Analysis of bond immunization strategies based on duration model

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Abstract. Due to the rapidly expanding bond market and the growing variety of bond types, investors must take on a lot of risks in the investing market. As a result, this article explains and examines how to benefit from the duration matching immunization method, namely the maturity immunity of bond portfolio to ensure return, in order to get consistent profit income and prevent interest risk. The investment portfolio is developed to determine the matching analysis of maturity and yield and maturity and investment cycle after the screening phase of the bond market. It is proposed and the flaws are highlighted in accordance with the interest rate fluctuations of the bond market with the maturity immunization portfolio, in order to give investors pertinent recommendations on how to maintain the stability of income and mitigate the loss of interest rate change.

Keywords: risk of financial investment, duration, immunization strategy.

1. Introduction

The bond market has always been a crucial element in the world of finance, providing financing channels for governments, corporations, and individuals while offering diverse investment opportunities for investors. However, like any financial market, the bond market is rife with complexity and potential risks. One of the most significant risks is interest rate risk, which can have a profound impact on bond prices.

In this case, the bond duration immunization strategy has emerged. This strategy aims to help investors manage interest rate risk within their bond portfolios, thereby reducing potential losses and enhancing returns. The core idea behind the bond duration immunization strategy is to mitigate the sensitivity of bond prices to interest rate changes by aligning the portfolio's duration with the investor's investment objectives. In this essay, we will delve into the importance of the bond duration immunization strategy and explore avenues for improvement in its practical application.

This essay will first introduce and analyze different types of financial investment risks and their causes. It is worth noting that the characteristics of the bond market make it vulnerable to fluctuations in market interest rates. When market interest rates rise, bond prices usually fall, and vice versa. This puts investors at serious risk, especially in long-term bonds. Therefore, the bond duration immunization strategy has become an indispensable tool to avoid this risk.

Secondly, this essay will introduce the basic principle of bond duration immunity strategy, including the definition and calculation method of bond duration, and how to reduce interest rate risk by constructing a portfolio with duration immunity. We will delve into the application of the duration immunity strategy in real investing, exploring how the price of a bond portfolio will change in the maturity year if interest rates change.

Finally, the limitations of this strategy are analyzed, and by delving into this topic, we hope to provide investors with more insights on how to effectively manage risk and achieve returns in the bond market. In a financial environment full of challenges and opportunities, it will become increasingly important to understand and apply the promotion path of bond duration immunity strategies.

2. Concept Definition and Literature Review

2.1. Concept Definition of Bond Market

The bond market, where bonds are issued and sold, is an important part of the financial market. Any country's financial system must include the bond market as a key element. The bond market is a crucial channel for the communication of central bank monetary policy since the yield curve of bonds serves as the benchmark for the rate of return of all financial goods in the social economy. The entire society can benefit from low-risk financing and investing tools that can be found on a unified, developed bond market. A unified and robust bond market is sometimes referred to as the financial market's cornerstone. Any danger that could result in a financial loss for a company or organization is referred to as a financial risk. Examples include financial market risk, financial product risk, and risk associated with financial institutions. The effects of risks taken by a financial institution frequently outweigh the impact on the institution itself and may endanger the stability of the entire financial system.

2.2. Literature Review

The theory of fixed-income portfolio investment has developed rapidly and become more mature in Western financial markets. Fabozzi (1985) istudied the traditional fixed-income portfolio strategy [1][2]. Maginn studied sensitivity analysis as a tool for active management strategies and extended it to consider the results of analysis on the valuation of expected returns; Fisher & Weil (1971) firstly proposed bond-immunization strategy. They pointed out that in order to immunize a bond portfolio, i.e., the holding return of the portfolio is at least not lower than the yield of the corresponding maturity implied by the forward rate, the basic strategy is to make the Macaulay duration of the portfolio equal to the planned holding period of the portfolio [3]. The model proposed by Ma (2006) is an important improvement in the nature of the immunized portfolio investment [4].

