Portfolio Allocation Optimization with US Equities

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Abstract. As a result of the worldwide pandemic, many industries are in need of growth stimuli, and the US equity markets, as the main growth engine, have intrigued investor interest. This article renders a performance analysis using the weights portfolio optimization given as a focal process that is skillfully implemented by financial practitioners. Each portfolio which includes Apple, IBM, Microsoft, Home Depot, Starbucks, NIKE, P&G, QUALCOMM, and JPM, gathers weekly return data from January 2017 to December 2020 and performs a Monte Carlo simulation. The amount of portfolio alternatives will be defined by the strategies being utilized, and it will choose ten stocks from a pool of 100 stocks to form a portfolio. Asset allocation strategies with Mean-Variance and Minimum Variance were constructed using the efficient frontier. Comparing a portfolio's asset allocation performances to that of the benchmark, the S&P 500 index, and an equally weighted strategy portfolio, with respect to asset allocation, the portfolio with the highest Sharpe ratio is the most optimal. The results in this study benefit investors and industry stakeholders in the post-covid era.

Keywords: Portfolio Optimization, US Stock Market, Asset Allocation Strategy, Sharpe Ratio

1. Introduction

Given that extraordinary worldwide epidemic and the largest economy's hyperinflation, the United States by definition enters a technical recession following two consecutive quarters of shrinking for GDP or gross domestic product [1]. The Federal Reserve reacted by saying that the five most historic Federal Fund rate rises according to St.Louis FED —rise in each FOMC meeting in a single year— had not been seen since 1981 or in the previous 41 years [2]. The US economy was going through a crisis. The American stock market has suffered. As a result, optimizing the portfolio allocation has become an even more pressing issue for investors seeking to reduce risk while boosting revenue.

The proposed mean-variance model, one of the most effective models to address the portfolio management issue, was based on the Modern Portfolio Theory developed by Harry Markowitz [3]. This approach seeks to diminish variance, which is typically used to calibrate the risk bound by the expected rate of return [4]. The expected return and risk of the portfolio are calculated in this model and invested jointly. The risk-expected return profiles of all feasible asset permutations are plotted together to create "the efficient frontier," a clustered region with a positive-slope boundary. Based on various strategies, the efficient frontier produces two optimal portfolios. Equally weighted strategy and two risk-based strategies, namely the maximum Sharpe ratio strategy and the minimum variance strategy, are used in this study. The equally weighted strategy has the competitive edge of easy implementation since it is independent from the approximation of stock return or optimization [5]. Previous studies used it as a benchmark for evaluating the performance of other portfolios, such as DeMiguel et al. [5] and Haley [6], concluding that no single optimizing model beats the equally weighted portfolio in an elongated period. Thus, this study will include the naïve portfolio as the classic efficient strategy. The maximum Sharpe ratio strategy applies the well accepted metrics which used to measure the asset allocation [7]. Theron and Vuuren [8] drew a comparison and concluded that the cumulative return of the maximum Sharpe ratio portfolio is ideal at most of the time among 4 strategies. The minimum variance strategy meets the assumption for risk management caused by the financial crisis [9]. Given that reliance on nothing but the covariance matrix, the estimation risk of the minimum variance portfolio weights is also found to be lower than other portfolios weights. Alternatively, estimation of 3 other portfolio weights utilized both the matrix and the expected returns [10]. The result revealed that the minimum volatility portfolio performed the worst during the study

period. Last but not least, this study generated a robustness test to demonstrate the validity of the conclusion.

This paper is structured into different sections: Section 2 lists the 10 stocks selected for the study as well as the data. The model methodologies and optimal portfolios are described in Section 3. The findings of the robustness test are presented in Section 4. And Section 5 will reach the conclusion and discussion.

