Option Pricing Practices Based on the B-S-M Model in Carbon Markets

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Abstract. Carbon trading refers to the buying and selling of carbon emission allowances, stimulating businesses to adopt low-carbon development strategies through price signals. By allocating limited emission allowances to emitters, the carbon market encourages the reduction of carbon emissions to achieve emission reduction targets. Market participants can compensate for emissions exceeding their own quotas by trading carbon emission allowances, thus minimizing the cost of emission reductions. The Chinese carbon market has a large trading volume and numerous development opportunities, but currently, the carbon financial market is not fully open. In comparison to Western countries, China lacks carbon emission derivatives such as futures, options, and swap contracts. The risk and return of carbon emission allowance options are estimated and managed using the B-S-M option pricing model. This paper utilizes data from the Shenzhen Carbon Emission Allowance Exchange as a sample to conduct option pricing practices for carbon emission allowances using the B-S-M model. The aim is to promote research and development in the carbon emission allowance market and contribute to the achievement of emission reduction goals.

Keywords: B-S-M model, Option pricing, carbon market, Carbon Emission Allowance Exchange.

1. Introduction

A market-based instrument created to meet carbon emission reduction targets is the carbon emission trading market. By allocating limited emission allowances to emitters, it encourages them to take measures to reduce carbon emissions. The participants in the market include governments, businesses, and individuals. These participants can buy and sell carbon emission allowances to offset emissions that exceed their allocated quotas, thereby achieving global greenhouse gas emission reduction goals while minimizing the cost of emission reductions. As the demand for trading expands, there is a growing need among market participants for financial derivatives of carbon emission allowances.

Carbon trading refers to the buying and selling process of carbon emission allowances. Buyers can use the purchased carbon emission allowances to offset their own carbon emissions, while sellers can derive economic benefits by selling carbon emission allowances. This market mechanism stimulates businesses to adopt low-carbon development strategies through price signals.

Carbon emission derivatives include futures, options, and swap contracts. The value of the underlying asset, which in this case is the price of carbon emission permits, determines the distinguishing feature of these financial instruments. The risks associated with these derivatives primarily include market risk, credit risk, and operational risk. Bachelier was the first to propose the relevant theory and methods for option pricing [1]. On top of this, Sprenkle took into account the time value of money and made the assumption that stock prices and returns follow a log-normal distribution [2]. Numerous experts and scholars have offered various modifications to the original model [3,4]. Fisher Black and Myron Scholes introduced a comprehensive option pricing model [5], and in the same year, Robert Merton independently proposed a more generalized model that expanded the applicability of the original model [6]. They created the B-S-M model from the B-S model by loosening the requirement that the underlying stock does not pay dividends during the duration of the option. The B-S-M model has had a significant influence on finance and is frequently used to price options in a variety of financial markets. The model is predicated on a number of hypotheses, including the existence of arbitrage possibilities, frictionless markets, continuous trading, and the

random walk assumption for stock prices. It provides a mathematical framework for calculating option prices, enabling investors to make rational decisions considering different factors. The B-S-M option pricing model has long been the central focus of option pricing theory, and Chinese scholars, based on its application, have conducted extensive research [7-11]. In the carbon market, the application of the B-S-M model can assist investors and companies in better estimating and managing the risks and returns of carbon emission allowance options. Through in-depth research on option pricing and risk, we can gain a better understanding of the trading mechanisms in the carbon market and provide support for achieving carbon emission reduction goals.

China has established the world's largest carbon emission trading market, with the Shenzhen Emissions Exchange being one of the earlier exchanges established. China's carbon market continues to expand in terms of scale, trading activity, and the number of market participants, with policy support gradually improving. Several scholars have provided guidance for China's carbon market [10]. In this context, this study utilizes the daily carbon trading data from the Shenzhen Emissions Exchange between January 2, 2018, and December 31, 2020, as the sample and conducts pricing practice using the B-S-M option pricing model implemented through Python software programming. China's future carbon emission market has significant potential for development in achieving emission reduction goals and sustainable development objectives. Through market-based mechanisms, carbon emitters can achieve emission allowances at the lowest cost, thereby encouraging more businesses and governments to participate in emission reduction efforts. The goal of this study is to further in-depth investigation of the carbon emission trading market and to make a contribution to this significant area.

2. Data and method

2.1. Data sample

In this study, the Shenzhen Emissions Exchange was selected from China’s first batch of voluntary greenhouse gas emission reduction trading organizations. As test data for pricing call option products, the daily carbon emission allowance transaction prices from January 2, 2018, to December 31, 2020, were gathered.

To ensure data quality, this study utilized the Akshare library in Python to scrape the data and performed cleaning to remove non-trading days’ information. In the end, a total of 599 high-quality sample data points were selected for the research (table 1).

