Index Model with Modern Portfolio Theory and its Application to Find Optimal Portfolio in the American Stock Market

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Abstract. A crucial framework in financial science, the Index Model and Modern portfolio theory provide investors with a scientifically sound method of asset allocation. This study applies solver tables for data manipulation and computing to an empirical analysis of the theory in the setting of the American stock market. Based on predetermined criteria, 10 well-known stocks from various industries were chosen, IM will use these optimization inputs to discover the regions of acceptable portfolios for the various five situations of the extra limitations after calculating the necessary optimization inputs for the entire Index Model according to the month and everyday observations. The definition of the efficient frontier and identification of the portfolio with the highest Sharpe ratio and lowest variance under five distinct restrictions were made possible thanks to the help of the Solver table. The results of the empirical investigation support the relevance of the index model and portfolio theory in the American financial market by demonstrating its applicability and usefulness for global investors. According to the results, investors are encouraged to diversify their portfolios in accordance with the efficient frontier's rules in order to maximize profits while minimizing risks.

Keywords: Index Model, Solver Table, Efficient Frontier, Sharpe Ratio, permissible portfolios.

1. Introduction

A straightforward asset pricing model called the single-index model (SIM) may be used to calculate a stock's risk and return. In Markowitz's mean variance circumstances. Thus, the model can alleviate the strain of significant input requirements and difficult calculations [1]. In their study, Varghese & Joseph discovered that the single index model outperforms the Markowitz model, when looking at each model's return equation and portfolio risk [2]. Investment strategies try to figure out what proportion of the portfolio either optimizes return for a given level of risk or minimizes risk for a given level of return [3], and MPT (Modern Portfolio Theory) proposes the idea of diversification, or "diversification" through the introduction of the statistical concept of a covariance, or correlation [4]. that is, spreading your investments in different industries can simultaneously increase returns and reduce risk. The purpose of this paper is to apply index model to analyze selected American stocks and find the optimal investment portfolio.

2. Index Model

In specifically, one of the single-index model's versions, the market model, was the first index model to garner widespread notice. Although this was initially covered in Markowitz, Sharpe refined and popularized it [5].

Assume, for simplicity, an equally – weighted portfolio $w_i = \frac{1}{n}$:

$$R_p = \frac{1}{n}\sum_{i=1}^{n}(\alpha_i + \beta_i \cdot R_M + e_i) = \frac{1}{n}\sum_{i=1}^{n} a_i + \left(\frac{1}{n}\sum_{i=1}^{n}\beta_i\right) \cdot R_M + \frac{1}{n}\sum_{i=1}^{n} e_i \quad (1)$$

From which follows:

$$\beta_p = \frac{1}{n}\sum_{i=1}^{n}\beta_i\,, \alpha_p = \frac{1}{n}\sum_{i=1}^{n}\alpha_i\,, and\ e_p = \sum_{i=1}^{n} e_i. \quad (2)$$

The portfolio's variance is:

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p). \quad (3)$$

The firm - specific risk is diversifiable:

$$\sigma^2(e_p) = \sum_{i=1}^{n} (\frac{1}{n})^2 \sigma^2(e_i) = \frac{1}{n} \overline{\sigma}^2(e). \quad (4)$$

Step 1: Calculate Expected Returns: calculate the expected return for each asset.

Step 2: Compute Beta Coefficients:

For each asset, calculate its beta coefficient using regression analysis or other methods, representing the asset's sensitivity relative to the market index (e.g., S&P 500).

Step 3: Calculate Specific Returns

Compute the specific returns for each asset (i.e., returns not explained by beta coefficients), typically calculated using the following formula:

$$Specific\ Return_i = Actual\ Return_i - (\beta_i \cdot Market\ Index\ Return) \quad (5)$$

Step 4: Risk and Return Analysis

Using beta coefficients and specific returns, conduct a comprehensive analysis of the risk and expected returns for assets or portfolios. The specific analysis steps may vary based on the research question and objectives.