2. Data

In this study, cross industries components' top 10 representative stocks are chosen based on market capitalization from Yahoo Finance, respectively AAPL.O, DIS.N, HD.N, IBM.N, JPM.N, MSFT.O, NKE.N, PG.N, QCOM.O and SBUX.O are the ticker for the ten equities. Adjusted closing prices for the period of January 4, 2017, through December 2020, are gathered and divided into a training data set and a test data set. The efficient frontier is built by calculating the average return and covariance matrix using the training set. The test set compares the cumulative returns of the chosen asset allocations to the return of the Index in order to assess how well they performed. Table 1 lists the fundamental details of the ten stocks which were selected. According to Table 2, the coefficient of each two components is greater than 0 and around 0.5. MICROSOFT CORP and APPLE INC present the highest correlation coefficient of 0.746 while the lowest correlation coefficient is 0.361 between PROCTER & GAMBLE and QUALCOMM INC. Figure 1 demonstrates the highest stock price $ 279.80 which conforms to HOME DEPOT INC whereas the lowest stock price is $ 27.14 of APPLE INC.

Table 1. Selected listed stocks

	Company
AAPL.O	APPLE INC
DIS.N	WALT DISNEY CO
HD.N	HOME DEPOT INC
IBM.N	International Business Machines Corp
JPM.N	JPMORGAN CHASE & CO
MSFT.O	MICROSOFT CORP
NKE.N	NIKE INC
PG.N	PROCTER & GAMBLE Co
QCOM.O	QUALCOMM INC/DE
SBUX.O	STARBUCKS CORP

Table 2. Correlation coefficient matrix of selected stocks

	AAPL	DIS	HD	IBM	JPM	MSFT	NKE	PG	QCOM	SBUX
AAPL	1.000									
DIS	0.450	1.000								
HD	0.576	0.526	1.000							
IBM	0.492	0.529	0.588	1.000						
JPM	0.489	0.633	0..591	0.624	1.000					
MSFT	0.746	0.498	0.656	0.572	0.542	1.000				
NKE	0.488	0.503	0.565	0.591	0.530	0.532	1.000			
PG	0.441	0.381	0.504	0.482	0.396	0.501	0.389	1.000		
QCOM	0.555	0.380	0.511	0.414	0.440	0.567	0.396	0.361	1.000	
SBUX	0.504	0.583	0.586	0.541	0.599	0.574	0.534	0.439	0.429	1.000

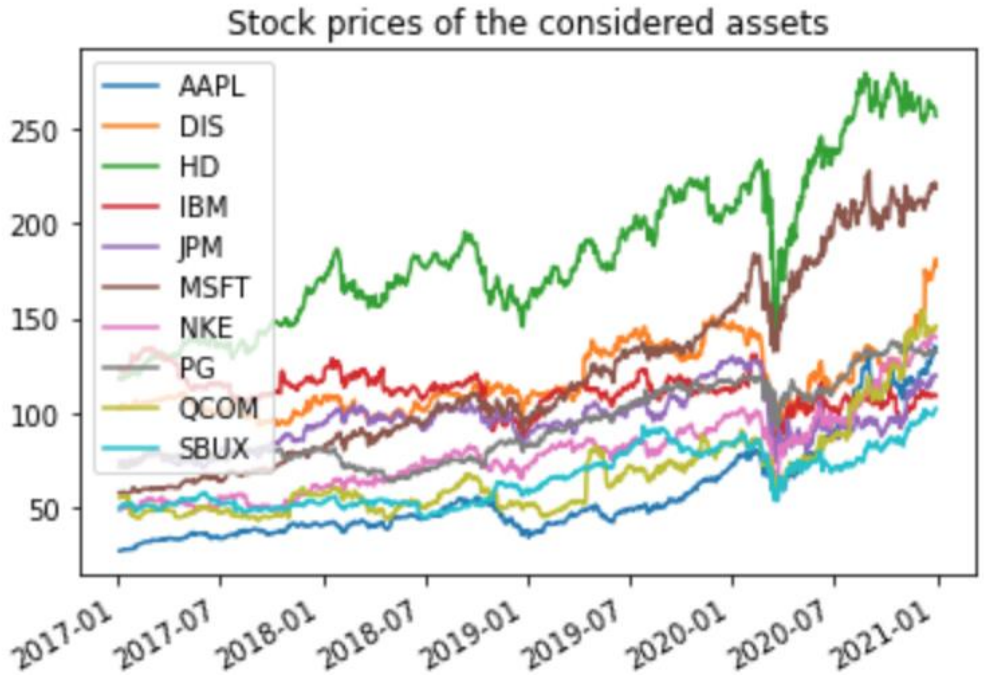


Figure 1. Prices of selected stocks

3. Methodologies

Diversification is a strategy that has existed for a while and makes a lot of sense. However, it wasn't until 1952, when Harry Markowitz's renowned article on portfolio selection was published, that the actual theory underlying this fundamental idea and its application to investments became apparent [3]. The article is the bedrock on which today is referred to as modern portfolio theory (MPT). The key finding of MPT is that investors should not only own portfolios but also pay attention to the correlations for the individual securities in those portfolios. The correlation coefficient function is defined as:

$$Cov(R_A, R_B) = \sum_{i}^{n} \sum_{j}^{n} P(R_{A,i} - R_{B,j})(R_{A,i} - ER_A)(R_{B,j} - ER_B) \quad (1)$$

$$\rho R_A R_B = \frac{Cov(R_A, R_B)}{\sigma R_A \sigma R_B} \quad (2)$$

Where $\sigma R_A, \sigma R_B$ are the standard deviations and $\rho R_A R_B$ is the correlation coefficient between asset A and B. Ostensibly, covariance demonstrates level of movement while correlation coefficient enables relative comparison among a number of assets. The mean variance model looks at how risk and expected return on investment are balanced. Following are the portfolio return and variance:

$$\mu P = \sum_{i} w_i \mu_i \quad (3)$$

Of which μP is the expected return of the i weighted portfolio. To build several portfolios, the Monte Carlo simulation is used. The average return on stocks and the covariance matrices were utilized in this study as parameters. This technique assists in simulating the operation process as random ones under specific conditions and using supplied parameters. One can determine the expected return and the portfolio's variance under the weights by stochastically generating weights for each asset in the portfolio. The data pool would be reflective of the most likely asset allocations and could be used to draw the efficient frontier by repeatedly simulating 1 million portfolios in the same way.

Three classic strategy portfolios are examined in this work: the equally weighted portfolio, the maximum Sharpe ratio portfolio and the minimum volatility portfolio that can be identified from the efficient frontier. According to research, investors like to distribute their funds evenly among different assets, and the simplest equally weighted portfolio is frequently chosen [11]. Take this into consideration while choosing an asset allocation strategy. For calculating and analyzing the risk-adjusted return on investments, the Sharpe ratio is frequently utilized [12]. The calculation looks like this:

$$Sharpe\ Ratio\ = \frac{R_p - R_f}{\sigma P} \tag{4}$$

Thus, the Sharpe ratio indicates the excess return for each unit of total risk taken.

4. Results

Under a distinctive aspect of Monte Carlo simulation, the paper creates the number of 100,000 random samples from a given set of stocks in order to depict the role of risk in the system. Figure 2 the scatterplot demonstrates the efficient frontier with a dotted blue line and 10 distinctive markers under the assumption of selection of a single stock with no diversification. Each portfolio under the efficient frontier has a corresponding portfolio having higher return located on the efficient frontier given an identical volatility level. Area beyond the efficient frontier is not achievable.

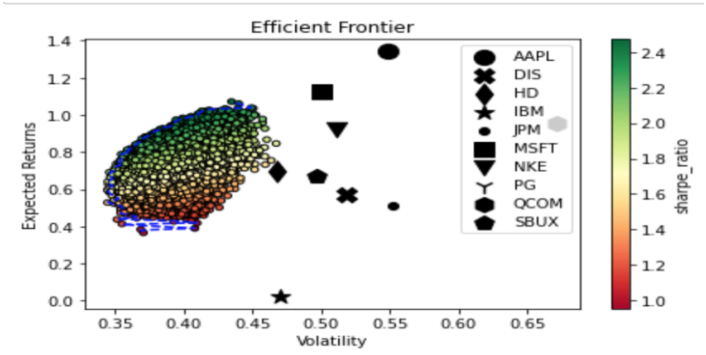


Figure 2. Monte Carlo results and Efficient Frontier

Combinations on the upper edge of the scatter plot have a higher Sharpe ratio. The optimal portfolios in terms of the well-performed risk-return trade-off can be found on the efficient frontier. Two portfolios are produced shown on Figure 3. Along with showing how a diversified portfolio strategy lowers risk. And furthermore, Figure 3 demonstrated how significant portfolio composition is. Maximum Sharpe ratio portfolio locates on the right end of the efficient frontier. The asset allocations of Maximum Sharpe ratio portfolio are: AAPL.O: 24.21%, DIS.N:0.20%, HD.N: 4.53%, IBM.N: 0.09% , JPM.N: 4.10%, MSFT.O: 30.42% , NKE.N : 26.70%, PG.N: 8.09%, QCOM.O: 0.97% and SBUX.O: 0.67%. APPLE.INC, MICROSOFT CORPS AND NIKE INC possess 81.33% of the weight in maximum Sharpe portfolio. The performance of returns is 102.23% and the volatility is 41.34% for maximum Sharpe portfolio and the Sharpe ratio is 247.29%.

