Comparison and Analysis of the Effectiveness of the State-of-the-Art in Option and Financial Pricing Models

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Abstract. Contemporarily, various models have been developed and used to price the returns of an asset or an option for decades. This paper will discuss and compare two option pricing models (the BSM model and the Monte Carlo Simulation) and three financial pricing models (the CAPM, the FF3 Model, and the FF5 Model). It will start with discussing the history, assumptions, and formulae of each model. Then, it will lead into how each model has changed over time, whether that be changes from the researchers themselves or other researchers within the community. Additionally, specifics of the change will be addressed, and the paper will outline what the new model has done to improve upon those concerns. Eventually, an empirical comparison will contrast the effectiveness and accuracy of each model. Ultimately, it will be seen that the BSM model and the MCS are useful in different time frames and the FF5 model is superior to the original financial pricing models as it builds on the original models' limitations. Even so, the BSM model, MCS, and the FF5 model still have their own limitations that vary from reality and can be improved on in future research. These results shed light on guiding further exploration of newer and more refined models, methodologies, and approaches aimed to enhance our ability to make informed decisions and manage risks with greater precision.

Keywords: BSM model, Monte Carlo simulation, CAPM, FF3 model, FF5 model.

1. Introduction

The Black-Scholes-Merton model (BSM model) was invented by Black, Merton, and Scholes in 1973. This popular model calculates prices for European options by using the time to expiration, stock price, expected volatility, the option's strike price, and expected interest rates. Twenty-four years later, in 1997, they were awarded the Nobel Memorial Prize in Economic Sciences [1]. Unfortunately, by that time, Black had passed so could not be a recipient, however, the Nobel committee did acknowledge his contribution [2]. The Monte Carlo Simulation (MCS) was invented by Von Neumann and Ulam in the 1940s. It is used to predict the possible outcomes of an uncertain event, such as the price of an option based on complete randomness [3], which is why it is named after the famous gambling location in Monaco. When comparing these two pricing models, it has been argued that the MCS is more favorable because it takes randomness into account. On the other hand, others have argued that the MCS has good results in the short term, while the BSM model has good results in the longer term. It is possible that these 2 conclusions are linked as statistics tell us, in the long-term, samples and trials will converge on the true value.

Moving onto financial pricing models, the capital asset pricing model (CAPM) was invented by Sharpe, Mossin, Lintner, and Treynor in the early 1960s to predict the expected rate of return for an underlying asset or investment [4]. In 1990, Sharpe was granted the Nobel Memorial Prize in Economics for this model [5]. The Fama-French 3-factor model (FF3) was invented in 1992 by Fama and French. It builds on the CAPM by adding additional factors to the original model. The 3 factors that the model takes into consideration are the performance of book-to-market values, small versus big companies, and excess return on the market [6]. In recognition of his notable contributions, Fama was granted the Nobel Prize in Economic Sciences in 2013 [7].

The various pricing models with unique characteristics not only managed to capture the author's attention but also sparked an interest to delve deeper. Through this learning experience, the importance of pricing models became apparent, and the author was genuinely captivated by its potential. In this paper, a third financial pricing model will be brought to attention but it is very similar

to the other two models because, once again, it builds on the limitations of the previous one. Given this, it can be assumed that the most recently developed model is the most effective. The important question now is what part of the model makes it superior to the previous. On this basis, this paper will start with an introduction to each model. The assumptions (if any) the model is based on and the formula(e) used will be brought into the light. It will explain how each model has changed over time and the what, who, and why. What part of the model changed or was improved. Who made the improvements, the researcher themselves or others from the community. Finally, why did they make this improvement. To conclude each type of pricing model, there will be an empirical comparison that contrasts the effectiveness and accuracy.

