A Study of Underwriting Decision Making Based on ARIMA Modeling with Composite Poisson Processes

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Abstract. Facing the problem of frequent occurrence of natural disasters, how to accurately predict the scale of loss and the probability of insolvency is the key for insurance companies to reduce risk and optimize underwriting strategies. This paper explores the multiple influencing factors of catastrophe risk and works to establish a practical risk prediction model. The model is divided into two parts: the first part uses ARIMA model to predict the risk loss, considering the influence of disaster factors, vulnerability, exposure, and disaster prevention ability on the risk size; the second part establishes a positive risk model based on the bankruptcy theory, and assumes that the insurance enrollment process is a composite Poisson process, and constructs a model of the claims process. The probability of insolvency for each year is estimated by substituting the predicted loss values into the claims model and calculating the initial and annual premiums. The study conducted a five-year risk prediction for the Philippines and Australia, and the results show that there is a significant difference in disaster risk management between the two countries, which provides a theoretical basis and practical guidance for subsequent underwriting recommendations.

Keywords: ARIMA, Bankruptcy Theory, Composite Poisson Process, Underwriting Decisions.

1. Introduction

Extreme-weather events continue to impact the lives of people around the world[1], and as the global economy grows, property owners and insurers alike are facing a crisis. In recent years, the insurance industry has seen an increase in claims for natural disasters compared to decades ago. At the same time, damages from extreme-weather events such as floods, dust storms, and droughts are expected to get worse, which will directly contribute to the rising cost of insurance coverage.

The insurance protection gap is widening worldwide. On the one hand, the reluctance of insurers to underwrite policies in areas prone to extreme weather events is reducing the profitability of the insurance industry, which continues to decline; on the other hand, even when insurers are willing to underwrite policies, the large premiums are causing a dilemma for policyholders. Thomas Blunck, CEO of Reinsurance and member of the board of directors of Munich Re, said that despite the fact that there were no extreme single losses globally in 2023, natural catastrophes continued to cause extremely high insured losses in the current year. The question of sustainability in the insurance industry is pressing[2].

For the purpose of modeling insurance underwriting decisions to predict risk losses and calculate probabilities of insolvency in order to provide a basis for insurance underwriting.

In this paper, the insurance process is abstracted as a composite Poisson process, a positive risk model is constructed based on the bankruptcy theory, and the instantaneous values of each year are predicted using ARIMA time series, and the probability function of bankruptcy (large loss) is determined by the characteristics of the Poisson process[3].

In this paper, regions located on different continents - the Philippines and Australia - are selected to demonstrate our model. The results of this paper's model demonstrated in two regions provide short-term insurance planning solutions for insurers in both regions.

2. The Basic Fundamental of Underwriting Decision Model

2.1. The Structure of Underwriting Decision Model

This paper abstract the claims process as a composite Poisson process and construct a positive risk model about time based on ruin theory. At the same time, the author learn the data of previous years and predict the future risk loss values by time series ARIMA (1, 1, 0), and only make short-term predictions due to the strong autocorrelation of the model. Combining the two previous methods, the author construct a risk probability function that predicts the annual probability of insolvency (large loss) by time, initial premium, annual premium, and payout ratio[4].

The author divide the decision model into two parts, the risk compensation prediction part based on the ARIMA model, and the positive risk prediction model constructed on the basis of bankruptcy theory and the composite Poisson process. The former will provide the corresponding compensation data for the latter, and we firstly predict the loss size through the time series, and then predict the probability of bankruptcy to provide a reference for the insurance company.

2.1.1. Components of Risk

The probability of disaster risk can be summarized as follows:

(1) From the perspective of risk, the magnitude of disaster risk can be interpreted as the risk of loss and uncertainty.

(2) We need to understand disasters from the perspective of the causal factors, and the magnitude of disaster risk is the probability of the occurrence of the causal factors

(3) We recognize the disaster risk as the result of the combined effect of the causal factors and vulnerability.

At the same time, considering human activities, the magnitude of a disaster is affected, and the specific role of human activities is to “amplify” or “mitigate” the impact of such a disaster. Adopting the four-factor theory, we define the probability of risk as a function of four indicators: Catastrophic Factors, Vulnerability, Exposure, and Disaster Prevention and Mitigation Capacity. As shown in Figure 1.