Table 1 Descriptive statistics

maximum	minimum	mean	variance
273.06	43.49	155.85	2912.71

2.2. Method

This study utilizes the Black-Scholes-Merton (B-S-M) pricing model to simulate the pricing of carbon emission rights options. The value of options is correlated to variables like the cost of the underlying asset, volatility, and risk-free interest rate in this model, which creates a world that is risk neutral.

The following seven premises form the basis of the B-S-M model:

(1) During the option’s lifespan, the underlying stock does not pay dividends or engage in any other type of distribution.

(2) Trading stocks or options has no related transaction expenses.

(3) The short-term risk-free interest rate is established and remains so for the duration of the option.

(4) Any buyer of securities is eligible to take out a loan of any size at the short-term risk-free interest rate.

(5) Short sales are permitted, and the seller is given an upfront payment equivalent to the stock's current market value.

(6) Only on the expiry date are call options eligible for execution.

(7) Stock prices move in a random manner, and all securities trading is ongoing.

This study will fit the call option pricing to the sample data using the B-S-M model, and the pricing formula is as follows:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{1}$$

$$d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{3}$$

3. Results and discussion

3.1. Parameter estimation

3.1.1 Risk-free interest rate

In China, the risk-free interest rate is typically defined as the one-year fixed deposit rate, one-year loan rate, or short-term government bond yield. The one-year fixed deposit rate of 1.5 percent from the People's Bank of China in 2018 is chosen as the risk-free interest rate since this study uses information from the Shenzhen Carbon Emission Rights Exchange for the period of January 2, 2018, to December 31, 2020.

3.1.2 Volatility

The standard deviation method is used to calculate volatility in this study. The calculation formula is as follows:

$$\text{Volatility}(\sigma) = \text{Sample Standard Deviation}(s) \times \sqrt{\text{Number of Trading Days per Year}} \tag{4}$$

The specific steps are as follows:

(1)Let's denote the daily closing price as P_t , and take the logarithm to obtain the logarithmic daily closing price $\ln P_t$

(2)Calculate the logarithmic daily returns (R_t) based on the values of $\ln P_t$ - $\ln P_{t-1}$.

(3)To get the sample standard deviation(s), calculate the sample standard deviation of the logarithmic daily returns. The following is the calculating formula:

$$s = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(R_t - \bar{R})^2} \tag{5}$$

(4)The number of trading days per year, based on the sample data, is taken as 200.

By conducting statistical analysis on the daily closing prices and logarithmic daily returns of the sample data, this study utilizes Python to analyze and present the results in the following charts (Fig 1, 2).

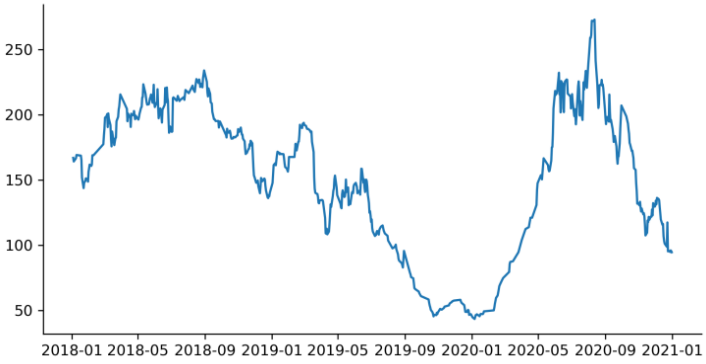

Fig. 1. Price fluctuation chart

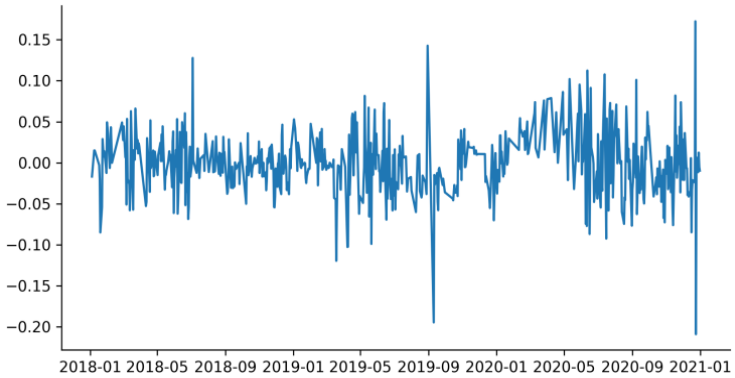

Fig. 2. Logarithmic daily returns chart

From Fig 1, it can be observed that the daily closing prices of carbon trading remained relatively stable during 2018 but exhibited significant volatility between 2019 and 2020. Fig 2 shows that except for abnormal fluctuations in September 2019 and December 2020, the returns mostly fluctuated within the range of -5% to 5%, indicating a relatively stable pattern.