2. Option Pricing Models

2.1. Black-Scholes-Merton Model

The BSM model is built on a few assumptions. First, the BSM model makes the assumption that the stock has a lognormal distribution:

$$\log S(t) = \log S(0) + (\mu - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}z(t) \quad (1)$$

Where t represents a time prior to maturity, S(t) is the price of the stock, z(t) is a standard normal random variable, and σ, also known as Sigma, is the expected volatility. The second assumption is that the stock adheres to a geometric Brownian motion:

$$dS = \mu S dt + \sigma S dW \quad (2)$$

Where μ is the average growth rate. Here, W(t) follows a geometric Brownian motion that is a continuous stochastic process where W(0), the Brownian motion at time 0, is equal to 0. W(t), the Brownian motion of any time t, has a normal distribution with a mean of 0 and standard deviation of t, for all t greater than 0 [8]. In the case where there are 2 Brownian motions, W(t) - W(s), the mean is the same but the standard deviation is t - s. Third, the underlying asset follows a random walk where at any moment in time, the price is as likely to move down as it is likely to move up. Fourth, the stock pays no dividends. Fifth, there are no transaction fees. Sixth, the expected interest rate (r) remains constant.

For call options, the payoff of an option is the gain or loss made from the option. Therefore, the payoff of a call option is

$$C(T) = \max[S(T) - K, 0] \quad (3)$$

Where K represents the strike price of the option and S(T) is the price of the underlying asset with strike price K and a maturity date of T in years. The BSM model price of the call option at time t before maturity is [9]

$$C(t) = S(t)N(d1) - e\text{-}rtKN(d2) \quad (4)$$

Where S(t) is the stock price and r is the risk-free rate of return (RFR). Regarding put options, the payoff of a put option is [9]

$$P(T) = \max[K - S(T), 0] \quad (5)$$

Where the same variables defined in Eq. (3) are used. The BSM model price of the put option at time t before maturity is [9]:

$$p(t) = e^{-rt}KN(-d2) - S(t)N(-d1) \quad (6)$$

Where the same variables defined in Eq. (4) are used. For both equations,

$$d1 = \frac{1}{\sigma\sqrt{T}} [\log(S(T)/K) + (r + \frac{\sigma^2}{2})(T-t)] \quad (7)$$

$$d2 = \frac{1}{\sigma\sqrt{T}} [\log(S(T)/K) - (r + \frac{\sigma^2}{2})(T-t)] = d1 - \sigma\sqrt{T} \quad (8)$$

Where N(d) has a standard normal distribution and is the cumulative probability distribution for variable d. Eq. (7) and Eq. (8) include the variables defined in Eq. (1), Eq. (3), and Eq. (4).

Over the years, there have been numerous modifications to improve the BSM model. One of those modifications is the attempt to improve upon the assumption of no transaction costs. It is proven that volatility is correlated to the transaction costs and volume of trade [10], one of which is assumed to be non-existent and one of which isn't included in the model. Therefore, researchers (e.g., Davis et al. [11]) have developed new models that include transaction costs in the picture. Moreover, Hull et al found that empirically as the time to maturity increases, the expected volatility decreases [12], which contradicts the assumption that volatility is constant. As a result, Hull et al created a new option pricing model that incorporated stochastic volatilities.

2.2. Monte Carlo Simulation

The main idea is to estimate the value θ by creating a pseudo-population that reflects the real world where there is the same sampling strategy, sample size, and other characteristics to maximize the accuracy of the simulation. First, perform the simulation to calculate $\hat{\theta}$ (estimated value of θ) n times, where n represents the count of trials and each trial is labelled $\hat{\theta}_n$. The Monte Carlo result of the sampling distribution of $\hat{\theta}$ under the pseudo-population conditions and the sampling procedures is the relative frequency distribution of the $\hat{\theta}_n$ values [13]. Naturally, researchers have developed a model that builds upon the MCS. An example of an improved MCS for discontinuity orientations was developed by Zheng et al which is based on the Fisher distribution. The Fisher distribution was chosen because it uses one parameter instead of multiple, the Fisher constant K. An advantage to this is that it is an integrable function. Being an integrable function means that it is easier to produce randomized data through the MCS. Using the improved model, a program named MCSDO was developed. Through this program, the orientations of generated discontinuities could be calculated, as well as the mean orientations of both discontinuities [14].

