A study of sex ratio in lampreys based on Lotka-Volterra modelsour Paper's Title Starts Here: Please Center

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Abstract. To elucidate the interactions between changes in adaptive sex ratios of lampreys and ecosystems, we conducted the following studies; first, we used linear regression to fit the relationship between food availability and sex ratios. Based on vortex modeling and biological knowledge, we simulated three scenarios: natural disasters, species invasions, and data from similar species in normal ecosystems. This led to the modeling of lamprey population growth. To further investigate the effect of sex ratio changes on ecosystem stability, we built an n-dimensional Lotka-Volterra model. Combining the relationship between food supply and sex ratio, we identified the mechanisms by which sex ratio affects ecosystem stability.

Keywords: Lotka-Volterra; seven-gill eel; linear regression; vortex model.

1. Introduction

With the advancement of human civilization, public concern over environmental and ecological issues is increasing day by day. Lampreys is one of the two extant groups of ancient vertebrates that are ecologically important for ecosystem connectivity and food web productivity [1]. It is widely distributed in estuarine and coastal waters of North America, the European North Atlantic and the eastern Mediterranean. In North America (Great Lakes), the species has become a successful invader because of its ability to gain a foothold in the Great Lakes and cause destructive impacts on the entire aquatic community [2]. Approximately 500 Great Lakes streams are infested with sea lampreys [3]. On the other hand, some biodiversity conservation impacts have threatened sea lamprey populations in Europe.

Sea lampreys harbor many biological mysteries, making their control a challenging problem. Recent studies have shown that lampreys can change their sex depending on the availability of resources [4]. In this paper, we mathematically explore the effects of proportional changes in the sex of lampreys adaptations on the ecosystem. The aim is to deepen our understanding of the interactions between changes in the sex ratio of lampreys and ecosystems and to provide valuable insights for ecological management and conservation.

2. Ecological implications of adaptive sex ratio variation in lampreys

2.1. The effect of food availability on the sex ratio of lampreys

According to the related research, the percentage of males can reach approximately 78% of the population where food availability is low, while the percentage of males has been observed to be approximately 56% of the population where food is more readily available. Because food availability plays a dominant role in the sex ratio, to simplify the model, we do not consider the effect of factors such as temperature in the environment on the sex ratio.

Therefore, we can use linear interpolation to fit the relationship between food availability and sex ratio.

To begin with, we use to describe the availability of food. It can be written as

$$\alpha = \frac{F_S - F_D}{Max\{F_S, F_D\}} \tag{1}$$

Where F_D is referred as the amount of food in demand; F_S is referred as the amount of food in supply. With the data above, we can obtain this equation:

$$R = -0.11\alpha + 0.67 \tag{2}$$

Therefore, we can describe the effect of food availability on sex ratio as the Figure 1.

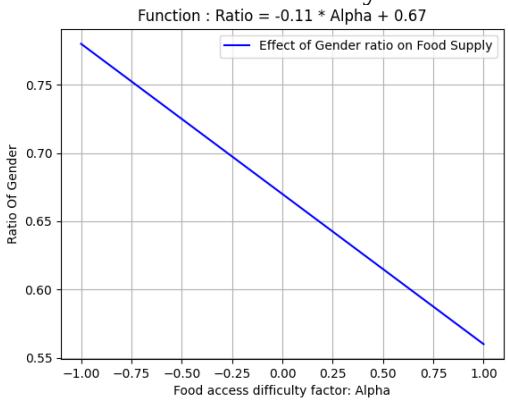


Fig. 1 The effect of food availability on sex ratio

2.2. Vortex-based modeling of population growth

2.2.1 Basic idea

The Vortex model, rooted in island biogeography theory and centered on an age matrix model, incorporates an Individual-Based Modeling (IBM) approach to describe individual migration. Through multiple simulations, it derives population outcomes under the combined influence of various deterministic and stochastic factors, demonstrating greater accuracy compared to deterministic population models. Brook et al. extensively validated and affirmed the accuracy and predictive capability of the Vortex model using a wealth of population ecology field data, providing substantial support for the widespread application of Vortex.

