Application Of Probabilistic Linguistic Term Sets and Prospect Theory in Multi-Attribute Decision Making: Aviation Industry Evaluation

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Abstract. With the increasing complexity and diversification of current decision objectives, partial data absence in evaluating certain decision criteria is common. Therefore, this paper introduces a fuzzy evaluation method using probabilistic linguistic term sets and integrates it with prospect theory. This integration allows reflecting decision-makers' preferences while addressing the issue of data absence for evaluation. Additionally, the paper employs the entropy-weighted Analytic Hierarchy Process (AHP) to evaluate other criteria, and all criteria are evaluated using the TOPSIS method, forming a comprehensive evaluation methodology. Finally, the proposed methodology is applied to assess the aviation industry level in eastern coastal provinces represented by Zhejiang Province. Finally, the proposed methodology is applied to assess the aviation industry level in eastern coastal provinces represented by Zhejiang.

Keywords: probabilistic linguistic term sets; prospect theory; entropy-AHP; TOPSIS.

1. Introduction

In 2021, Zhejiang Province's "The 14th Five-Year Plan" highlighted the aerospace industry's significance as a symbol of national strength and a strategic area for national defense, technological innovation, and economic development. This plan emphasizes the industry's role in boosting Zhejiang's scientific and technological prowess, industrial competitiveness, and high-quality economic growth. Consequently, evaluating the aerospace industry becomes crucial. However, the evaluation faces challenges due to the industry's specialized nature and the unavailability of many relevant data. The paper terms these issues as semi-fuzzy problems, where some indicators have missing data that can be addressed through fuzzy evaluation methods. The solution proposed involves using fuzzy evaluation for the incomplete data indicators (fuzzy indicators) and established methods like entropy weight-AHP for the complete ones (precise indicators). Selecting the appropriate evaluation methods for the fuzzy indicators is a key challenge in resolving these semi-fuzzy evaluation problems. The paper also compares the aerospace industry development in Zhejiang with other eastern coastal provinces like Shandong, Hebei, Liaoning, Jiangsu, Fujian, Guangdong, and Hainan to ensure a comprehensive and scientific evaluation.

Fuzzy set theory was first proposed by Zadeh in 1965[1] . Later, fuzzy evaluation branch methods evolved to include the use of linguistic variables to represent fuzzy concepts, which is one of the easier methods to understand. In 1974, Zadeh [2] proposed the concept of linguistic variables, but the problem of choosing among several linguistic terms arose. Therefore, Rodriguez et al.[3] combined linguistic term sets and hesitant fuzzy sets[4] together and proposed the hesitant fuzzy linguistic term set, which contains multiple linguistic term variables with the same weight. However, due to the limitation of linguistic term weights, hesitant fuzzy linguistic term sets are not applicable when different degrees of importance need to be reflected in real problems. Therefore, Pang et al.[5] introduced probabilistic information based on hesitant fuzzy linguistic term sets and proposed Probabilistic Linguistic Term Sets (PLTS), which takes into account the diversity and flexibility of

linguistic information expression and reflects the decision maker's preference for these linguistic variables. Recently, Liu et al.[6] proposed a combination of PLTS and OWD and developed the Probabilistic Linguistic Terminology OWD (PTLOWD) metric, which enriches the distance theory in the context of probabilistic linguistic items. There are also other extensions and studies on OWD measures[7-11] The OWD metric has been developed in the context of the Probabilistic Languages Project. The analysis of the existing literature shows that the above measures do not take into account the subjective value perceptions caused by the expected gains and losses of the decision makers, and cannot reflect the decision maker's preferences. Therefore, another research hotspot is to combine relevant theories that can reflect decision-making preferences, such as prospect theory, with fuzzy set theories, such as probabilistic language[12], which can effectively make up for this deficiency.

Consider the above background，the text introduces three key innovations:

1.A new method to address decision-making and evaluation in scenarios lacking data for essential indicators, applicable to various "semi-fuzzy" evaluation problems.