2.3. Comparing the Black-Scholes-Merton Model and Monte Carlo Simulation

Priyatna et al. compared the BSM model to the MCS through an application called SOCA using the Python programming language. Priyatna et al extracted data from yahoo finance to predict the option price using the MCS and BSM methods. In this study, Priyatna et al also calculated an error value where they subtracted the predicted option price of the two models from the market price.

Table 1. Comparison of the BSM Model and MCS to the Market Price of a Tesla call option.

Month	Call Option			Error	
	BSM Model	MCS	Market Price	BSM Model	MCS
1	1.42	4.16	7.70	6.28	3.54
3	7.06	3.18	8.50	1.44	5.32
5	7.35	1.66	13.30	5.95	11.64

From Table 1, it can be seen that the error value generated from the BSM model in the first month is greater than the MCS error value. From those numbers, it can be seen that when predicting the price

of the Tesla stock call option, the MCS has better outcomes than the BSM model. However, in the relatively longer term, the third and fifth months, it is flipped. The BSM model has a better outcome than the MCS. Therefore, one can conclude the MCS produces reliable outcomes when calculations are performed in the short term, while the BSM model produces reliable outcomes when calculations are performed in the long term [15].

3. Financial Pricing Models

3.1. Capital Asset Pricing Model

Since CAPM is an addition of the mean-variance model by Markowitz it follows the assumption that investors are reluctant to take risks and will be willing to trade off returns for a safer investment. This means investors will maximize their expected returns (ER) when given a specific value for variance and minimize the variance when given a specific value for ER. On top of this, CAPM adds some additional assumptions. First, CAPM assumes that all investors focus on their own risk and return to maximize their utility. Depending on how much return the investor is willing to trade off for less risk and their utility function, they will select different portfolios. Second, every investor possesses the opportunity to lend or borrow at an RFR. Third, investors hold their assets for the same amount of time. Fourth, all investors have homogenous expectations for their investment's future return and risk. Fifth, there are no transaction costs. Sixth, interest rates are constant. Seventh, investors can buy or sell shares of any portfolio they hold. Eighth, all investors have access to the information of the portfolio they are investing in. Finally, markets are in equilibrium and an individual investor does not have the ability to affect market prices [16]. To calculate the ER of an asset or investment using the CAPM, one uses the following equation:

$$ER = R_f + \beta(ER_m - R_f) \tag{9}$$

Where ER signifies the expected return of asset (or investment), R_f signifies the RFR, β signifies the beta (risk) of the asset relative to the market, and ER_m signifies the expected return of the market portfolio. Finally, by taking the definitions above, $ER_m - R_f$ is the expected return of the market portfolio minus the RFR, which is known as the market risk premium.

3.2. Fama-French 3-Factor Model

The formula to calculate the ER of an asset or investment using the FF3 model is

$$R_t = R_{ft} + \beta_1(R_{Mt} - R_{ft}) + \beta_2(SMB_t) + \beta_3(HML_t) + \epsilon_t \tag{10}$$

Where t is the time. R_t is the expected rate of return, R_{ft} is the RFR, and R_{Mt} is the total return of the market portfolio. Therefore, $R_{Mt} - R_{ft}$ is the total return of the market portfolio minus the RFR, which is also known as excess return on the market portfolio. Small minus big (SMB_t) is the size premium, which is the performance of small-cap firms compared to large-cap firms at time t, and high minus low (HML_t) is the value premium, which is high-value assets/investments compared to low-value assets/investments at time t [17]. Finally, $\beta_{1,2,3}$ refers to the factor coefficients, which are the asset's risk relative to the market.