Using Python, a histogram of the distribution of logarithmic daily returns was plotted and a curve was fitted (Fig 3).

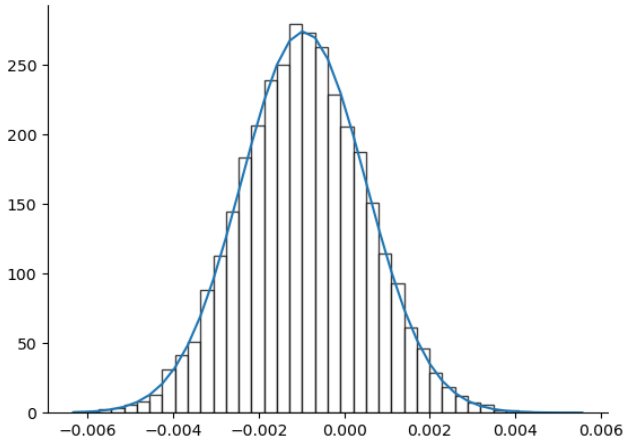

Fig. 3. Logarithmic daily returns distribution chart

Figure 3 shows that for the years 2018 to 2020, the stock's logarithmic daily returns (R_t) roughly follow a normal distribution without noticeable heteroscedasticity. This finding shows that the data initially fulfil the fundamental presumptions of the Black-Scholes-Merton method for valuing options.

By using Python, the calculated volatility is σ=0.5390.

3.2. Model pricing results

The beginning price of the underlying asset in this experiment, designated asS_0=94.59, was set at the closing price of carbon trading on December 31, 2020.

The strike price was set equal to the initial asset price, denoted as K=94.59.

The time to expiration of the asset was set as T=0.5 years.

$$d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln(94.59/94.59)+(1.5\%+0.5390^2/2)0.5}{0.5390\sqrt{0.5}} \approx 0.21$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.21 - 0.5390\sqrt{0.5} \approx -0.17$$

$$C = S_0N(d_1) - Ke^{-rT}N(d_2) = 94.59N(d_1) - 94.59e^{-1.5\%*0.5}N(d_2) = 14.60$$

The price of the option is determined to be C=14.60 by entering the aforementioned information into the Black-Scholes-Merton options pricing model.

3.3. Discussion

This paper utilizes the daily carbon emission trading prices from the Shenzhen Carbon Emission Rights Exchange between January 2, 2018, and December 31, 2020, to conduct pricing analysis. The estimated value for a call option on carbon emission rights on December 31, 2020, was found to be C=14.60. However, due to the lack of large-scale implementation of carbon emission options trading in China, these results serve as a reference for market participants to better manage carbon emission risk. The results of this study can direct interest parties in developing more specific carbon emission policies and associated environmental investment choices, assisting in the growth of a low-carbon economy and lowering greenhouse gas emissions.

The market for carbon emission rights is anticipated to grow further as initiatives to combat climate change continue to gain pace. Although the Chinese market for carbon emission derivatives trading is still immature, we can speculate about its future trends:

(1)Internationalization of the carbon emission rights market: With increasing cooperation among governments worldwide, the carbon emission rights market will further integrate to form a unified global carbon market. This will enhance the transparency and stability of carbon emission rights prices, providing more risk management tools for businesses.

(2)Innovation in carbon emission derivatives: As the market develops, Chinese carbon emission derivatives will continuously innovate to meet the diverse needs of market participants. For example, it will be possible to design cross-period and cross-commodity swap contracts, as well as structured products, to offer more risk management solutions for enterprises.

(3)Technological advancements: With the advancement of financial technology, carbon emission derivatives trading in China will become more efficient and convenient. For instance, blockchain technology can enhance the transparency and security of carbon emission rights trading, while artificial intelligence and big data can provide market participants with more accurate pricing and risk management tools.

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4. Conclusion

This paper utilizes a sample of data from the Shenzhen Carbon Emission Rights Exchange and applies the B-S-M option pricing model for valuation, implementing it through Python programming. In the context of the Chinese carbon market, this study aims to advance the growth of the market for carbon emission rights and investigates how the B-S-M model may be used to estimate and manage the risks and rewards of carbon emission rights alternatives. By delving into the study of option pricing and risk, it contributes to a better understanding of the trading mechanisms in the carbon market and supports the achievement of carbon emission reduction goals.

However, this paper has some limitations. Firstly, it only employs sample data from the Shenzhen Carbon Emission Rights Exchange for valuation, without including data from other carbon exchanges, which might restrict the sample selection to some extent. Secondly, the assumptions in the B-S-M model may deviate from actual conditions, potentially impacting the results. Additionally, this paper does not consider other factors affecting the carbon market, such as policy changes and international trade, which present areas for further research.

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