2.The development of the PTPLTOWD operator, an extension of the PLTOWD operator, integrated into the TOPSIS method to better reflect decision-makers' preferences.

3.A specific indicator system for evaluating the aviation industry in coastal provinces, offering a model for similar semi-fuzzy evaluation situations.

2. Basic Theory

2.1. Probabilistic Linguistic Term Ordered Weighted Distance Measures

Current information distance measures for probabilistic linguistic term sets are complex and cumbersome. In this context, the PLTOWD is introduced[6] operator. PLTOWD can simplify the operation between probabilistic linguistic term elements and thus improve the efficiency of the operation. Its operation process is as follows:

Assuming that $L_1(p)$ and $L_2(p)$ are two PLTS, the

$$PLTOWD(a,b)=\left(\sum_{j=1}^{n}\omega_j\left(d_{PLT}\left(a_{\sigma(j)},b_{\sigma(j)}\right)\right)^{\lambda}\right)^{\frac{1}{\lambda}} \tag{1}$$

is the ordered weighted distance between the set of probabilistic linguistic terms a and b, where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ is the weight vector associated with the PLTOWD metric satisfying $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$. The subscript $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$ such that $d(a_{\sigma(j-1)},b_{\sigma(j-1)})\geq d(a_{\sigma(j)},b_{\sigma(j)})$, where $d_{PLT}(a_j,b_j)$ is the distance between the probabilistic linguistic terms a_j and b_j .

2.2. Prospect theory

Prospect theory describes the psychological changes in a decision maker's mind when considering gains or losses, thus showing changes in subjective value perceptions and reflecting the decision maker's preferences. The specific form of the prospect theory value function is as follows[12] :

$$v(\Delta x)=\begin{cases}(\Delta x)^{\alpha}, & \Delta x\geq 0,\\ -\theta(-\Delta x)^{\beta}, & \Delta x<0,\end{cases} \tag{2}$$

Δx is the magnitude of deviation of x from a reference point x_0 ; $\Delta x\geq 0$ represents gains; and $\Delta x<0$ represents losses. α,β reflects the decision maker's sensitivity to gains and losses, while θ is the loss aversion coefficient. According to Tversky and Kahneman[13] According to the analysis and research of Tversky and Kahneman, $\alpha=\beta=0.88,\theta=2.25$ is in line with human decision-making psychology.

2.3. Prospect theory - PLTOWD

Step 1: Selection of reference points

For a neutral decision maker, the decision reference point is $D^z = \frac{1}{n}\sum_{j=1}^{n} d_{PLT}(a_{\sigma(j)}, b_{\sigma(j)})$ regardless of whether the distance metric is a cost or a benefit.

Step 2: Calculate the distance from each scenario to the reference point

In this step, we need to calculate Δx in Equation 2, which represents the distance of each scenario in the foreground theory from the reference point.

For a neutral decision maker: $\Delta x = d(a_i, D^Z) = \{(P_{i1}, D_1^z), (P_{i2}, D_2^z), \cdots, (P_{im}, D_m^z)\}$

Step 3: Foreground value and weight calculation

The foreground values are calculated as follows: using D^z as the reference point, the foreground value function for each scenario is calculated using Equation (2). In order to make all the foreground values positive, the larger the value the better, and convert all the foreground values as follows:

$$V_j^{'} = max\{V_1, V_2, \cdots, V_n\} - min\{V_1, V_2, \cdots, V_n\} + V_j \tag{3}$$

The weights associated with the weighted distance measure for probabilistic linguistic terms are calculated as follows. The foreground values computed in the different cases are transformed and normalized to satisfy $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

$$\omega_j = \frac{V_j}{\sum_{j=1}^{n} V_j} \tag{4}$$

Step 4: Determination of indicator weights

Determining the index weights. In this module, we construct a multi-objective planning model to minimize $d_\omega(P_{ij}, P_{*j})$ and maximize $d_\omega(P_{ij}, P_{\#j})$ to obtain the weights of the attributes.