The FF3 model was critiqued as an inadequate model for ER because it overgeneralized the returns related to profitability and investment into 3 factors [18]. In 2014, Fama and French enhanced the FF3 model by introducing supplementary components, resulting in a total of 5 factors. This upgraded version is known as the Fama-French 5-Factor model (FF5). Aside from the original three factors, SMB, HML, and excess return on the market, the new model improves the accuracy by adding the concepts that companies that report higher earnings for the future have higher stock market returns and companies are more prone to encountering stock market losses when they direct profits toward major growth projects.

3.3. Comparing the Fama-French 3-Factor Model and Capital Asset Pricing Model

Sehrawat et al. put forward the hypothesis that the FF3 model outperforms the CAPM in explaining the cross-sectional variations (CSV) in the mean returns of different stocks within the Indian stock market. They made this hypothesis through the backing of numerous empirical studies by other researchers such as Taneja who found a high association between the value and size factors in the Indian stock market. Taneja concluded that either factor could be included to improve the model [19].

Table 2. Adjusted R^2 for the CAPM in the Indian Stock Market.

Dependent Variable	Adjusted R^2 for CAPM
S/L-Rft	0.6350
S/M-Rft	0.7063
S/H-Rft	0.7317
B/L-Rft	0.8973
B/M-Rft	0.9237
B/H-Rft	0.8190

Table 3. Adjusted R^2 for FF3 in the Indian Stock Market.

Explanatory Variables	Dependent Variable	Adjusted R^2 for FF3
(Rmt-Rft), SMBt	S/L-Rft	0.7842
	S/M-Rft	0.8427
	S/H-Rft	0.9149
	B/L-Rft	0.9035
	B/M-Rft	0.9249
	B/H-Rft	0.8211
(Rmt-Rft), HMLt	S/L-Rft	0.6385
	S/M-Rft	0.7267
	S/H-Rft	0.8037
	B/L-Rft	0.9189
	B/M-Rft	0.9268
	B/H-Rft	0.9219
HMLt, SMBt	S/L-Rft	0.3061
	S/M-Rft	0.4382
	S/H-Rft	0.6103
	B/L-Rft	0.1100
	B/M-Rft	0.2310
	B/H-Rft	0.4708
(Rmt-Rft), SMBt, HMLt	S/L-Rft	0.7929
	S/M-Rft	0.8567
	S/H-Rft	0.9726
	B/L-Rft	0.9272
	B/M-Rft	0.9277
	B/H-Rft	0.9224

Table 2 shows the adjusted R^2 values using the CAPM where only the market factor was taken as an independent variable. Table 3 then shows the adjusted R^2 values using FF3 by incorporating the value and size factor into the original market factor. It can be seen that when comparing Tables 2 and 3, the adjusted R^2 values increased, therefore, using 2 factors to explain asset returns is better than one. Furthermore, when all three factors were incorporated, it seems that the adjusted R^2 was greater than CAPM in all instances, and the majority of factors displayed statistical significance. These results proved that the FF3 model has a stronger presence in the Indian stock market compared to CAPM [19].

3.4. Comparing the Fama-French 5-Factor Model and Fama-French 3-Factor Model

Gokten et al. tested FF3 and FF5 within the context of the Turkish stock market in the time frame of June 2000 to May 2017. For all models they utilized an equal-weight market portfolio, to explain the CSV in the returns of stocks. For the following tables, S is sorted from tiniest to largest portfolio size where S1 is the tiniest and S4 is the largest. The book-to-market value is ordered from low to high where BM1 is the lowest and BM4 is the highest. The profitability is ordered from major to minor where PO1 is major profitability and PO4 is minor profitability. The investment is ordered from cautious to risky where INV1 is cautious and INV4 is risky [20].

Table 4. Comparing the t-statistic of alpha coefficients of FF3 and FF5 for 16 size-profitability portfolios.