Domestic theoretical and applied research on fixed-income securities is still in the primary stage, which is mainly due to the underdevelopment of China's financial derivative market and the small variety of financial derivative products. Male addition, due to the slow progress of the domestic interest rate marketization and the immature development of the treasury bond market, it is difficult to compile a dynamic yield curve.

For bond portfolio investment research, Bertocchi M. & Moriggia V, J. (1996) explored bond portfolio investment analysis methods and stochastic planning applications [5]. In recent years, some people gradually began to study the problem of interest rate risk immunization in the treasury bond market with more mature methods, such as Xue Yifei (1999) introduced and used the M-model to conduct empirical research on interest rate immunization in the treasury bond market [6].

The combined use of empirical and normative analysis is one of the characteristics of this paper. Empirical analysis and normative analysis are the two research methods of economics research is also the basic analysis method used in this paper. These two methods of analysis are complementary to each other. Using a combination of empirical analysis and normative analysis to study China's fixed-income securities drawing on and absorbing foreign empirical research ideas, methods and conclusions to make up for the shortcomings of the traditional domestic normative analysis to improve the understanding of China's fixed-income securities market theory and practice will have a profound impact.

3. The Risk of Financial Investment

3.1. Type of the Risk of Financial Investment

(1) Market risk

The risk of changes in a derivative's price or value as a result of negative changes or abrupt variations in the market price of the underlying asset is known as market risk. Market interest rates, exchange rates, stock and bond quotes, as well as other variables, can all affect the market price of the underlying assets. Market risk calculation methods mainly include Variance-Covariance Approach, Historical Simulation Method and Monte-Carlo Simulation [7].

(2) Credit risk

The possibility that the counterparty won't meet its commitments as they become due is known as credit risk. Credit risk, also known as default risk, is the potential that a counterparty, securities issuer, or borrower won't be able or willing to uphold the terms of a contract for a variety of reasons, leading to a default and a loss for the bank, the investor, or the counterparty. The credit risk associated with each OTC derivative transaction Each OTC derivative transaction varies as a result of the various settlement mechanisms.

(3) Liquidity risk

According to the Guidelines on Liquidity Risk Management for Commercial Banks published by the CBRC in 2009, liquidity risk is the possibility that a commercial bank, even one that is solvent, won't be able to raise enough money promptly or affordably to keep up with the growth of its assets or pay off its debts as they become due.

(4) Industry risk

Industry risk is the possibility that a specific industry's output, output, and production efficiency may be impacted by a certain industry as a result of the existence of unknowable elements. The likelihood of losses resulting from departure from the anticipated results of production, operation, investment, or credit granting to a certain industry is made possible by the existence of some unpredictable circumstances. Cyclical risk, growth risk, industrial correlation risk, market concentration risk, industry barrier risk, and macro policy risk are some of the variables that might affect industry risk.

(5) Personnel risk

Personnel risk is distinct from the insurance category of risk; it results from subpar business management and system shortcomings and raises the chance of employee harm to the company's interests. Direct and indirect causes—which may come from internal or external factors—can result in personnel risk.

(6) Stock investment risk

The chance that a stock investor would lose money as a result of a decline in stock price after purchasing a stock is known as stock investment risk. It is commonly understood to mean that there was a discrepancy between the selling price and the anticipated price or that the actual dividend fell short of the expected minimum. The trading price on the stock market frequently undergoes dozens of adjustments per day; it may go up for a profit or down for a loss, or even go up or down for a few days. Investors anticipate to receive a big return at the same time, but it is unavoidable that they will also have to endure the accompanying huge dangers. Opportunities and hazards in the stock market always exist, develop, and recede at the same time. The risk associated with stock investments can be split into two categories: general risk and individual risk.

(7) Political risk

Both macropolitical risk and micropolitical risk are categorized as political risk. Macro-political risk is the term used to describe situations in which foreign-invested businesses suffer financial losses as a result of changes in the political climate of the host nation or in interactions with other nations. These occurrences include theft, expropriation, nationalization, political intrusion, regime change in the host nation, civil war, violent conflict, social unrest, and degradation of ties between the host nation and its home nation or other third parties.