$$\omega_j = \frac{\{\sum_{i=1}^{m}[d(P_{ij}, P_{*j}) - d(P_{ij}, P_{\#j})]\}^2}{\sum_{j=1}^{n}\{\sum_{i=1}^{m}[d(P_{ij}, P_{*j}) - d(P_{ij}, P_{\#j})]\}^2} \tag{5}$$

Step 5: Construction of PTPLTOWD

In this study, the PLTOWD metric obtained from Eq. (1) in Section 2.1 is combined with the weights obtained from the foreground theory processing to obtain the foreground theory-based PLTOWD operator (PTPLTOWD). Here, φ_j is the weights obtained from the prospect theory also the OWD position weights and ω_j is the metric weights.

$$PTPLTOWD(a, b) = \left(\sum_{j=1}^{n} \varphi_j \left(\omega_j d_{PLT}(a_{\sigma(j)}, b_{\sigma(j)})\right)^{\lambda}\right)^{\frac{1}{\lambda}} \tag{6}$$

2.4. Comprehensive Model

The PTPLTOWD metric proposed in Eq. (6) is introduced into the TOPSIS method to realize the orderly aggregation of diversified information, reflecting the subjective value perception of the decision maker, and thus obtaining a new evaluation framework, i.e., PTPLTOWD-TOPSIS. which is a widely used multi-attribute decision-making method.

In short, it is the application of the operators proposed in Section 2.3 to the TOPSIS model, and the process is shown in Figure 1.

Since the entropy weight-AHP method is relatively common, it will not be repeated in this paper, and the specific process is shown in the Figure 2.

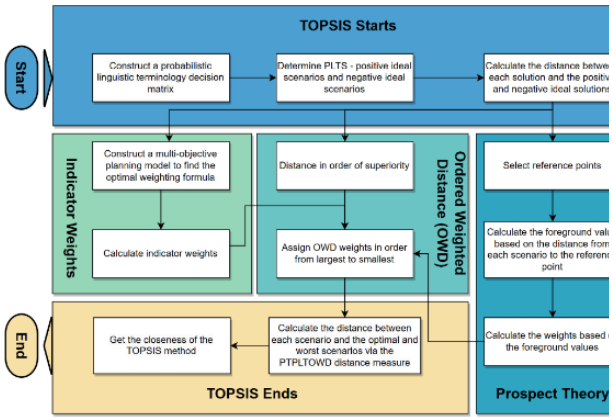


Figure 1 Flowchart of the PTPLTOWD-TOPSIS

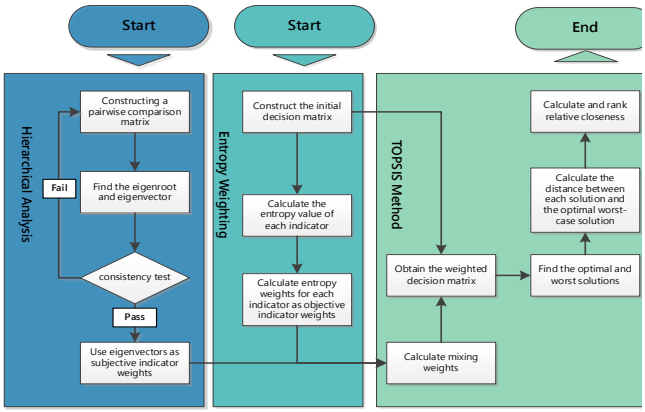


Figure 2 Flowchart of entropy weight-AHP-TOPSIS

Based on the above, noting that the relative closeness obtained from PTPLTOWD-TOPSIS model is R_1 ,and the relative closeness obtained from entropy weight-AHP-TOPSIS model is R_2 , then the final combined relative closeness is:

$$R = \alpha R_1 + \beta R_2 \tag{7}$$

α, β are the coefficients of the decision maker's trust in the two components of the indicator, respectively, which sum to 1 and can be adjusted according to the decision maker's preferences.