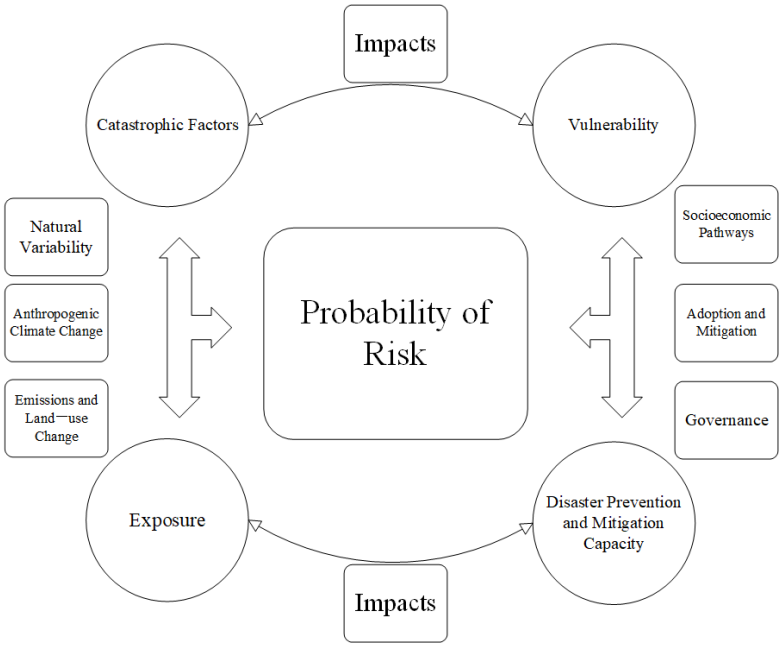


Figure 1. Schematic representation of risk composition

2.1.2. Positive Risk Model

Assuming that the insurance enrollment process is a composite Poisson process[5], the enrollment process is a series of counting processes, the strength of the individual processes is, and the traditional insurance surplus process is λ . Using this as a basis, the model of the claims process is

$$U(t) = u + ct - S(t) \tag{1}$$

Constructed by bringing in the value of the loss at risk predicted above. Where t is time, a is the initial insured capital, and c is the linear growth rate i.e. the amount insured per year, assuming that the amount insured per year remains constant.

The modeling of the claims process shows that the surplus accumulates at a rate c on the one hand, while on the other hand there are constant claims that require expenditures on property. Overall, the process is not a continuous process, but one that keeps jumping and changing, with the image trending upwards in terms of the long-term trend.

The paper use u, c and $\hat{c}$ for initial deposit and annual premium and percentage of loss covered. Consider the following equation

$$\begin{aligned} U(t) &= u + ct + \hat{c}(A(t) - B(t)) \\ A(t) &= \sum_{i=1}^{N_{X(t)}} X_i, B(t) = \sum_{i=1}^{N_{Y(t)}} Y_i \end{aligned} \tag{2}$$

In equation (2), $A(t)$ denotes the hazard mitigating factor and $B(t)$ denotes the hazard increasing factor of the four components, where $N_X(t)$, $t \geq 0$ is the Poisson process with parameter $\lambda_X > 0$,while $N_Y(t)$, $t \geq 0$ is the Poisson process with parameter $\lambda_Y > 0$.

$X_i, i \geq 1$ is a sequence of nonnegative independent random variables taken on the interval $(0, +\infty)$ with distribution function F_X, and mean value μ_X. Y_i, $i \geq 1$ is a sequence of nonnegative independent random variables taken on the interval $(0, +\infty)$ with distribution function F_Y, and mean value μ_Y.

This risky process equation is a right-continuous stochastic process that has smoothly independent increments and satisfies

$$E[A(t) - B(t)] = \lambda_X t \mu_X - \lambda_Y t \mu_Y \tag{3}$$

For a risky process $U(t)$, there exists a function $g(r)$ such that satisfies

$$E[\exp(-r(ct + A(t) - B(t)))] = \exp[tg(r)] \tag{4}$$

Where

$$\begin{aligned} g(r) &= -cr + \lambda_X[M_X(-r) - 1] + \lambda_Y[M_Y(r) - 1], \\ M_X(r) &= \int_0^\infty e^{rx} dF_X(x), M_Y(r) = \int_0^\infty e^{ry} dF_Y(y) \end{aligned} \tag{5}$$

Where $M_X(r) = \int_0^\infty e^{rx} dF_X(x)$, $M_Y(r) = \int_0^\infty e^{ry} dF_Y(y)$ are the moment generating functions of X and Y, respectively.