Alternatively, the Minimum volatility portfolio, marked on the left end of the efficient frontier, comprises of AAPL.O: 2.45%, DIS.N:0.39%, HD.N: 13.41%, IBM.N: 12.77%, JPM.N: 0.80%, MSFT.O: 3.20%, NKE.N: 7.37%, PG.N: 38.71%, QCOM.O: 4.85% and SBUX.O: 16.05%. PROCTER & GAMBLE Co, HOMEDEPOT INC and STARBUCKS CORP make up 68 percent of the weight in Minimum Volatility portfolio and the performance returns is 59.72% and the Sharpe ratio is 173.18% for the Minimum volatility portfolio. Also, the minimum volatility portfolio demonstrates a lower volatility of 34.48% given the volatility of maximum Sharpe portfolio equals 41.34%. WALT DISNEY CO gained petty weight in both maximum Sharpe portfolio and minimum volatility portfolios, the weights are accordingly 0.20% and 0.39%. International Business Machines Corp which accounts for 0.09% in maximum Sharpe ratio portfolio shifts significantly to 12.77% in minimum volatility portfolio (Details are shown in the following Tables 3 and 4, respectively).

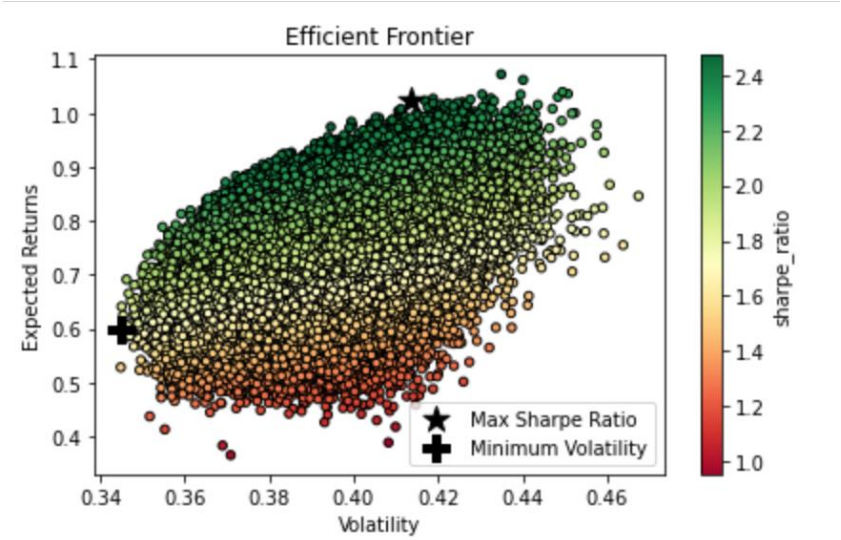


Figure 3. Maximum Sharpe ratio portfolio & minimum volatility portfolio

Table 3. Asset allocations in max Sharpe and min volatility portfolios

	Maximum Sharpe portfolio	Minimum Volatility portfolio
AAPL.O	24.21%	2.45%
DIS.N	0.20%	0.39%
HD.N	4.53%	13.41%
IBM.N	0.09%	12.77%
JPM.N	4.10%	0.80%
MSFT.O	30.42%	3.20%
NKE.N	26.70%	7.37%
PG.N	8.09%	38.71%
QCOM.O	0.97%	4.85%
SBUX.O	0.67%	16.05%

Table 4. Performance of the optimal portfolios

	Returns	Sharpe Ratio	Volatility
Maximum Sharpe	102.23%	247.29%	41.34%
Minimum Volatility	59.72%	173.18%	34.48%