3. Portfolio Efficient Frontier

The ideal portfolio of hazardous assets that the market can provide is called the efficient frontier. According to Figure 1, the entire frontier (solid and dashed lines) is called the minimum variance frontier. A rational investor would choose a portfolio with higher expected returns for the same risk, so the main focus is the upper half of the solid border, which can call the Markowitz efficient frontier.

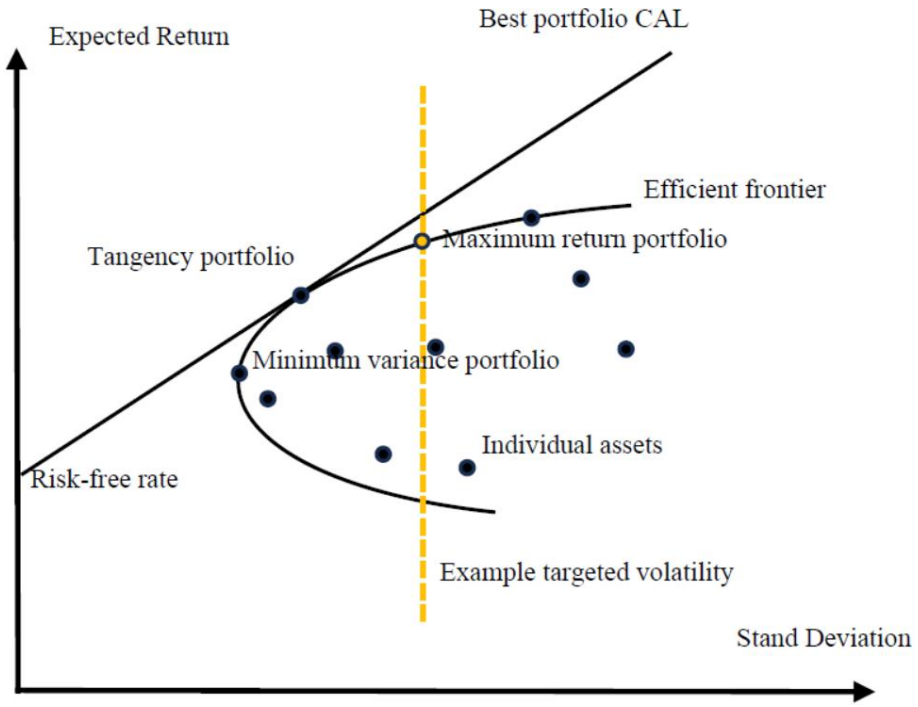


Figure 1. Portfolio Efficient Frontier

Photo credit: Original

4. Literature Review

In several international marketplaces and studies, experimental evaluations have been done to determine the applicability and utility of the Modern Portfolio Theory (MPT) and the Index Model. This is the justification given as to why academics may build global portfolios using the index approach. The Markowitz model has a more complex and challenging calculation compared to the single index model, which has a simpler calculation [6]. Here are some applications of the model. In her analysis, Nalini used the BSE Sensex as the market index and 15 equities from different sectors in the S&P index. It was discovered that portfolio diversification can lower risk [7].

Here is also an application which is using index model here for stocks and different portfolios to showed out the conclusion of the usefulness in the real market. The single index technique employed with a few BSE shares was detailed by R. Nalini (2014). Gaining a practical knowledge of the single index model is the study's goal. Information was collected between 2009 and 2014. The monthly closing prices of fifteen prescriptions were utilized in the study. Only four of the portfolio's fifteen stocks—or companies—are featured. It was discovered that high return equities frequently signal portfolio instability. Before choosing anything Review of Pharmaceutical adverse outcomes Volume 14 Normal Issue 03 2023 408 putting money decisions, this should be considered. According to the research, when returns are strong, investors should evaluate a script's risk while making investment decisions. Research suggests that the majority of the funds might only purchase one type of stock. It would have been preferable to optimize the creation of the study portfolio. Most of the funds are being utilized to purchase this potentially risky stock. How to educate investors on the Sharpe index methodology was looked at in research. By assembling a portfolio of diverse high-return equities, the completed risk analysis may be diversified. The study's findings suggest that micro level research has a greater influence on fund managers' ability to make wise investment decisions.