Therefore, the population growth model for lampreys based on the Vortex model can better align with the characteristics of lampreys, resulting in more accurate results.

2.2.2 Model building

The life cycle of sea lampreys can be broadly categorized into four stages, each exhibiting significant variations in reproductive and survival capabilities. Thus, we establish the lamprey age matrix based on the Leslie Age Matrix, as follows:

$$\begin{bmatrix} F_1 & F_2 & \cdots & F_i & F_{i+1} & \cdots & F_{n-1} & F_{n+1} \\ S_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{i+1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{n-1} & 0 \end{bmatrix} \tag{3}$$

Where F_i and S_i are the reproductive rate and the survival rate of biological individuals from age $i-1$ to age i, respectively. $i = 1, 2, \cdots n$. n represents the maximum age at which lampreys normally survive.

Among them, the reproduction rate of individuals who have not reached the age of sexual maturity is 0. S_i can be expressed as follows:

$$S_i = 1 - M_i \tag{4}$$

M represents the mortality rate, the probability of an individual dying in a given period of time, which is equal to the percentage of the number of dead individuals in the population. M_i is the mortality rate of biological individuals from age $i-1$ to age i, which can be formulated as

$$M_{(t_1,t_2)} = \frac{\ln\left[\frac{e^{k_{t_2}} - e^{k_{t_0}}}{e^{k_{t_1}} - e^{k_{t_0}}}\right]}{t_2 - t_1} \tag{5}$$

$M_{(t_1,t_2)}$ represents the mortality rate from t_1 to t_2. For example, $M_{(0,1)}$ is the mortality rate from age 0 to age 1, that is M_1. K is the growth coefficient of lampreys; t_0 refers to the birth of the lampreys, and the equation of growth can be written as

$$W_t = W_\infty \left[1 - e^{-k(t-t_0)}\right]^b \tag{6}$$

Where W_t is the body length predicted; W_∞ is the maximum body length; b is coefficient of conversion between length and weight.

N_t presents the number of populations in overlapping part between generations, which can be obtained as

$$N_t = N_0 \cdot e^{rt} \tag{7}$$

Where N_0 is the initial population; t is the duration of simulation; r is the innate rate of increase, which can be calculated as

$$r = \frac{\ln(R_0)}{T} \tag{8}$$

Where R_0 is the net growth rate; T is the time spent on the first reproduction, which can be described respectively as

$$R_0 = \sum_{i=1}^{n} (F_i \cdot S_i) \tag{9}$$

$$T = \frac{\sum_{i=1}^{n} (i \cdot F_i \cdot S_i)}{\sum_{i=1}^{n} (F_i \cdot S_i)} \tag{10}$$

Where F_i and S_i can be obtained from the lamprey age matrix above.

2.2.3 Model solution

We simulate the dynamics of population size in three different scenarios: the first scenario simulated a situation in which food was plentiful ideally, the second scenario simulated an actual situation in reality incorporating various sources of randomness, and the third scenario simulated a situation in which food was very scarce.

More important, to better understand the influence of sex ratio variation in lampreys, we established control groups in three different scenarios. Typically, sex ratios in the biological realm are differentiated at birth, with a nearly equal distribution of males and females (1:1). Here, we apply the method of controlling variables, assuming that the sex inheritance in lampreys follows a similar pattern. Through comparative analysis, we investigate how the population's ability to adjust sex ratio impacts the population size.

Through the process above, we can figure out the results, and observe the population dynamics over time as the Figure 2:

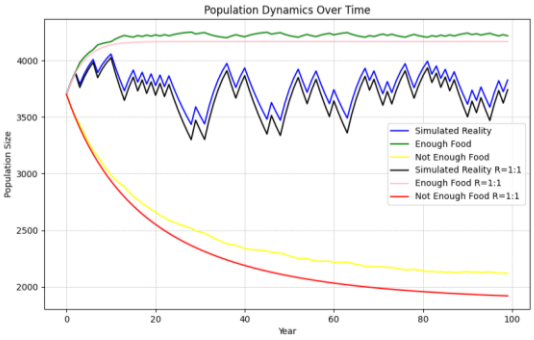


Fig. 2 Population Dynamics Over Time

Through the observation of the population dynamics curve, we can draw the following conclusions:

Under the condition of adequate food supply, the population size of sea lamprey first increase and then stabilize at the level of environmental capacity.