2.5. Evaluation system for the level of the aviation industry

According to the scientific construction of high-quality development evaluation index system proposed by Li Jinchang et al. , this paper constructs a semi-fuzzy evaluation index system for the evaluation of the aviation industry, as shown in the Figure 3.

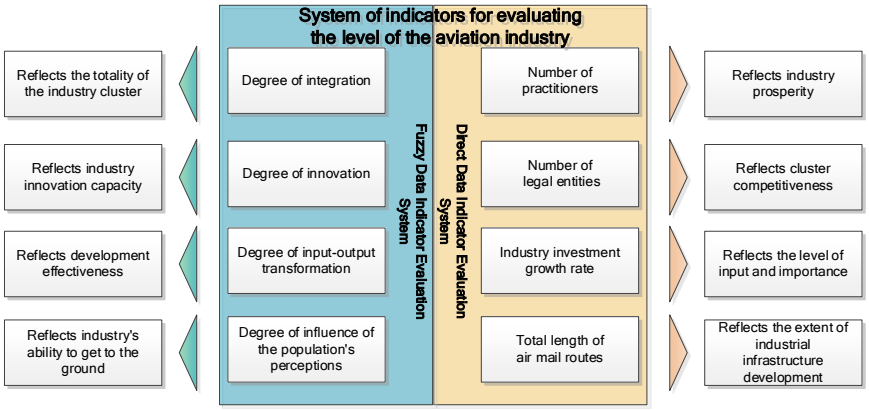


Figure 3 System of indicators for evaluating the level of the aviation industry.

3. Example applications

The direct data source of this paper is CNKI's Statistical Database of China's Civil Aviation Industry and Economic and Social Development, and the selected indicators are the number of employees, the number of legal entities, the growth rate of industry investment and the total length of airmail routes, and the selected years of existing data are 2020, which is the year that is closest to the current situation.

The source of fuzzy data for this paper is the Delphi method i.e. the expert scoring method, for this purpose a questionnaire was used to get the scores from the experts. In this case, the experts' opinions will occupy the majority of the final results summarized in the questionnaire, and the students and other people together will occupy a small percentage, which is 90% and 10% respectively.

In this example, we consider eight provinces on the east coast of China, namely Zhejiang, Shandong, Hebei, Liaoning, Jiangsu, Fujian, Guangdong and Hainan, which are named $A_1 - A_8$. The fuzzy indicators are named as c_1 - degree of input, c_2 - degree of innovation, c_3 - degree of transformation of inputs and outputs, and c_4 - degree of influence of residents' perception. The direct data indicators are named as e_1 - Employees, e_2 - Number of Legal Units, e_3 - Growth Rate of Industry Investment, e_4 - Total Length of Air Mail Route.

And for the fuzzy part of the evaluation, we used the Delphi method to obtain data for these indicators. Each expert on the evaluation team assessed each indicator attribute using a five-level linguistic terminology - independently assessed using a seven-level scale: good, better, fair, poor and poor. For convenience, the set of five linguistic terms is noted at $S=(s_0,s_1,s_2,s_3,s_4)$. Based on the linguistic assessment of each expert on each attribute indicator of each evaluation object, the decision matrix for evaluating probabilistic linguistic terminology information is obtained, as shown in Table 1.

3.1. Evaluation of fuzzy indicators

Step 1: Construct a probabilistic linguistic terminology decision matrix based on the decision information $R=[P_{ij}]_{m\times n}$. The details are shown in Figure 4. The final combined probabilistic linguistic terminology decision matrix obtained from experts, students and others is listed in Table 1, with 90% of experts and 10% of students (others are categorized as students).

