematics [9].

With the development of financial markets to the present day, modern financial markets have changed their face. Many experts and scholars at home and abroad have studied modern financial markets through physics and created a nonlinear system based on modern financial markets and with the characteristics of modern financial markets through the data obtained from the study. The study of financial markets through this system has revealed that modern financial markets are indeed regular, but there are also uncertainties that exist, and these uncertainties make the application of financial mathematics hindered. For example, the financial economy itself is changing from moment to moment, and creating a monetary model for the rapidly changing financial economy requires complex analysis and practice. First, it is necessary to deeply analyze and understand the characteristics of stochastic models such as ambiguity and multiplicity, and to find out some patterns and mechanisms of change from them. Secondly, it is necessary to deduce the laws of change in practical terms and clarify the monetary conditions and characteristics of the embodied financial market economy. On the basis of this, the monetary systems of different countries are used as the basis for unified research, so as to understand the characteristics of the demand for money around the world, and to make supply based on this, and to have a clear understanding of the direction of the volume of liquidity around the world. With a perfect monetary model, it is possible to study and analyze the model to provide the financial market with reliable data and accurate information on interest rates and tax rates in the

financial market. On this basis, a multi-level and multi-dimensional data model is created to organize and integrate the actual situation of the financial market with the production resources, and on this basis, the quality of financial market services is improved with the help of modern financial digital theory. As it can be seen by the above model of the establishment of the monetary system, in order to better play the role of financial mathematics in a situation where the financial market is affected by unstable factors, it is necessary to rigorously complete all the tedious operations and improve the effectiveness of the application of financial mathematics [10].

2.5.2 Financial market supervision and management mechanism

With the development of financial market, people pay more and more attention to the use of financial mathematics, but still lack of perfect market supervision mechanism. Although the existing market supervision mechanism plays a certain role, but in the actual application ignores the actual situation and characteristics of the market, not combined with the application of financial mathematics links and characteristics, the work effect is poor. Therefore, considering that the modern financial market area often appears the phenomenon of unstable economic development, coupled with China's financial market has not formed a more mature development mechanism, facing more complex professional problems may make irregular and unprofessional behavior, this situation should create a perfect market supervision and management mechanism, using effective supervision to enhance the standardization and scientific nature of the application of financial mathematics. First, it is suggested that government departments should timely and accurately intervene in the field of financial mathematics application in the financial market, provide accurate ways for the effective supervision and management of the market, study and analyze the shortcomings and problems of the market, set up special supervisory work organization, arrange professional personnel to scientifically carry out the supervision and management activities of the market, clarify whether there are problems of industry standardization and financial mathematics application, and help the market to enhance various. The level of professionalism in the application of financial mathematics in all areas of work. Second, combined with the characteristics of the modern financial market, put forward the standardization of various functions, unify the process requirements of the application of financial mathematics in the field of industry, so that enterprises can operate in accordance with various systems and norms, and also need to enhance the transparency and openness of relevant supervision and management, all institutions and enterprises must obtain the corresponding certificate of conformity, and the work and behavior performed are standardized, so that the modern financial industry toward a more scientific and rigorous direction and enhance the effect of financial mathematics applications [8].

2.5.3 The perfect financial mathematics system

The construction of the financial mathematics system of the modern financial market still needs to be strengthened, its system is not sound enough, the system is not perfect, China's research on the theoretical and practical part is still superficial, therefore, the financial mathematics should be studied in depth and give full play to the value of financial mathematics. The healthy development of modern financial market cannot be achieved without a perfect financial mathematics system, modern financial enterprises should strengthen the attention to financial mathematics and actively build a financial mathematics system. Financial enterprises can create a financial mathematical system by studying economic tax rates and using market laws to create a more in-depth review of the industry's development status through a perfect mathematical model.

3. Conclusion

Through the study, the role of financial mathematics in the modern financial market is crucial. Financial mathematics can not only provide a strong theoretical basis for the healthy development of the financial market, but also adapt to the development of the financial industry and meet the needs of the financial industry. The emergence of financial mathematics has solved many problems for the

financial market, and the flexible use of mathematical models to deal with financial problems plays an important role in the rational allocation of resources and investment forecasting. As described in this paper, the financial market should also be completed for the changing environment of financial mathematical models, as well as financial market supervision and management mechanisms and other aspects. However, it is undeniable that the methods and results of mathematical theory research will be enriched on the basis of all the current theoretical frameworks, so that mathematical models can improve their application in the financial field and better interpret reality.

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