In the real world, the population size of sea lamprey fluctuates up and down around the horizontal line of the initial level.

In the presence of very inadequate food, sea lamprey population size continuously reduce first and then stabilize at a very low level.

Through comparisons, we observed that in various scenarios, the population size of lampreys with a 1:1 sex ratio is significantly lower than that of populations where sex ratio adjusts with environmental changes. This strongly indicates that the ability to adapt sex ratio is advantageous for the population development.

3. Impact on ecosystem stability

A Lotka-Volterra model was developed to analyse the effect of the sex ratio of sea lampreys on the stability of the ecosystem. Autonomous Lotka-Volterra ecosystems when n populations are in a mutually antagonistic relationship:

$$\frac{d\rho_i(t)}{dt} = \rho_i(t)\left(b_i - \sum_{j=1}^{n} a_{ij}x_j\right) \tag{11}$$

Where $\rho_i(t)$.denotes the density of population i at moment t, b_i>0, a_{ij}>0, and a_{ij}>0, (i≠ j),i,j=1,2,...,n.

Considering the system on this basis, the following conclusions can be drawn:

3.1. Impact 1

$$\begin{cases} b_k > \sum_{j=1,j\neq k}^{n} a_{kj}\frac{b_j}{a_{jj}}, \forall k < n, \\ b_n < \sum_{j=1}^{n-1} a_{kj}v_j^*, \\ \det\left(a_{ij}\right) > 0, \end{cases} \tag{12}$$

$$b_k = \sum_{1\leq j\leq n, j\neq k} a_{kj}\cdot\rho_j \tag{13}$$

There is a unique positive solution, which is that when the inequality is satisfied, for any positive initial value, ρ_j will go extinct, while other populations are able to persist.

3.2. Impact 2

If the condition is satisfied

$$(H_1) b_i > \sum_{j=1,j\neq i}^{r} a_{ij}\frac{b_j}{a_{jj}}, i = 1,2,\cdots,r \tag{14}$$

$$(H_2)b_k < \sum_{j=1}^{r} a_{kj}\rho_j^*, k = r+1, r+2, \cdots, n \tag{15}$$

Where $(x_1, x_2, \ldots, x_r)$ is a linear algebraic system.

$$b_i = \sum_{j=1}^{r} a_{ij}\rho_j, i = 1,2, \cdots, r \tag{16}$$

A positive solution , (H_3) of,exists for n - r positive constants such that

$$\begin{aligned} &max\left\{\left(\sum_{j=r+1}^{n} a_{ij}\alpha_j\right)\left(b_i - \sum_{j=1,i\neq j}^{r} a_{ij}\frac{b_j}{a_{jj}}\right)^{-1}, i = 1,2, \cdots, r\right\} \\ &\leq max\left\{\alpha_k a_{kk}\left(\sum^{r} a_{kj}\frac{b_j}{a_{ij}}\right)^{-1}, k = r+1, \cdots, n\right\} \end{aligned} \tag{17}$$

Then any solution satisfying the initial conditions of system (11) satisfies.

$$\rho_i(t) \to \rho_i^*, \text{ when } t \to \infty, i = 1,2, \cdots, r \tag{18}$$

$$\rho_k(t) \to 0, \text{ when } t \to \infty, k = r+1, r+2, \cdots, n \tag{19}$$

That is, it follows that $\rho_1 \ldots \rho_r$ populations persist and $\rho_{r+1} \ldots \rho_n$ go extinct.