Dependent Variables	FF3 model	FF5 model
S1OP1	3.72	3.83
S1OP2	2.58	2.36
S1OP3	2.49	2.36
S1OP4	0.81	0.51
S2OP1	1.81	1.62
S2OP2	2.35	1.93
S2OP3	2.65	1.52
S2OP4	1.79	0.10
S3OP1	2.55	0.71
S3OP2	0.59	−1.03
S3OP3	2.01	1.04
S3OP4	1.94	0.11
S4OP1	2.12	−0.09
S4OP2	1.81	−0.38
S4OP3	2.47	0.52
S4OP4	0.45	−1.83

From the data in Table 4, the most successful model is the FF5 model. Notably, among the 16 alpha coefficients in the FF5 model, three display statistical significance at 5% (bolded values). However, in contrast, the FF3 model showcases significant results for 9 of the alpha coefficients. This implies that the FF3 model has a large portion of unexplained CSV of ER [20].

Table 5. Comparing the t-statistic of alpha coefficients of FF3 and FF5 for 16 size-investment portfolios.

Dependent Variables	FF3 model	FF5 model
S1INV1	1.69	1.64
S1INV2	2.10	1.75
S1INV3	2.29	2.38
S1INV4	2.30	1.88
S2INV1	2.54	1.80
S2INV2	2.87	2.36
S2INV3	1.53	0.01
S2INV4	1.63	−0.26
S3INV1	2.02	0.55
S3INV2	1.65	0.19
S3INV3	2.36	0.30
S3INV4	2.60	1.00
S4INV1	1.46	−0.41
S4INV2	1.42	−0.55
S4INV3	1.84	−0.72
S4INV4	1.61	−0.52

Table 6. Comparing the t-statistic of alpha coefficients of FF3 and FF5 for 16 size-Book-to-Market portfolios.

Dependent Variables	FF3 model	FF5 model
S1BM1	1.74	1.69
S1BM2	2.05	1.76
S1BM3	2.23	1.94
S1BM4	2.47	2.23
S2BM1	0.86	0.52
S2BM2	2.43	2.00
S2BM3	1.43	0.49
S2BM4	2.02	0.55
S3BM1	1.64	−0.19
S3BM2	1.94	0.32
S3BM3	1.26	0.32
S3BM4	3.12	1.93
S4BM1	1.03	−0.64
S4BM2	1.80	−0.51
S4BM3	1.59	−0.70
S4BM4	2.04	−0.20

Although the values calculated through the FF3 model in Table 5 may look convincing, eight alpha coefficients displayed significance. Then in Table 6, seven alpha coefficients displayed significance. Therefore, FF3 cannot accurately outline the nuances in the portfolios' monthly excess returns. However, as shown in both tables, only two of the sixteen alpha coefficients are significant for FF5. Gokten et al.'s study shows that, generally, it can be concluded that when explaining the common variations in Turkish stock returns, FF5 outperforms FF3 [20].

4. Limitations and Prospects

There are a few limitations to the BSM model. First, the biggest drawback is the assumptions that the model is built on. Assumptions such as constant interest rates and no transaction costs are limitations because they do not materialize in the real world [21]. An example of this is the assumption that there are no dividend payments, which is completely different from the true case, where approximately 75% of companies in the S&P 500 pay a regular dividend. Therefore, the Black-Scholes model may lack the ability to truly reflect the accurate future cash flow of an investment due to model rigidity. Second, Hull et al conclude that when the price of the stock and the price's volatility are positively correlated, options that are in-the-money are overpriced, while options that are out-of-the-money are underpriced. All of which is overvalued as the time to maturity increases [12]. Additionally, the BSM model does not take into account all types of options. Although this means that there are only certain options one can calculate, at least it does not affect the accuracy when calculating European options. Then there are two main limitations of the MCS. First, the MCS can't account for anomalies in the market. This means that events such as recessions, bear markets, or any other kind of financial crisis that influence the final results will not be included. Second, since the MCS is data-intensive, it is troublesome to produce accurate results. The word troublesome is used here because an abundance of empirical data has to be gathered for the results to be an accurate estimation. Or the analyst could trade off the realisticness of the results to make several assumptions in order to fill in the lack of such empirical information.