Despite those applications, the index model is still using during the covid 19 period by some researchers to find out the result and made portfolios in different stock markets. Sharpe's Single Index Model was used by Indian academics to choose the Nifty 50 firms for the period of April 2019 to March 2022. According to the study, the ideal portfolio has 10 equities during the duration of the investigation. The results are only applicable to the research period and might be affected by the use of different time periods, indices, or models. With this knowledge, investors may choose investments wisely [8]. Using of index model in India stock market is also represented by Chintan A. Shah. With building an Optimal Portfolio with the Sharpe Index Model for the Top 15 Securities on the BSE. The Markowitz Model has substantial practical limitations in the Indian stock market due to the stringent criteria involved in calculating the expected returns, standard deviation, variance, and covariance of each asset to every other investment in the portfolio. This method has been made simpler by Sharpe Model, which links a security's return to a single Market index, some kinds of portfolios can be constructed better using Index model [8]. For global using, such as Indonesian stock market, researchers Partono, Thomas used index model to do the Optimal Portfolio Forming Analysis by using 33 sample firms from the Indonesian Most Trusted firms, with a total sample of 27 companies [9].

However, this part can find a lot of different research for Indian Stock markets and some other countries such as Indonesia, even for Spanish stock market index model had been used by Bilbao-Terol, A., Pérez-Gladish, B., & Antomil-Lbias, J. With 31 Spanish mutual funds of the Fondos de Inversión Mobiliaria (FIM) type taken into consideration, an ideal fuzzy compromise portfolio was created using quarterly return data for the years 1999 to 2003 [10].

Therefore, Through the researching, this article found that in the sound stock market of the United States, there is still a lack of application and portfolio construction of the index model in different situations, and this model should not be abandoned. Especially when studying specific situations, it may be simpler and more efficient than other methods.

In this context, this study hopes to make use of the simple features of index model and its ability to avoid risks and maximize returns, select the most famous stocks in the American stock market,

analyze their three-year data, and provide a modern analysis of the complexity of portfolio theory in the field of American stock market.

5. Empirical Analysis

The empirical analysis conducted for this study captures the distinctive features of the US's securities market. Given 20 years' worth of daily data on the S&P 500 index's total returns (ticker symbol: "SPX"), 10 stocks from the same industry (Yahoo! Finance), a 1-month annual Fed Funds rate (ticker symbol: "FEDL01"), and risk-free rate. Using these optimization inputs for the Index Model, determine the regions of acceptable portfolios (efficient frontier, least risk portfolio, optimal portfolio, and minimal return portfolios frontier) for the following five scenarios with the additional limitations: (1) This additional optimization constraint was developed to closely resemble Regulation T by FINRA (https://www.finra.org/rules-guidance/key-topics/margin-accounts), which authorizes broker-dealers to let their customers to take positions that are supported to a minimum of 50% by their account equity:

$$\sum_{i=1}^{11}|w_i| \leq 2; \tag{6}$$

(2) The goal of this additional optimization constraint is to mimic some arbitrary "box" weight restrictions that the client may provide:

$$|w_i| \leq 1, for\ \forall i \tag{7}$$

(3) A "free" problem to demonstrate how the efficient frontier and the region of allowed portfolios in general appear when there are no extra optimization restrictions;

(4) Short positions are prohibited for U.S. open-ended mutual funds; for further details, see Section 12 (a) (3) of the Investment Company Act of 1940, which is accessible at https: //www.law.cornell.edu/uscode/text/15/80a-12. The purpose of this extra optimization restriction is to imitate the usual constraints present in the mutual fund sector in the United States:

$$w_i \geq 0, for\ \forall i \tag{8}$$

(5) Finally, this section wants to determine if adding the wide index to our portfolio has a favorable or unfavorable impact. To that end, this part would like to take into consideration an additional optimization constraint, $w_1 = 0$.