3.2. Reason of the Risk of Financial Investment

3.2.1. External reason

(1) Economic environment

The economic environment is the main external factor affecting the financial risk of financial enterprises, specifically including the national economic policy, the level of social and economic development, and the structure of the market economy. Among them, the national economic policy is the government in order to implement macro-control of social and economic development and formulate policy guidelines, and the introduction of policies will inevitably affect the development of certain industries. The economic environment is always in a complex and changing state, which is actually the market economy cannot change the objective phenomenon, to a certain extent, also determines the direction of the development of financial enterprises, if the financial enterprises are not able to follow the changes in the economic environment and make adjustments to their own business strategy, then it will inevitably bring serious financial risks to the enterprise.

(2) The outbreak of the financial crisis

The outbreak of the financial crisis may lead to financial enterprises' shortage of funds or even bankruptcy, and the interest rate hike policy implemented by the national financial management department to stabilize the economy has also increased the interest expenses and financing risks of enterprises. From another point of view, the financial crisis will also affect the capital behavior of investors to some extent, making the financial services enterprises to increase the risk of investment and unable to achieve the expected business objectives. In the era of global economic integration, the outbreak of financial crises abroad will affect the market economic situation in our country, so that the financial risk of our enterprises increased, consumers in any geographical area of the world due to the financial crisis and change their regular capital behavior, may affect the sales of enterprise products, affecting the conversion process of goods to funds. In addition, the financial crisis will also make the flow of funds show uncertainty, seriously disrupt the business plan of enterprises, make the exchange rate of various countries have abnormal fluctuations, reduce the ability of enterprises to repay debts and business reputation, and the investment behavior of enterprises will be changed as a result.

3.2.2. Regime reason

Difficulties in bank loan financing Bank loan financing refers to the loan business that enterprises handle with financial institutions in order to obtain the funds needed for business operations, and generally speaking, the loan methods mainly include credit loans and mortgages, which are also the main financing business carried out by financial enterprises at present. However, due to the lack of effective collateral, evaluation department service is not standardized, the lack of suitable guarantors and other issues, resulting in financial institutions are not willing to lend to these enterprises, in addition, the financial enterprise internal financial system is not sound, as well as the environment in which the internal and external problems such as unsatisfactory and so on also make its capital inflow is also troubled, in particular: first of all, the tightening of the monetary policy, the monetary policy has a direct impact on the day-to-day operations and investment behavior of the financial enterprise and investment behavior, tight monetary policy will increase the difficulty of enterprise financing, financial enterprises will increase the cost of financing, greatly weakening the market competitiveness of its financial enterprises and solvency, so that the enterprise's capital flow cycle problems, and once the financing behavior of the financial enterprise is restricted, then its investment behavior will also be affected; Secondly, the internal financial management system is not perfect, financial management system Whether perfect is also a key factor affecting the financial risk of financial services enterprises, if the internal management of the enterprise is chaotic, then the distribution of interests between the various departments of the financial enterprise and the management of funds will have serious problems, the efficiency of the use of funds will also be greatly reduced as a result of this, and the integrity of the enterprise's finances and security cannot be guaranteed, and it is difficult to attract the inflow of external funds.

4. Immunization Strategy

4.1. Duration

Duration is also called Macaulay Duration, which represents the maturity of a set of discrete cash flows, weighted on the percentage of their present value, Frederic Macaulay first proposed the concept in 1938.[8] It provides investors with information on how long it usually takes to receive all of a bond's cash flows. It can be calculated as the sum of all the multiples of the present value of each cash flow and the corresponding time divided by the current bond price. The formula is given by formula 1.

$$\text{Macaulay Duration} = \sum_{t} t\frac{\text{PV}(\text{CF}_t)}{\text{P}} \tag{1}$$

t is the respective time period, PV is the present value of the coupon payment, P is the current bond price, CF is the cash flow. Or

$$Macaulay\ Duration = \frac{\sum_{t=0}^{n} \frac{t*C}{(1+y)^t} + \frac{n*M}{(1+y)^n}}{current\ bond\ price} \tag{2}$$

C is periodic coupon payment, y is the yield to maturity, M is the maturity value, n is the total number of periods.