Due to the harnessed nature of the function g(r)[6], there exists a unique positive solution to the equation $g(r) = 0$, denoted R, and called the moderating factor. The final expression for the risk probability is obtained as follows

$$\phi(u) = \frac{e^{-Ru}}{E[exp(-RU(t) \mid T < \infty)]} \text{-1}, \quad u \geq 0 \tag{6}$$

Where the probability a of the investment going bust (large loss) is a function of the initial premium u, where R is the moderating factor, and the constraints $T < +\infty$ indicate that this is a finite time process. E denotes the mean value of the corresponding indicator from the beginning to the end of the projection year.

Equation (6) provides a quantitative standard of comparison for calculating the short-term probability of risk of underwriting insolvency that can be intuitively calculated over a short period of time, and then provide recommendations to underwriters based on the magnitude of the probability of insolvency.

2.2. General Description of the Model

According to the law of Poisson distribution of the composite process, we can actually estimate the corresponding risk probability and the degree of harm, which provides a theoretical basis for our calculation.

After constructing the above model, we can determine the initial premium and the annual premium cost according to the level of development of different countries, and then we can estimate the probability of insolvency (huge loss) in each subsequent year, so as to qualitatively give the underwriting recommendation[7].

Specifically, we determine the year in which we will start the forecast, use the data of that year as a reference to give the initial premium, and then, based on the development of the selected region, give the annual premium and compensation ratio to be paid in each subsequent year, and analyze it by regressing the forecast on a yearly basis to give the probability of insolvency (huge loss) each year.

3. Demonstration Results of The Model in Two Regions

3.1. Selection of Research Subjects

From the perspective of how catastrophe insurance operates, the determination of catastrophe insurance-related indicators, such as the compensation ratio, principal and annual premium, is closely related to the country's geographic conditions, the degree of development and other factors, and in terms of the selection of the research object, we would like to select representative countries to analyze and compare, and to be able to highlight the differences between the two.

The countries this paper selected are the Philippines and Australia.

At present, the Philippines is a typical developing country, the economy has been rebounding in the past 20 years, and it is located in Asia, which experiences more natural disasters every year, so the amount of data is sufficient to support our analysis and ensure the accuracy of the prediction.

Australia, on the other hand, is one of the developed countries, with the highest economic and technological strengths in the world, a comprehensive insurance system, and is located in the southern hemisphere, with a variety of climatic conditions, and suffers from natural disasters every year, so the relevant data can support our analysis.

3.2. Analysis of Experimental Results

The author selected data from 2000 to 2022, prior to which the data are imperfect and highly volatile due to the state of the art. Considering that the data for the year 2023 is not perfect, we have projected a five-year period from 2024 to 2028.

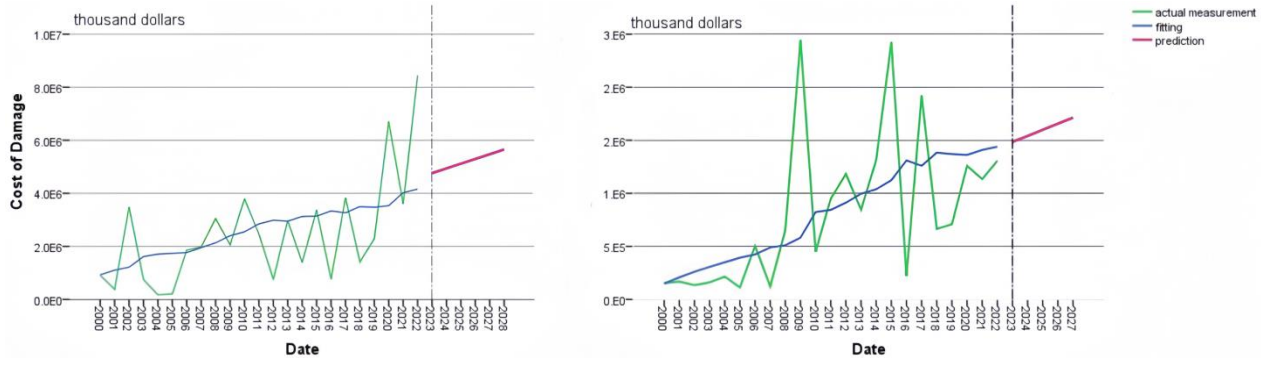


Figure 2. Loss prediction maps. Philippines on the left and Australia on the right

The author imported the time-series data of the previous years in SPSS software, and the images of risk loss forecasts for the Philippines and Australia are shown in Figure 2. Projections from the year corresponding to the dotted line. The actual curves show strong volatility in risk losses from year to year, and the fitted curves do not show strong seasonality or cyclicity, but the overall long-term trend of growth[8]. Depending on the country's exposure, the value of risk losses for the Philippines will grow from US, 1.541 trillion in 2024 to US 1.771 trillion in 2028, an increase of about the percentage of 15.0 Australia's loss at risk will grow from 4.761 trillion in 2024 to 5,654 trillion in 2028, an increase of about the percentage of 18.7. Considering the base of claims in the country as a whole, while the rise is not huge, the projected economic losses still pose a significant test for the nation's finances.