3.3. Impact 3

If positive integer r<n, and for any positive integer k>r there exists an $i_k < k$, such that any j<k has

$$\limsup_{t\to\infty} \frac{\int_t^{t+\omega} b_i(s)ds}{\int_t^{t+\omega} b_{i_k}(s)ds} < \liminf_{t\to\infty} \frac{a_{kj}(t)}{a_{i_k j}(t)} \tag{20}$$

then the population is extinct for i > r. Also if there are

$$\liminf_{t\to\infty} \int_t^{t+\omega} \left[b_i(s) - \sum_{i\neq j}^{r} a_{ij}(s)\rho_{j0}(s)\right]ds > 0 \tag{21}$$

Where ρ_j is a positive outcome of the system, then for i<r part of the population is surviving.

In response to the above conclusions, we discuss only two populations, the sea lamprey and the hosts of the sea lamprey collectively referred to here as fish, for which a two-dimensional model is proposed to analyze the effect of the sex ratio of the sea lamprey on the stability of the ecosystem.

$$\frac{dx_1}{dt} = r_1 x_1\left(1 - \frac{\delta_3 x_1}{N_1} - \frac{\delta_1 x_2}{N_2}\right) \tag{22}$$

$$\frac{dx_2}{dt} = r_2 x_2\left(-1 + \frac{\delta_4 x_2}{N_2} - \frac{\delta_2 x_1}{N_1}\right) \tag{23}$$

Where δ_3, δ_4 are Logistic in order to respond to the presence or absence of self-blocking effect of x_1 and x_2, which take the value of 0 or 1. The significance of this is in order to respond to the presence or absence of dynamic equilibrium, x_1 is the predator, x_2 is the predator, r_1 is the relative growth rate of x_1, r_2 is the relative death rate of x_2, δ_1 is the relative coefficient of the predation of x_2 by x_1, and δ _2 denotes the efficiency of converted energy for x_2 predation on x_1 [5]. The model combined with the change of the sex ratio coefficient obtained from the previous problem, we can get that the sex ratio is related to the overall predation ability of the species and the efficiency of energy conversion.

The sea lamprey invaded the Great Lakes of North America, its size is much larger than the local lamprey and the production is also large, however, the host fish did not evolve to fight against it, resulting in a part of the host fish died in the sea lamprey attack, according to the survey shows that a sea lamprey can kill nearly 21 kilograms of fish, so the following analysis of the sea lamprey as a predator, will be predators that is, generalized to the fish. The following analysis of the three cases discusses the effect of sex ratio on the stability of the ecosystem.

Case 1:

When the sex ratio under the influence of δ_1, δ_2 does not change the number of the two populations, it is called the equilibrium point of the ecosystem, which is globally and gradually stabilised (in Lyapunov's sense), and the equilibrium point is located in the first quadrant, and it can be concluded from the Figures 3 and 4 that the two populations will be stabilised in the end in the course of the competition between each other, that is, the ecosystem of the seven-gill eel is in the ecosystem that is more similar to the ecosystem in which it is native. The stability of the ecosystem is conducive to the stability of the environment in which human beings live and produce, and it has the ability to resist external disturbances so that its own structure and function can be maintained in the original state, and can be quickly restored to its original state after being damaged by external disturbances.

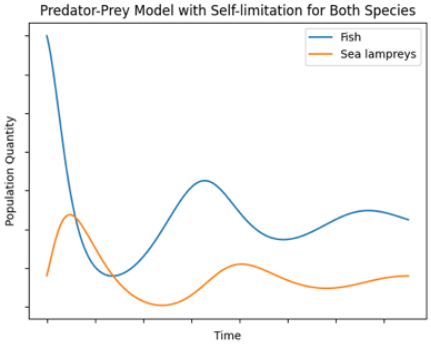


Fig. 3 Predator-Prey Model

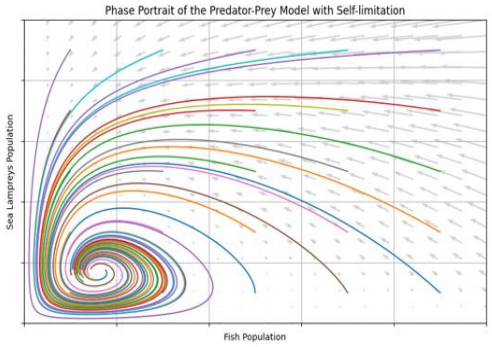


Fig. 4 Phase Portrait of the Predator-Prey Model with Self-limitation

Case 2:

When δ_3, δ_4 are equal to 0 means that there is human capture of the population, at this time the sex ratio changes, resulting in the two populations far from the maximum environmental capacity N_1, N_2, which is reflected in the population as shown in Figure 5.