Step 2: Determine the positive ideal scenario for PLTs L_* and the negative ideal scenario for PLTs $L_{\#}$

	c_1	c_2
A_1	$\{s_0(0.035),s_1(0.041),s_2(0.372),s_3(0.366),s_4(0.186)\}$	$\{s_0(0.041),s_1(0.221),s_2(0.192),s_3(0.546),s_4(0)\}$
A_2	$\{s_0(0.024),s_1(0.558),s_2(0.047),s_3(0.36),s_4(0.012)\}$	$\{s_0(0.018),s_1(0.564),s_2(0.233),s_3(0.18),s_4(0.006)\}$
A_3	$\{s_0(0.006),s_1(0.564),s_2(0.233),s_3(0.006),s_4(0.192)\}$	$\{s_0(0.372),s_1(0.204),s_2(0.419),s_3(0),s_4(0.006)\}$
A_4	$\{s_0(0.378),s_1(0.384),s_2(0.041),s_3(0.006),s_4(0.192)\}$	$\{s_0(0.186),s_1(0.384),s_2(0.419),s_3(0.006),s_4(0.006)\}$
A_5	$\{s_0(0.395),s_1(0.029),s_2(0.378),s_3(0.186),s_4(0.012)\}$	$\{s_0(0.029),s_1(0.407),s_2(0.558),s_3(0),s_4(0.006)\}$
A_6	$\{s_0(0.018),s_1(0.024),s_2(0.407),s_3(0.54),s_4(0.012)\}$	$\{s_0(0.024),s_1(0.395),s_2(0.215),s_3(0.36),s_4(0.006)\}$
A_7	$\{s_0(0.035),s_1(0.389),s_2(0.384),s_3(0.18),s_4(0.012)\}$	$\{s_0(0.035),s_1(0.389),s_2(0.569),s_3(0),s_4(0.006)\}$
A_8	$\{s_0(0.029),s_1(0.198),s_2(0.215),s_3(0.546),s_4(0.012)\}$	$\{s_0(0.198),s_1(0.401),s_2(0.035),s_3(0.36),s_4(0.006)\}$

	c_3	c_4
A_1	$\{s_0(0.407),s_1(0.209),s_2(0.204),s_3(0.18),s_4(0)\}$	$\{s_0(0.041),s_1(0.389),s_2(0.564),s_3(0),s_4(0.006)\}$
A_2	$\{s_0(0.024),s_1(0.395),s_2(0.575),s_3(0),s_4(0.006)\}$	$\{s_0(0.018),s_1(0.389),s_2(0.407),s_3(0.18),s_4(0.006)\}$
A_3	$\{s_0(0.186),s_1(0.569),s_2(0.239),s_3(0),s_4(0.006)\}$	$\{s_0(0.012),s_1(0.006),s_2(0.791),s_3(0.186),s_4(0.006)\}$
A_4	$\{s_0(0.012),s_1(0.761),s_2(0.221),s_3(0),s_4(0.006)\}$	$\{s_0(0.024),s_1(0.186),s_2(0.779),s_3(0.006),s_4(0.006)\}$
A_5	$\{s_0(0.035),s_1(0.389),s_2(0.569),s_3(0),s_4(0.006)\}$	$\{s_0(0.209),s_1(0.575),s_2(0.209),s_3(0),s_4(0.006)\}$
A_6	$\{s_0(0.024),s_1(0.029),s_2(0.401),s_3(0.54),s_4(0.006)\}$	$\{s_0(0.024),s_1(0.558),s_2(0.233),s_3(0.18),s_4(0.006)\}$
A_7	$\{s_0(0.029),s_1(0.389),s_2(0.569),s_3(0.006),s_4(0.006)\}$	$\{s_0(0.209),s_1(0.204),s_2(0.395),s_3(0.186),s_4(0.006)\}$
A_8	$\{s_0(0.029),s_1(0.384),s_2(0.215),s_3(0.366),s_4(0.006)\}$	$\{s_0(0.024),s_1(0.198),s_2(0.227),s_3(0.54),s_4(0.012)\}$

Figure 4 Decision Matrix for Integrated Probabilistic Language Terms

Step 3: Calculate the distance between each solution A and the positive and negative ideal solutions $d(P_{ij},P_{*j})$ and $d(P_{ij},P_{\#j})$.

