Operations Management of A Single-Period Inventory Model

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Abstract. Single-period systems: the decision is just a one-time purchasing decision where the purchase is designed to cover a fixed period of time and the item will not be reordered. e.g., the newsvendor and Cash Reserve Plan. The newsvendor tries to decide how many newspapers to stock on the newsstand before observing the demand.

Keywords: “Overage Cost and Underage Cost”; “Optimal Order Decision”; “Cash Reserve Decision”.

1. Introduction

The newsvendor tries to decide how many newspapers to stock on the newsstand before observing the demand.

Decide how many newspapers to put on the newsstand before observing demand. That newsvendor faces the cost of over- and under-subscription if he orders too many or too few copies, he faces the cost of over- and under-subscription.

2. Profit Model

If the daily purchase of newspapers is too little, not enough to sell, will make less money; if the purchase of too much, cannot sell out, will lose money. Therefore, there is an optimal amount of purchase that makes the most profit. Therefore, the purchase quantity should be determined according to the demand.

Cost price C, selling price p. Sell one copy and make p-c dollars. Buy x copies at the beginning, so you have spent cx dollars, but these x copies are not necessarily sold, and how many you buy depends on the demand, which is d. So, the money earned is p multiplied by the decimal of d and x. Then the net profit is the money earned minus the cost.

(1) COST=C.X, SALE=P.d

We want to maximize so,

$$\pi(x)=P\cdot E\min\{x\cdot d\}-c\cdot x$$

(2) CO=C CU=p are the overage cost and underage cost .

We want to minimize the cost

$$C(x)=C\cdot E(x-d)+ +PE(d-x)+$$

(3) Write models for maximizing profit and minimizing loss and prove that they are equivalent. The equation that maximizes profit is expressed as a function that minimizes loss, so maximizing profit is equivalent to minimizing loss.

Let c be the unit cost, p > c the selling price and s < c the salvage value. If x units are ordered, then min (x, d) units are sold

Let Ed= μ , var d=σ^2

$$\pi(x)=p\min(x,d)+sE(x-d)^{*}+-cx$$

We use the fact that min (x,d)=d-(d-x)*+ we can writer the expected profit as:

$$\pi(x)=(P-C)\mu-G(x)$$

$$G(x)=CE(x-d)^{+}+(p-c)E(d-x)^{+}$$

So, they are equivalence.

3. Optimal Order Decision

It is convenient to think of C-S as the per unit overage cost and of P-S as the per unit underage cost. Cause without loss of generality, we assume zero salvage value and that excess demand is lost.

Sometimes the underage cost is inflated to take into account the ill-will cost associated with unsatisfied demand. Then allow the π(x)=(P-C)μ-G(x)maximizing π(x) as that of minimizing the expected overage and underage cost G(x) then G(x) can be written as G(x) = E[g(x − d)]. Since g is convex and convexity is preserved by linear transformations and by the expectation operator it follows that G is also convex. Cause the distribution of d is continuous, we can find an optimal solution by taking the derivative of G and setting it to 0. Since we can interchange the derivative and the expectation operators, G (x) = CP r(x − d > 0) − (p-c)P r(x − d > 0).Setting the derivative to zero reveals that,

$$F(x) = pr(d \le x) = \frac{p-c}{p-c+c} = \frac{p-c}{p}$$

4. Cash Reserve Plan

Redemption is a financial product that will be paid to the purchaser at maturity Principal and interest, Redemption of funds is uncertain and banks need to determine the level of cash reserves for each WIP to ensure that they have enough cash to repay investors if they want to redeem their products.

So, this means that a fixed order quantity needs to be set to prevent excessive holding costs and to prevent lost sales due to under-ordering.

$\frac{Cic}{Cfr}=\frac{1}{10}$, From newsvendor problem we know that the best ordering level is order $=\mu+\Phi^{-1}(\frac{Cu}{C0+Cu})\sigma$, $P=\frac{Cu}{C0+Cu}$.

So we have $P=\frac{Cu}{C0+Cu}=\frac{Cic}{Cfr}=\frac{1}{\frac{Cic}{Cfr}+1}=\frac{1}{11}$

By the graph we can see,

$$\mu = -1.415106,\ \sigma = 84.7487$$

Suppose it is normally distributed we have,

$$F^{-1}(\frac{1}{11})=\sigma\Phi^{-1}(\frac{1}{11})+\mu$$

So optimal cash reserve $=\mu+F^{-1}(\frac{1}{11})\sigma$.

Consider the "0" as a cut-off point, the part that exceeds it as "redemption" and the part that is less than it "the client does not redeem the funds, but instead makes further additions".

the potential unit opportunity loss (for investment) of the idle capital is Cic and the unit penalty cost resulted from the fail of redemption is Cfr. And $\frac{Cic}{Cfr}=\frac{1}{10}$.

If have too much cash in the bank, Will lose some of the opportunity loss, which means that the money is sitting in the bank and not generating any income.

The ratio of losses resulting from not being able to redeem in the event of a shortage of funds to losses resulting from a loss of investment income due to excess funds is 10:1. The former is much greater than the latter.

$$\frac{Cic}{Cfr}\uparrow,\ \ P\downarrow, F^{-1}(P)\downarrow$$

If keep the cost of penalty (Cfr) unchanged, while opportunity loss (Cic) decrease, it will lead to the increases of optimal reserve.

The best probability (p) decreases $\frac{Cic}{Cfr}$ as rises, leading to a decrease in the best ordering level ($F^{-1}(P)$).

So optimal reserve decrease.

$$\text{Similarly, if } \frac{Cic}{Cfr}\downarrow, \quad \text{then p} \uparrow, \quad F^{-1}(P)\uparrow$$

If keep the cost of penalty (Cfr) unchanged, while opportunity loss (Cic) increases, it will lead to the decrease of optimal reserve.

The best probability (p) rises $\frac{Cic}{Cfr}$ as decreases, leading to a rise in the best ordering level ($F^{-1}(P)$).

So optimal reserve increase.

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