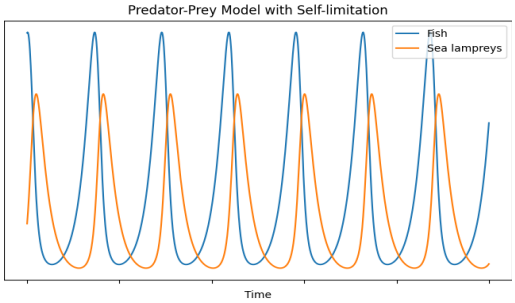


Fig. 5 Predator-Prey Model with Self-Limitation

The two populations form a reciprocal trend, reflecting the fact that the number of Sea lampreys is directly proportional to the growth rate of Fish in terms of its ability to prey on Fish, and inversely proportional to δ_1, and the number of Fish is directly proportional to the mortality rate of Sea lampreys, r_2, and inversely proportional to its ability to feed on predators, δ_2. This means that under the strong condition of weak meat, reducing the reproductive rate of Fish leads to a decrease in Sea lampreys, reducing the predatory capacity of Sea lampreys leads to an increase in it, the mortality rate of Sea lampreys leads to an increase in Fish, an increase in the capacity of Fish to feed Sea lampreys leads to a decrease in Fish, and the number of Fish is inversely proportional to his ability to feed predators R is inversely proportional to δ_2, and it is easy to conclude that the number of Fish decreases faster when the male share is larger.

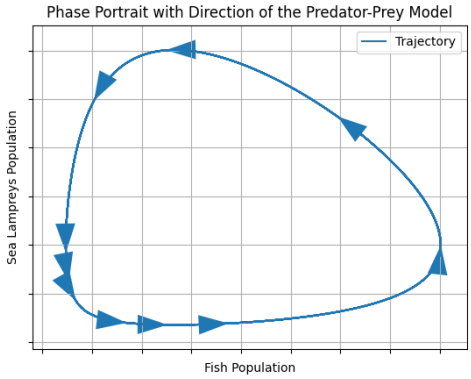


Fig. 6 Phase Portrait with Direction of the Predator-Prey Model

(More arrows in the phase diagram indicate more time spent, with one arrow for every hundred dots)

Looking at the Figure 6, the number of Fish is inversely proportional to their ability to feed predators δ_2, the less food there is, the more males there are, the more males there are, the more adaptable they are and the less food they need, so the bottom half of the diagram above (especially the bottom left corner) has more arrows than the rest of the diagram, which side by side reflects that males are more adaptable, and that females need more nutrients and prey more on the Fish. The lower half of the upper panel (especially the lower left corner) has more arrows than the rest of the panel, reflecting the fact that males are more adaptable, females need more nutrition and prey on Fish.

Changes in the sex ratio play a key role in regulating changes in its own population size, and this ability belongs to the self-regulation ability of the ecosystem, which can enable the ecosystem to maintain or restore its own structure and function in a relatively balanced state and enhance the stability of the ecosystem in the case of weak external disturbing factors.

4. Summary

We used the vortex model, a mathematical model commonly used for fish prediction in biology today, to simulate and predict lamprey populations by using specialized biological software to obtain prediction curves, which has the advantage of accounting for genetic mutations as well as other factors. We used cellular automata to compare the effects of different sex ratios on parasitic behavior. We used the Lotka-Volterra model to give n-dimensional scenarios and analyzed its dynamical system to give stability points as well as phase trajectory plots.

Due to the large number of species data required, some of the data we took instead of the same organisms, there may be errors here, but the results are still good after testing. After that we give the n-dimensional Lotka-Volterra model, which is simplified to a two-dimensional model in order to discuss the model more effectively, and there may be errors here.

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