For (1) the Regulation T by FINRA (IM):

$$\sum_{i=1}^{11}|w_i| \leq 2; \tag{9}$$

Table 1. the Regulation T by FINRA

IM	SPX	ADBE	IBM	SAP	BAC	C	WFC
Panel A							
Min Var (%)	138.69	-9.85	0.03	-9.68	-8.19	-15.39	-0.28
Max Sharpe (%)	48.2	35.9	-14.5	2.0	-0.6	-34.8	4.0
Panel B							
	TRV	LUV	ALK	HA	Return	StDev	Sharpe
Min Var (%)	11.23	-2.17	-1.77	-2.62	6.07	11.96	0.5075
Max Sharpe (%)	28.4	3.6	17.7	10.2	18.9	21.0	0.90

With this constraint for index model, according to the Table 1 and Figure 2, min variance of the SPX is occurred to be the largest for 138.69%, while the maximum Sharpe ratio can be occurred for 48.2% here as well.

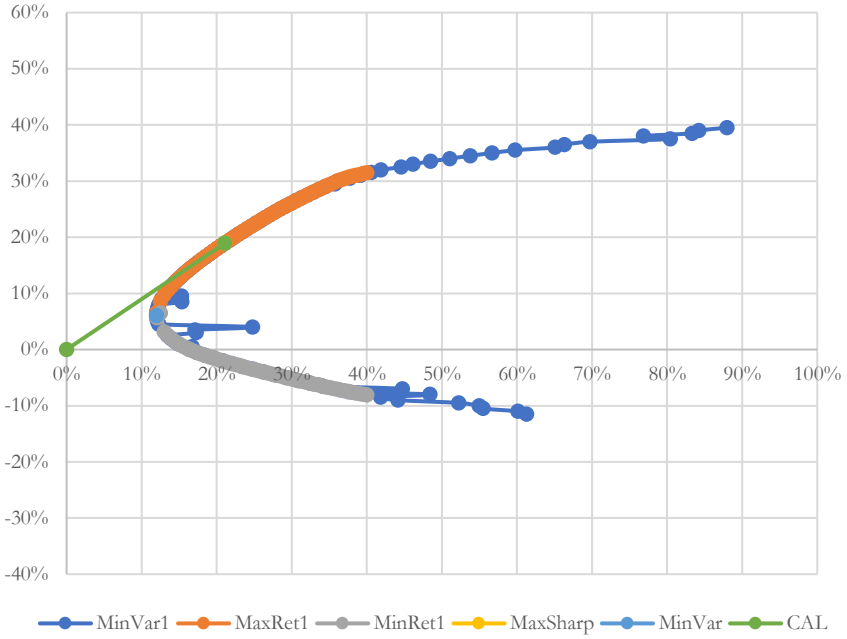


Figure 2 the Regulation T by FINRA, Index Model - Constraint 1

Photo credit: Original

For (2) Arbitrary "box" Constraints (IM):

$$|w_i| \leq 1, for\ \forall i \tag{10}$$

Table 2. Arbitrary "box"

IM	SPX	ADBE	IBM	SAP	BAC	C	WFC
Panel A							
Min Var (%)	100.0	-6.6	8.5	-7.0	-6.5	-14.0	3.7
Max Sharpe (%)	54.9	38.3	-22.7	3.0	-2.3	-37.6	4.3
Panel B							
	TRV	LUV	ALK	HA	Return	StDev	Sharpe
Min Var (%)	23.1	0.7	0.1	-2.1	6.9	12.5	0.55
Max Sharpe (%)	28.6	4.0	18.7	10.9	19.8	22.0	0.90

This constraint shows the strategy of investment portfolio which state investor want each of the stock be stable and have a small variance. According to the Table 2 and Figure 3 this is actually suitable for the investors with longterm buying. This portfolio can see the Sharpe ratio is higher than the first one, 54.9%.