3.3. Characterization of Residual Plots

The autocorrelation function plots and partial autocorrelation function plots given by the SPSS software are as Figure 3.

The suitable fitting model is of ARIMA (1, 1, 0) type. From the distribution plots of the ACF and PACF functions, it is clear that the time-series data are stable and autoregressive after first-order differencing. That is, the differencing of one moment is only related to the differencing of the previous moment. From a realistic point of view, the factors leading to disasters, such as the causative factors, are functions of time, with large differences in different moments, so more consideration should be given to the approaching years when forecasting through time series, which also reflects the reasonableness of the forecasting period of five years.

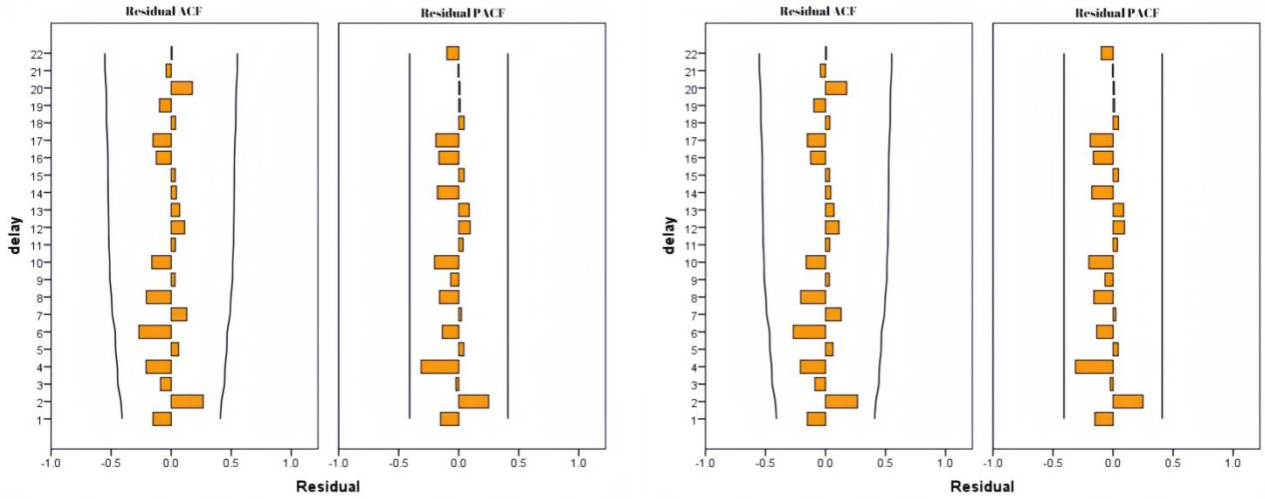


Figure 3. Data from the Philippines on the left and Australia on the right

For Philippines: Considering the economic development of the Philippines and past cases of catastrophe insurance payouts, the author delineates the relevant parameters as equation(7). And the model in this paper predicts the bankruptcy probability of the local insurance industry in the next five years as shown in the Figure 4.

$$u = 1.5, \hat{c} = 0.45, c = \begin{cases} 1.2, & \text{for plan 1} \\ 1.5, & \text{for plan 2} \\ 1.8, & \text{for plan 3} \end{cases} \tag{7}$$

To conclude: With fixed payout ratios and insurance principal, the probability of insolvency (large loss) decreases as the amount of annual coverage continues to increase, and with a risk probability of 34 percent until 2027 in Plan 3, we can conclude that the risk of implementing a short-term catastrophe insurance underwriting effort in the Philippines is acceptable with reasonably set annual premiums.