Step 4: Get the indicator weights $\omega=[0.755,0.001,0.032,0.212]$.

Step 5: Choose a reference point. The reference point for the neutral decision maker is chosen for the case of this paper. For the neutral decision maker, the reference points for the distance measures based on costs and benefits are $D^z=\left(\frac{1}{n}\sum_{j=1}^{n}d_{ij}^{+},\frac{1}{n}\sum_{j=1}^{n}d_{ij}^{-}\right)$.

Step 6: Calculate the distance measure between each scenario and the positive and negative ideal reference points.

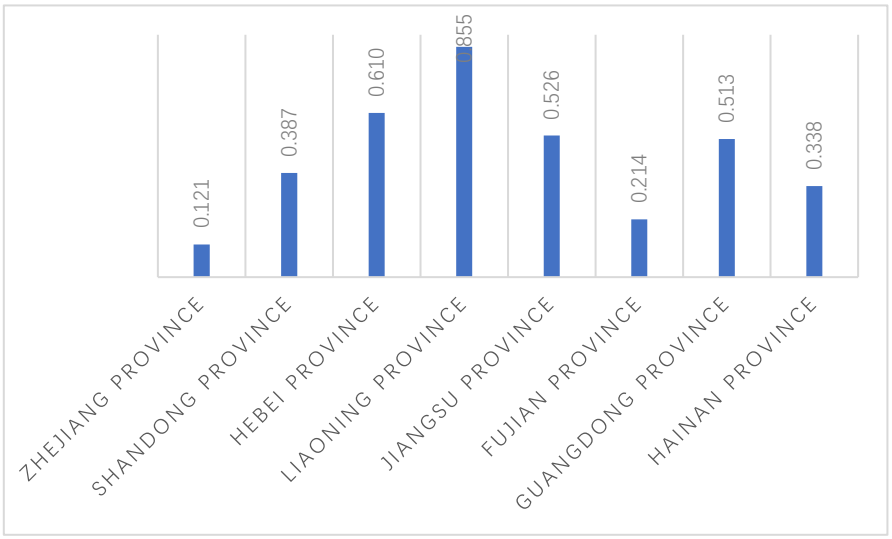


Figure 5 Relative closeness of fuzzy evaluation section

Step 7: From the distance from the reference point obtained in step 6, the prospect value for each distance is obtained.

Step 8: Calculate the relative weights of probabilistic linguistic term distance measures under cost-based and benefit-based distance measures, and then assign each weight to each indicator distance according to the OWD rule.

Step 9: Calculate the ordered weighted distance between each scheme and the positive and negative ideal solutions $PTPLTOWD^{'}(A_i, A^{+})$ and $PTPLTOWD^{'}(A_i, A^{-})$.

Step 10: Calculate the relative closeness according to PTPLTOWD. In order to visualize the relative size more, a diagram is made as shown in Figure 5.

3.2. Evaluation of numerical indicators

Step 1: Construct the initial decision matrix with m evaluation indicators and n each evaluation object.

And the data is normalized so that it maintains the same trend.

Step 2: Calculate the entropy value of each indicator H_i , and get the entropy value vector. And calculate the entropy weight based on the entropy value to get the weight of each indicator $\omega_1 = [0.316, 0.130, 0.124, 0.430]$.

Step 3: Construct the pairwise comparison matrix $A = \begin{bmatrix} 1 & 3 & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{3} & 1 & \frac{1}{7} & \frac{1}{5} \\ 3 & 7 & 1 & 2 \\ 2 & 5 & \frac{1}{2} & 1 \end{bmatrix}$.