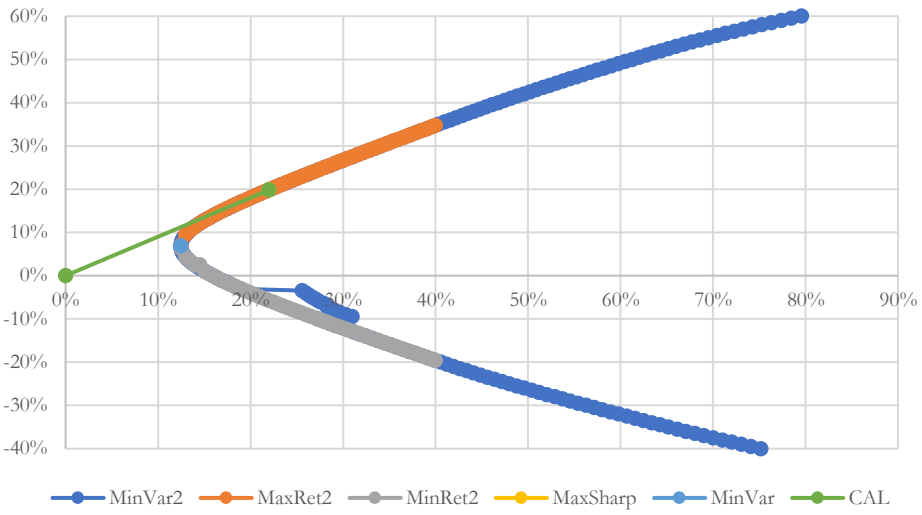


Figure 3 Arbitrary "box", Index Model - Constraint 2

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For (3) A Free Problem (IM):

Table 3. A Free Problem

IM	SPX	ADBE	IBM	SAP	BAC	C	WFC
Panel A							
Min Var (%)	144.4	-10.7	-0.6	-10.4	-8.6	-15.7	-1.4
Max Sharpe (%)	54.9	38.3	-22.7	3.0	-2.3	-37.6	4.3
Panel B							
	TRV	LUV	ALK	HA	Return	StDev	Sharpe
Min Var (%)	11.0	-3.0	-2.3	-2.7	5.8	11.9	0.49
Max Sharpe (%)	28.6	4.0	18.7	10.9	19.8	22.0	0.90

Free Problem Constraint is just for the investors with no investment performance. As here this section can see all the stocks are calculated for the best return. According to the Table 3 and Figure 4, the Min Variance for SPX is higher than the other constraints which is 144.4%.

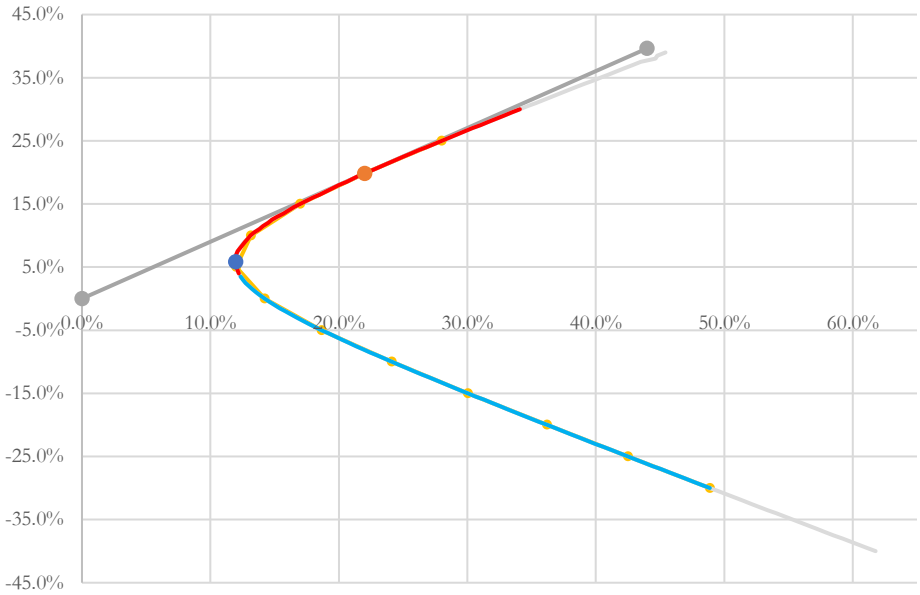


Figure 4. A Free Problem, Index Model - Constraint 4

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For (4) Not allowed to have any short (IM):

$$w_i \geq 0, for\ \forall i \tag{11}$$

Table 4. Not allowed to have any short

IM	SPX	ADBE	IBM	SAP	BAC	C	WFC
Panel A							
Min Var (%)	83.6	0	0	0	0	0	0
Max Sharpe (%)	0	40.8	0	0	0	0	0
Panel B							
	TRV	LUV	ALK	HA	Return	StDev	Sharpe
Min Var (%)	16.4	0	0	0	7.8	14.6	0.53
Max Sharpe (%)	26.3	0	20.5	12.4	17.3	23.4	0.74