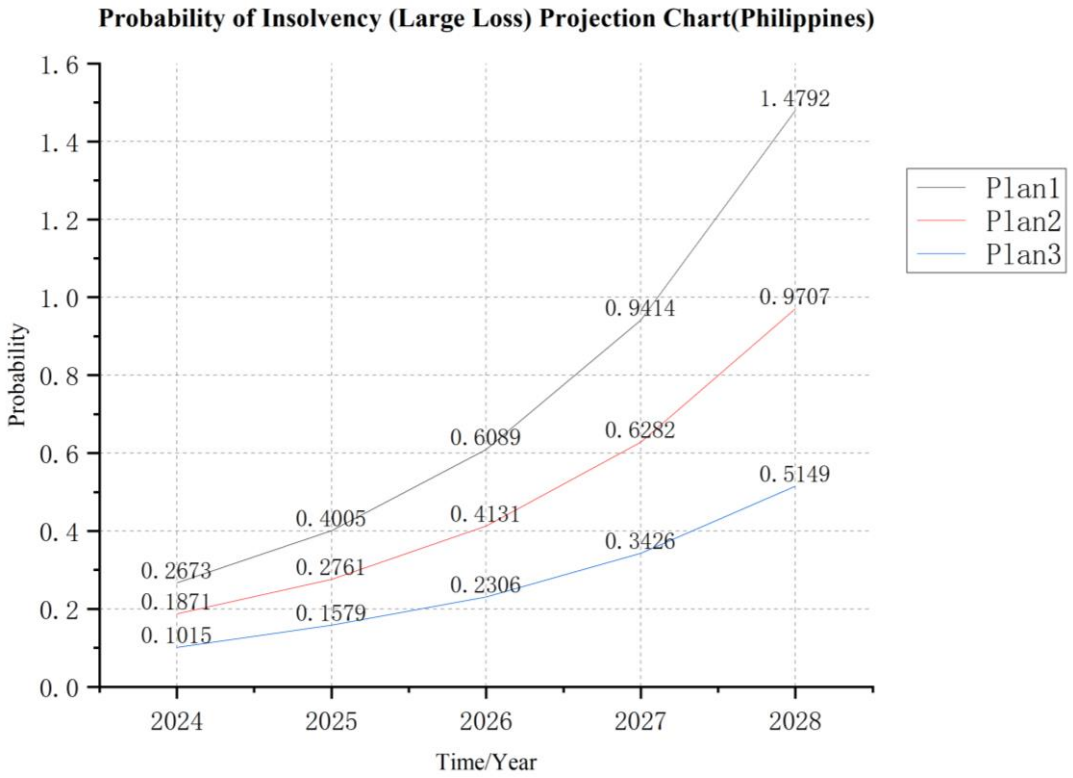


Figure 4. Philippines

And for Australia: Taking into account the developed situation in Australia, we have slightly adjusted the determination of each parameter, we have increased the price of the principal and the annual premium, and for the time being, slightly adjusted the fixed benefit percentage as equation (8). And the model in this paper predicts the bankruptcy probability of the local insurance industry in the next five years as shown in the.

$$u = 4.7, \hat{c} = 0.46, c = \begin{cases} 4.5, & \text{for plan 1} \\ 4.7, & \text{for plan 2} \\ 5.0, & \text{for plan 3} \end{cases} \tag{8}$$

To conclude: The model found a pattern similar to that of the Philippines, where raising the annual premium helps to reduce the probability of an insurer sustaining an insolvency (large loss) and can unilaterally ensure the growth of the insurer, and similarly, we determined through modeling that short-term (1-3 years) underwriting risk is acceptable.

Due to the differences in regional conditions and economic development status of countries in different regions, the change curve of the forecast model in this paper will be different. The specific insurance strategy needs to be determined according to the actual local situation. More specific insurance strategies are summarized in 3.4 Figure 5.