Macaulay Duration shows how sensitive bond prices are to interest rate changes, so it is a measure of the interest rate risk of bond prices.[9] This is so that the cash flow will be more significantly impacted by changes in interest rates for a longer period of time the longer the maturity of bonds. On the whole, longer Macaulay duration means more sensitivity to changes in interest rate, thus the bond is riskier [9]. Conversely, shorter Macaulay Duration means less sensitivity to changes in interest rate, so the bond is less risky.

4.2. Immunization Strategy

The immunization strategy matches the maturity of assets and liabilities to minimize risk and its effect on interest rates. To find the weight of each bond in a portfolio, it is necessary to figure out the duration of each bond first. It is possible to apply it to guarantee that the value of a portfolio of assets would rise or fall in proportion to a specific set of liabilities, keeping the equity portion of capital constant. Bond prices will decline and reinvested earnings will increase if interest rates increase, and vice versa if interest rates decline, bond prices will increase and reinvested earnings will decrease.

5. Problems with Immunization Strategy in Practical Application

5.1. Case Analysis of "Andy Plan to Receive €10,000,000 after 3 Years to Open a Bookstore"

Here's an example to analysis this strategy. Suppose that Andy plans to get €10,000,000 in three years to open a bookstore, assuming that the current market interest rate is 8%, the following is his bond investment strategy. The structures of both Bond A and Bond B are summarized in Table 1.

Table 1. Bonds parameter

Types of bonds	Face value	Current Price	Annual Coupon	Remaining Life	Yield to Maturity
A	€100	€98.15	6	1 years	6%
B	€100	€104.62	9	6 years	9%

Since Bond B is zero coupon bond, its Macaulay Duration is exactly its equals to it remaining life, which is 5 years.

Using the formula 1 to calculate the duration of bond B, data are presented in Table2 below:

Table 2. Calculation data of bond B

Time	CF	DF	PV=CF*DF	T*PV
1	9	0.9434	8.4906	8.4906
2	9	0.8810	8.0010	16.002
3	9	0.8396	7.5566	22.6698
4	9	0.7921	7.1288	28.5152
5	9	0.7473	6.7253	33.6265
6	109	0.7050	76.8407	461.0442
total			114.743	570.3483

So, MD=570.3483/114.743=4.97years

For the two types of bonds A and B, Andy need to calculate the appropriate proportion to form a portfolio to make sure that the average duration is equal to 3 years.

To achieve this, ABC need to calculate from

$\omega_1 MD_B+\omega_2 MD_A=3$

$\omega_1+\omega_2=1$

Then get the result of ω1=49.62%, ω2=50.38%.7

If calculated according to the market interest rate of 8%, the present value of the €10,000,000 bookstore startup capital that Andy needs after 3 years is:

€10,000,000/ (1+8%)3=€7938322.41

In other words, according to the portfolio of bond A and Bond B, the current investment of €7,938,300, at the current market interest rate of 8%, three years later Andy can own €10,000,000.

According to the ratio of bond A to 49.62%, the investment amount of this bond is €3938995.58, and the number of bonds is: €3938995.58/98.15=40132.41units.

According to the ratio of bond B to 50.38%, the investment amount of this bond is €3999326.83, and the number of bonds is: €3999326.83/104.62=38227.18units.

Then immunity would such a bond portfolio achieve under the change of rate as Table 3 shown

Table 3. price calculation of the portfolio at different interest rates

Process	The market value of the bond portfolio at the duration point		
The market interest rate one year from now	7%	8%	9%
The value of bond A at maturity in three years $=106*40132.41*(1+r)^2$	4870445.198	4961906.961	5054219.53
The reinvested interest price of bond B at the end of year 1 $=9*38227.18*(1+r)^2$	393896.6854	401293.6448	408759.413
The reinvested interest price of bond B at the end of year 2 $=9*38227.18*(1+r)$	368127.7434	371568.1896	375008.6358
Interest on bond B at the end of year 3 $=9*38227.18$	344044.62	344044.62	344044.62
The selling price of bond B at the end of year 3 $=109*38227.18/(1+r)^3+9*38227.18/(1+r)^2+9*38227.18/(1+r)$	4023358.404	3921233.15	3822718.00
The price of the bond portfolio at year 3	9999872.651	10000046.57	10004750.20

From Table 3 we can see that:

As the market interest rate rises, the value of A bond also keeps rising.