Once the two asset allocations have been determined, the second stage, the portfolio return computation, can be completed. Using the test set from 1 January 2021 to 1 February 2021 and the stock weights, the cumulative returns can be calculated. Benchmark is a measure of the S&P 500's performance over the same time period. Table 5 illustrates how the study's results outperformed the

general market. Comparing these returns against the benchmark's -1.11%, the maximum Sharpe ratio portfolio's cumulative return is -0.794%, the least volatility portfolio's is -4.95%, and the equally weighted portfolio is -2.3% (shown in Table 5). The returns of the minimum volatile portfolio had the worst return performance ranking among the four strategies, while the cumulative return of the maximum Sharpe ratio portfolio skyrocketed over the last 20 days and exceeded benchmark even in negative returns.

Table 5. Performance based on 4 strategies

	Max Sharpe	Min Volatility	Equally Weighted	Benchmark
Returns	-0.794%	-4.95%	-2.3%	-1.11%

This paper conducted a robust test to produce the optimal portfolios given two replaced assets. The substitute assets are CITIGROUP and NVIDIA which supplanted JPMORGAN CHASE & CO and WALT DISNEY CO. Repeat the Monte Carlo simulation and the cumulative return computation next. Alternatively, 1000,000 random combinations have been generated while the optimal portfolios locate on the upper edge of the efficient frontier. Accordingly, the weights for NVIDIA are 9.36% and 0.42% in maximum Sharpe ratio portfolio and minimum volatility portfolio. Given that the erstwhile top large weight is 30.41 % for MICROSOFT CORP in Table.4, the large weight in Max Sharpe portfolio shown in Table.7 is 24.31% for APPLE INC. After a substitute, the highest weight in the Minimum Volatility Portfolio after substitution is 37.20%, up from PROCTER & GAMBLE's previous percentage of 38.71%.

Gather together all the returns from the three portfolios and contrast them with the benchmark index at the conclusion. The findings, which are supported by preceding data and demonstrate that the least variance portfolio has the lowest cumulative return, are shown in Table 6 and Table 7. As a result, the procedure and the results are indeed dependable and effective.

Table 6. Asset allocation with 2 stocks supplanted in the two optimal portfolios

	Max Sharpe	Min Volatility
AAPL.O	24.31%	4.99%
NVDA.O	9.36%	0.42%
HD.N	0.90%	9.48%
IBM.N	1.34%	13.85%
C.N	0.41%	2.76%
MSFT.O	19.62%	3.12%
NKE.N	24.19%	11.09%
PG.N	11.91%	37.20%
QCOM.O	7.44%	3.39%
SBUX.O	0.54%	13.71%

Table 7. Retentive portfolio performances

	Max Sharpe	Min Volatility	Equally Weighted	Benchmark
Returns	-1.490%	-5.14%	-2.3%	-1.11%

5. Conclusions

In summary, this paper investigated three portfolios (equally weighted, minimum variance, and maximum Sharpe ratio) comprising ten selected US stocks. Using Yahoo Finance data, the cumulative return of the stocks over a specific time period can be calculated. Then, using the Monte Carlo simulation, one hundred thousand random portfolios were simulated, and the two optimal portfolios return, and volatility were calculated. The data was visualized by plotting the portfolio return versus the volatility, and the efficient frontier was found. The asset weights in the two best portfolios were

known beforehand. The cumulative portfolio returns, as well as the equally weighted portfolio, were accessible using the test set data. When compared to the benchmark S&P 500 index, the minimum variance portfolio had the lowest return. And the market index is difficult to beat because of its diversity in the downside market. The highest positive correlation coefficient between APPLE INC and MICROSOFT CORP, 0.746, accounts for the most weight in the maximum Sharpe ratio portfolio. Apparently, the maximum sharpe ratio strategy appears to provide the best return. Because of the trade-off between portfolio risk and portfolio return, the Maximum Sharpe ratio portfolio has the highest volatility.

Before the ramifications of the pandemic, the minimum volatility strategy can barely outperform the classic equally weighted strategy. Even though all of the portfolios have negative returns, and the macroeconomic investment environment is typically not ideal, the significant downside risk that comes with a wider range of industries is mitigated by switching to passive managed index portfolios, which are representative across cycles. Thus, there is sufficient evidence to support the conclusion that, given the information in the argument, an index fund is a suitable asset allocation strategy while confronting downside risk.

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