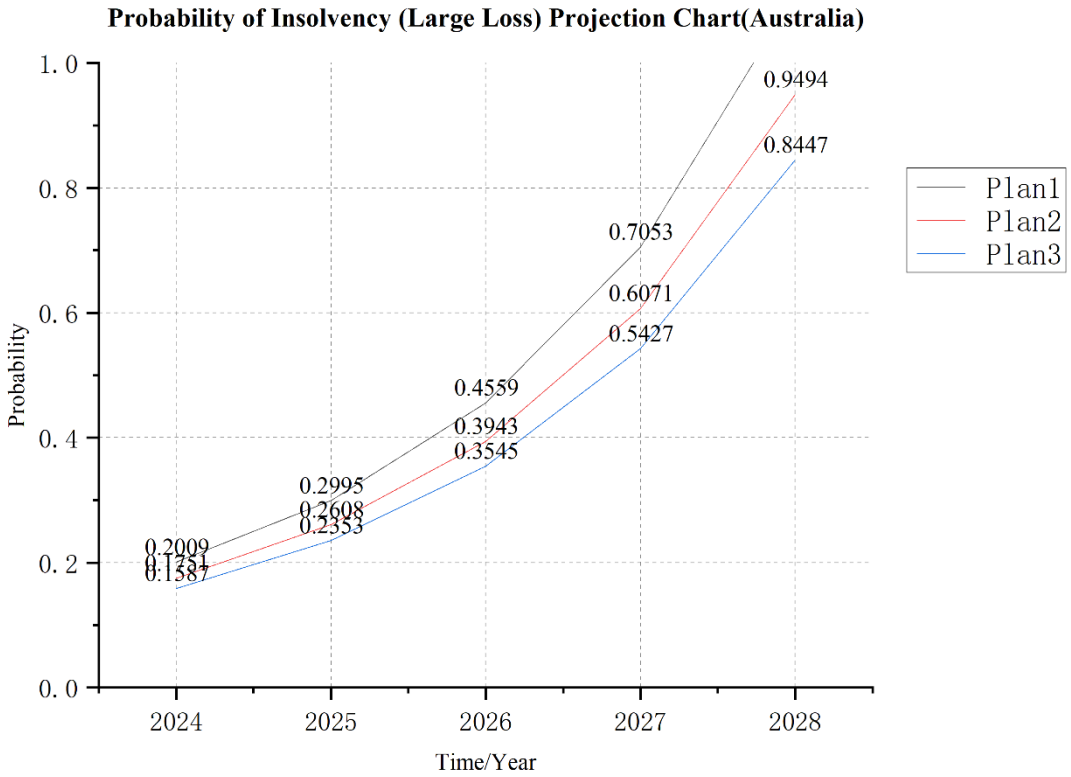


Figure 5. Australia

3.4. Findings

(1) Setting the initial premiums high helps to maintain a low risk of insolvency (large losses) over a short period of time by comparing the trends in the probability of risk for both from 2024 to 2026, but as the time lengthens, the probability of insolvency (large losses) for both reaches a high level, which suggests that the environmental conditions are deteriorating over time, and therefore the underwriting program needs to be adjusted at short intervals for longer-term underwriting[9].

(2) The projections are made from the perspective of the insurer and not from the consumer's point of view. The sustainability of the development is not well ensured if premiums are raised for this reason, and in our subsequent work, the author will consider the three variables of the initial premium, the annual premium, and the benefit ratio to find out the ratio of their utility in order to obtain the optimal solution[10].

4. Conclusions

This paper developed a widely applicable assessment model for determining the magnitude of the probability of a region's future risk from extreme weather hazards, and obtained a sufficient amount of data to support the analysis through the World Hazard Data Management (WHDM) repository.

For the construction of the model, the authors abstract the claims process as a composite Poisson process and construct a positive risk model in accordance with bankruptcy theory. At the same time, the authors predicted the size of disaster loss in the next few years through time-by-series ARIMA (1, 1, 0), and combined the two, introduced the reconciliation coefficient, and finally fixed the expression of risk probability.

Considering that the insurance industry has a strong relationship with the economic level of the country, the authors chose the Philippines and Australia, which represent developed and developing countries on the one hand and are subject to more extreme weather disasters on the other hand, as a way of demonstrating our model and giving specific recommendations, i.e., investing in the maintenance of the property or purchasing insurance separately to diversify the risk.

In the model, the property owner is not in a completely passive position, and the author have taken into account the protective factor of human activity in the model's components, so that the insured, in order to ensure that their interests are not jeopardized as much as possible, can carry out regular maintenance and repair of their insured buildings and other property, thus reducing the severity of the damage caused by the deterioration of the environment.

At the same time, the insured can classify the insured wealth according to our predicted development law, insure a percentage of the property in different insurance companies, and choose short-term insurance, so that the insurance company does not need to bear too much risk, which is conducive to the negotiation of the reduction of premiums, and at the same time, can protect the insured's property.

The economic loss data of natural disasters provide the basis for the prediction of this model. The demonstration examples in the two regions of this paper provide different underwriting strategy suggestions for regions with different levels of development, which has certain feasible value. Considering that there are many unstable factors in natural disasters, and even some regions may face the risk of not being hit by disasters for many years, it is difficult to guarantee the robustness of results in long-term prediction. Future research directions can focus on improving the accuracy of natural disaster prediction to ensure the reliability of model results.

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