The value of bond B itself, as the market interest rate rises, the selling price of bond B after three years will continue to decline, which is determined by the inverse relationship between the market interest rate and the net price.

The reinvested value of the interest on bond B, as the market interest rate goes up, the reinvested value of the interest on the bond goes up.

5.2. Problems that Exits when Applying Immunization Strategy

5.2.1. Assumptions of immunization strategy

However, from the previous calculations (Table2), this immunization strategy (Duration matching) is based on the following assumptions:

-There is no default risk for bond

-There is no taxes or transaction fee

-The yield curve is horizontal and moves horizontally, which means that the rate of return is always the same. And the yield curve moves only before the time a bond is purchased as well as the time it is actually counted into the holding period.

5.2.2. Errors in the assumptions of the duration model

Using Formula 2 to determine duration, every cash flow of the bond is discounted at the same return rate, "y". This suggests that the underlying assumption of duration is a level yield curve. Yet, this doesn't truly capture the shape of the term structure of interest rates, deviating from actual scenarios. Ideally, intricate fixed-income computations should consider the full-term structure. Cash flow present values should be determined using the zero-coupon yield tailored to each specific maturity. This method of calculating duration is termed Fischer-Weill duration [10]. Nonetheless, given the varied and intricate assets and liabilities in banks, estimating duration this way becomes challenging, with its precision being questionable.

5.2.3. Difficulties and costs in the operation of duration models

Theoretically, duration-based interest rate immunity is a dynamic process that requires financial institutions to continuously re-balance their asset holdings to guarantee that the length of their portfolios is consistent with the duration of their liabilities. Owing to the continually changing of portfolio proportions and there are too many transaction expenses, it is really challenging to balance and form a portfolio. Because of this, Therefore, most financial institutions will only make changes on a regular basis, such as once every three months. It can be observed that the transaction costs of acquiring full interest rate immunity and dynamically maintaining immune positions have a substitution relationship. As a result, this approach is only trustworthy in the short run. With the change of time, each asset and its duration will change, and the duration will also change with the change of interest rate. In the case of constant changes in market returns, it is necessary to re-balance the bond portfolio, but frequent re-balancing will increase the transaction cost and thus reduce the return level after balance. On the other hand, it also requires a large amount of labour source to re-balance the portfolio. If the initial desired effect is to be maintained, additional work is needed to rebuild the portfolio when there are significant fluctuations in market interest rates.

6. Conclusion

First of all, the application of duration immunity strategy can significantly reduce the interest rate risk faced by the bond portfolio. According to the results of the empirical study, using the duration immunity strategy to adjust the position structure of the bond portfolio can effectively reduce the loss of the portfolio under the change of interest rate.

Besides, the application of immunization strategies needs to fully consider investors' risk appetite and market environment. Investors' risk appetite directly affects the allocation of the duration immunization strategy, and the changes of the market environment require timely adjustment of the strategy to adapt to the new situation.

In addition, the ways to improve the effectiveness of the application of immunization strategies include but are not limited to the following aspects. First, strengthen the in-depth study of bond markets and interest rate movements to more precisely determine how to deploy duration immunization strategies. Secondly, establish a sound risk control mechanism to monitor and adjust the duration allocation of the bond portfolio in a timely manner. Finally, investor education and

information transmission should be strengthened during the implementation of the immunization strategy to improve investors' understanding and acceptance of the strategy.

This study has certain theoretical and practical value for the application and improvement of the long-term immunization strategy in practice. Through in-depth research on the implementation of the long-term immunization strategy and existing problems, it can provide references for relevant practitioners and investors to help them better apply the long-term immunization strategy to manage the bond investment risk.

Author Contributions

All the authors contributed equally and their names were listed in alphabetical order.

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