Step 4: Find the eigenvector $w_2 = [0.162, 0.060, 0.491, 0.288]$, which is our desired weights, based on the pairwise comparison matrix and perform a consistency test.

Step 5: Calculate the hybrid weights. This paper defaults to balancing subjective and objective, which have equal importance in this evaluation, noting $\alpha = 0.5$. Then $w = 0.5w_1 + 0.5w_2 = [0.239, 0.095, 0.307, 0.359]$.

Step 6: The original data matrix $R = (r_{ij})_{m \times n}$ is normalized and weighted to obtain a weighted decision matrix.

Step 7: Take the above matrices to get the optimal and worst solution vectors: $A^{+}=[0.239,0.095,0.307,0.359]$,$A^{-}=[0,0,0,0]$.

Step 8: Calculate the distance from each solution to the optimal and worst solutions.

Step 9: Calculate the distance according to the above to get the relative closeness $R=\frac{d^{-}}{d^{-}+d^{+}}$. In order to show the relative size more intuitively, make a diagram as shown in Figure 6.

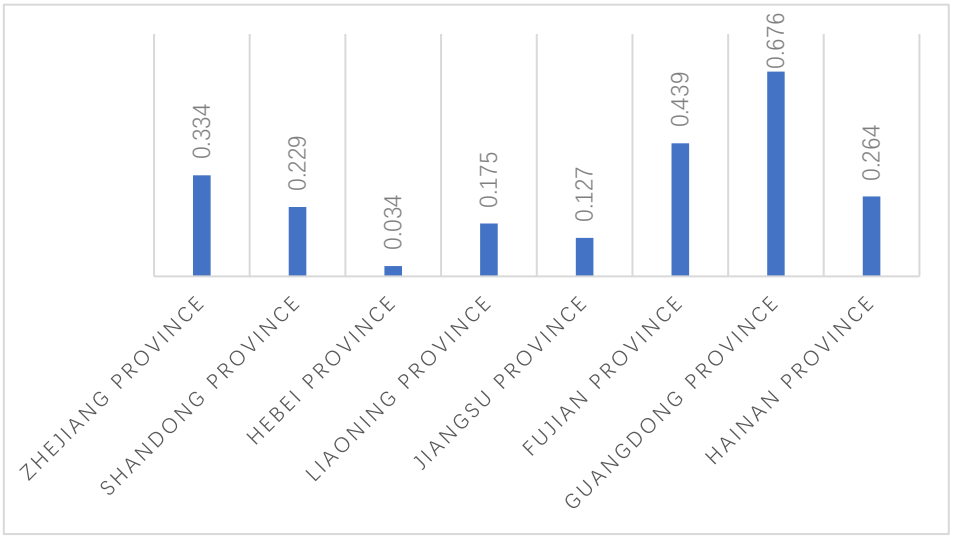


Figure 6 Numerical evaluation of the relative closeness of partial

3.3. Analysis of results

In this case, this paper defaults to the decision maker's trust in the two evaluation methods being equal, i.e.,$\alpha=\beta=0.5$, then the final combined closeness result is shown in the Figure 7.