According to the Table 4 and Figure 5, each Min Variance of the stocks must be larger or equal to 0. For the portfolio choosing, a lot of the stocks are not chosen to cause of the constraint.

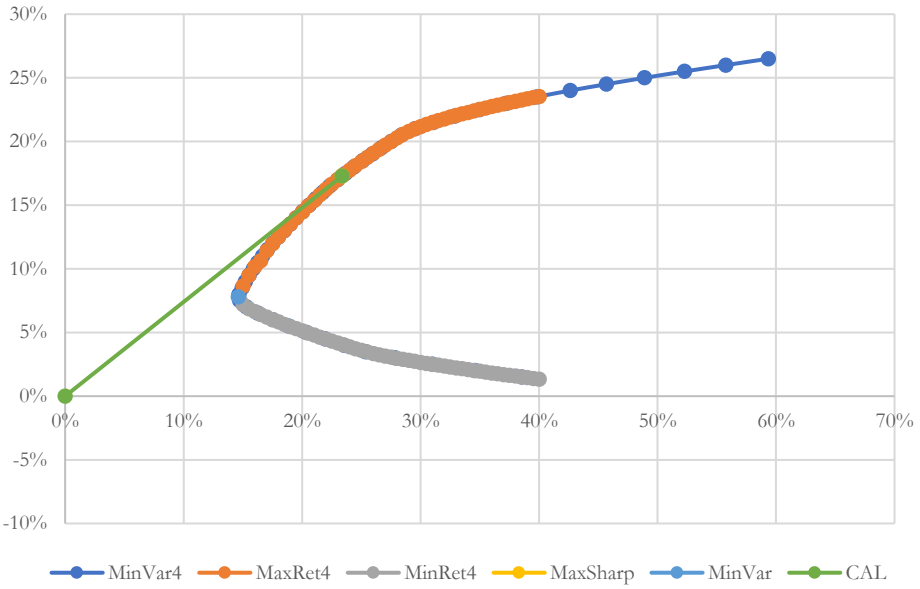


Figure 5. Not allowed to have any short, Index Model - Constraint 4

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For (5) Inclusion of the broad index (IM):

$$w_1 = 0 \tag{12}$$

Table 5. Inclusion of the broad index

IM	SPX	ADBE	IBM	SAP	BAC	C	WFC
Panel A							
Min Var (%)	0	2.7	29.1	0.6	-1.7	-10.1	15.2
Max Sharpe (%)	0	52.5	-20.6	8.6	0.8	-41.2	9.5
Panel B							
	TRV	LUV	ALK	HA	Return	StDev	Sharpe
Min Var (%)	50.4	9.1	5.4	-0.7	9.3	16.6	0.56
Max Sharpe (%)	42.1	8.5	25.2	14.5	23.8	26.7	0.89

According to the Table 5 and Figure 6, this constraint SPX can't be chosen for the investment portfolio, so all other stocks are chosen for constructing the portfolio.