Recalling, one of the assumptions in the CAPM is that every single investor has access to the information regarding the portfolio they are investing in. However, in the real market, investors cannot get the full information about the portfolios. This implies the decisions an investor is going to make are going to be different in the sense that the information they are exposed to varies. Furthermore, investors are inherently different, so every investor has their own opinion and return distribution. In cases where investors have experienced big profits or losses, it is likely they are willing to take more risks to, for example, win back losses. All of these factors and variables will cause assumptions to differentiate from reality, which means the market portfolio is not chosen by everyone [22]. Furthermore, the main limitation of the FF3 Model is similar to CAPM's. It is the failure to consider the ongoing persistence of short-term momentum irregularities. Fama and French acknowledge the limitations of the model. It has been shown that the intercepts are much more negative for investors with short-term losses and much more positive for investors with short-term profits. Therefore, some scenarios are not explained by the FF3 model. Although the FF5 model is an expansion of the FF3 model to fill in some of the gaps, the newly improved model still has some limitations. In the scenario where a small stock firm invests an abundance even though there are low profits to be made, the model struggles to accurately include the low average returns. Fama and French observed that when profitability (RMW) is incorporated, the intercept of the small-cap portfolio in the top accrual (AC) quintile gravitates toward zero, while the other four small-cap portfolios' intercepts move from values that are mostly alike from zero to big positive values. The FF5 model's issues are not exclusive to microcaps. Within the lowest AC quintile, all portfolios have higher intercepts after the addition of RMW, whereas within the highest AC quintile, three of the five portfolios have more negative intercepts [18].

When comparing these two pricing models, some have argued that the MCS is more favorable than the BSM model for calculating the value of options with multiple sources of uncertainty because

it takes randomness into account. On the other hand, others have argued that each model is suitable for different time frames where the MCS is suitable for predictions made in the short term, while the BSM model is suitable for predictions made in the long term. Currently, nothing proves that these two conclusions cannot coexist, in fact, they may be linked because, in the long term, samples and trials will converge on the true value. This implies that there is a possibility for the MCS to also be suitable to calculate long-term option prices. Given that the results of an MCS depend on the number of simulations, the results can still be optimized by performing more simulations. Future researchers can study if there is a minimum number of simulations that can calculate the true value, which may allow the MCS to be superior at calculating long-term option prices than the BSM model. Scholars could also conduct studies that compare FF5 to other models such as the arbitrage pricing theory. Researchers could even take the strengths of other pricing models and add them to the FF5 to create an improved model. Additionally, researchers could examine how each of the five factors influences portfolio returns and risk. They can investigate the implications of constructing portfolios with different factor loadings and analyze the performance of size, value, profitability, and investment-based investment strategies. Outside of the five factors, researchers could test how changes in economic conditions affect factor returns and whether the model's performance varies during different economic cycles.

5. Conclusion

To sum up, this paper intends to compare different pricing models and their effectiveness. One of the two main findings is that the BSM model and MSC both are effective models, but they are good at calculating different time frames. The MCS is suitable for the short term, while the BSM model is suitable for the long term. On this occasion, it is possible one model may be suitable for all timeframes, but that is yet to be empirically proven as it may be very troublesome. Future studies could dive into these two models and see if there is a possibility for one to come out as superior. The other main finding is that FF3 and CAPM both struggle to effectively explain the CSV in portfolio returns. The most suitable model is the FF5 model, but it is not the best so it can be perfected. After all, the current model does have some limitations such as the inability to accurately include the low average returns. In the future, one could investigate how to improve the current model to make it more well-suited to explain the CSV in portfolio returns.

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