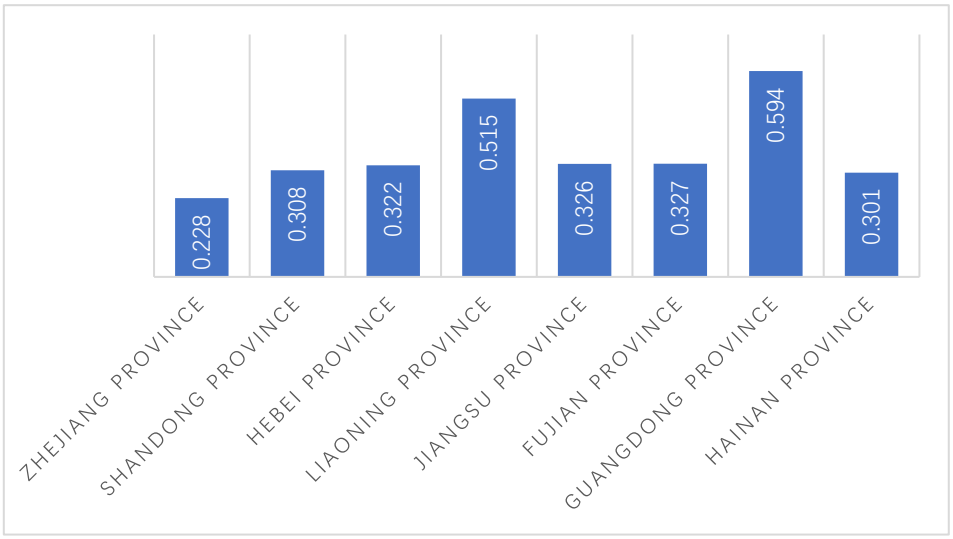


Figure 7 Combined relative closeness

We can find that the provinces on the way can be roughly categorized into three groups: the first group is represented by Liaoning and Guangdong, which are far ahead; the second group is represented by Zhejiang, which is obviously lagging behind; and the rest of the provinces are categorized into the third group, which is above and below the average level and with not much difference.

Looking deeper into the reasons, we can find that Liaoning's high relative closeness is due to the fact that although the industry's vitality in the exact numerical evaluation section is average, the degree of integration and innovation in the fuzzy evaluation section is surprisingly high, which is also very much in line with our expectations. For Guangdong, its degree of integration and innovation is

not as outstanding as Liaoning's, but it is still very good, coupled with the fact that Guangdong's infrastructure development and industrial activity are both very good, which results in Guangdong's excellent performance in both parts of the evaluation. Shandong, Hebei, Jiangsu, Fujian and Hainan have in common that they are all slightly higher on one side and slightly lower on the other. Except for Hebei, which is slightly skewed, their overall relative proximity is at the average level, so we will not discuss them in detail here. The results of the above evaluations are very much in line with our expectations, and at the same time prove the scientificity and feasibility of the method.

The difference in the degree of trust in the two parts of the evaluation system will create different evaluation results, and in today's fuzzy evaluation system has been quite mature, so this paper adopts the fuzzy evaluation and numerical evaluation of the same status of the degree of trust, but also for the ultimate goal of the method proposed in this paper to solve the evaluation of semi-fuzzy, to solve the evaluation problem, proposed a feasible reference program.

4. Conclusion

This paper introduces a new PLTOWD measure based on prospect theory, extending the existing PLTOWD measure to align more closely with decision makers' preferences. Prospect theory, which highlights how irrational behaviors arise in risky decisions due to factors like sensitivity to gains and losses, risk attitudes, and reference points, is utilized to enhance the PLTOWD measure's rationality and validity by incorporating decision makers' psychological traits.

Furthermore, a novel evaluation method is proposed, combining fuzzy and numerical evaluations to address semi-fuzzy evaluation issues. This method, the probabilistic language combination TOPSIS comprehensive evaluation method based on prospect theory, is applied to the aviation industry, yielding significant results. These results not only demonstrate the method's practicality and effectiveness but also its potential as a reference in similar evaluations. This work enriches fuzzy evaluation theory and methods while providing a valuable tool for the aviation industry and related areas.

Looking ahead, the application of prospect theory and other psychological theories like regret theory, which considers the emotional impact of decision-making, is anticipated to grow in various fuzzy evaluation domains. These theories, enhancing subjectivity and flexibility in evaluations, indicate the vast potential and future development of fuzzy evaluation methods, especially when combined with traditional numerical methods to cater to diverse scenarios and needs.

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