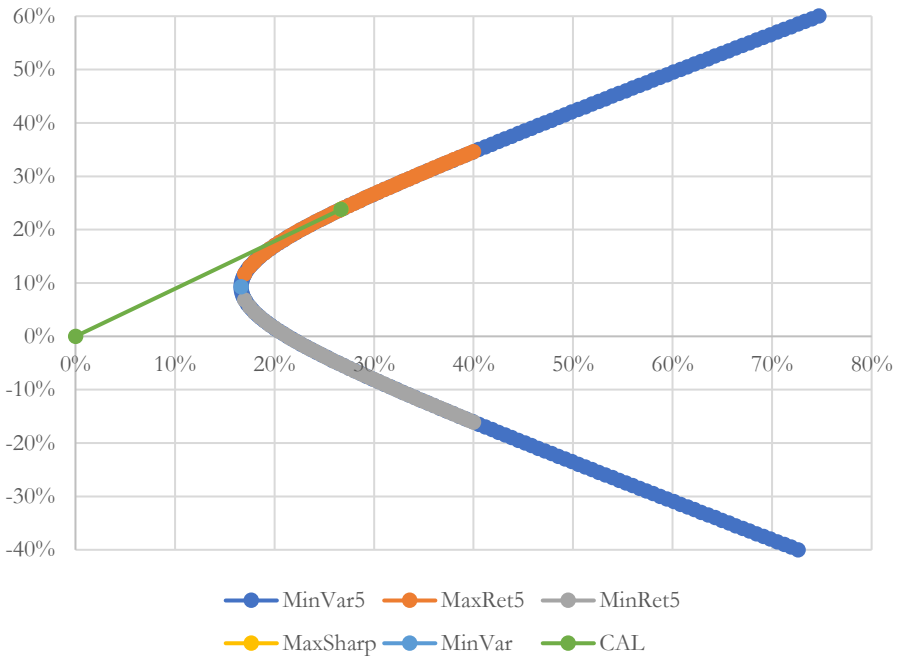


Figure 6. Inclusion of the broad index, Index Model - Constraint 5

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It's understood that every investor puts rewards first is a simplification. These results are solely the result of the underlying premises of the empirical investigation. The weights of constraint 2 could be preferred by a risk-averse investor since it imposed a weight limit on each stock. However, there are other investors who don't cleanly fall into the reward- or risk-seeking groups. These investors would choose a suitable strategy based on the efficient frontier shown in constraint 3, balancing it with their individual risk tolerance and investing preferences. Constraints 1 through 4 may be appropriate for some investors who must adhere to certain guidelines.

6. Conclusion

In the empirical analysis, each stock data is analyzed to determine and set a portfolio's maximum sharpe ratio and minimal variance. The best investment portfolio for a given circumstance or set of investment preferences is then found using the index model and various limitations, and the best investment portfolio is then outlined. The findings demonstrate that an investor aiming to maximize return for a given risk can choose constraint 5, where the Sharpe ratio is the greatest and the SPX index is not utilized to build the portfolio. Where this portfolio has the stocks data here: SPX with 0 Min Var and 0 Max Sharpe, ADBE with 2.7% Min Var and 52.5% Max Sharpe, IBM with 29.1% Min Var and -20.6% Max Sharpe, SAP with 0.6% Min Var and 8.6% Max Sharpe, BAC with -1.7% Min Var and 0.8% Max Sharpe, C with -10.1% Min Var and -41.2% Max Sharpe, WFC with 15.2% Min Var and 9.5% Max Sharpe, TRV with 50.4% Min Var and 42.1% Max Sharpe, LUV with 9.1% Min Var and 8.5% Max Sharpe, ALK with 5.4% Min Var and 25.2% Max Sharpe, HA with -0.7% Min Var and 14.5% Max Sharpe, with the largest return ratio of 9.3% of all constraints.

Investors desire to modify their holdings in accordance with various restrictions and situations to get various risk-return combinations. The findings demonstrate that the index methodology may successfully raise the weight of equities that boost returns. This aids investors in placing their funds in securities with favorable risk and return constraints. Notably, the effective portfolio beats the standard portfolio. The potential for bigger returns at higher risk or equivalent returns at lower risk as contrasted with a typical securities portfolio.

Investment is a topic, and the study shows that portfolio optimization is not perfect. Since there would be ongoing change in the real market, our analysis of historical data cannot predict how the

market would perform going forward. Investors should take a complete approach, taking into account the market forecast, their unique goals, their risk tolerance, and other elements.

However, investors may include these models into their investing strategies, which can truly enable them to perceive the expected hazards and help them to create a better balance between the risk and return more intuitively.

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