Effectiveness and Limitation of Markowitz Mean-variance Model: Evidence from Hang Seng Index

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Abstract. As China's economy has expanded rapidly, its financial system has gradually improved, new financial businesses have evolved, and new theories and technologies have also been developed. The Markowitz mean-variance model, the cornerstone of asset pricing theory, is utilized as an example in this paper to demonstrate how the Markowitz portfolio theory may be applied to assess a portfolio of five stocks in the Hang Seng data, a significant indicator of the Hong Kong stock market. Empirically, the optimal portfolios with the highest Sharpe ratios and lowest variances are identified. Their expected returns, standard deviations, and Sharpe ratios are then evaluated by comparing them to equal-weight portfolios and giving the effective frontier. The author further demonstrates the guiding significance of Markowitz's portfolio theory in the Chinese investment market through the assessment of the findings. Finally, in the context of the Chinese securities market, identify and analyze the model's constraints and offer suggestions.

Keywords: Mean-Variance model; Quantitative Investment, Hang Seng Index; Effective Frontier.

1. Introduction

The laws of the financial environment are diverse due to a variety of factors, a mature financial environment always has some constant laws, so investors can summaries these laws and principles and use them to guide a series of decisions in order to reap the corresponding or excessive economic benefits. Although investors can also summarize the laws of the market through data collection and analysis, the simple application of logic and rules for profit necessitates the selection of a realistic investment model, and the quantitative investment studied in this paper is a solid option. Compared to typical active investment choices, the advantages of the quantitative investment approach are more obvious.

The core of quantitative trading is essentially the precise customization of a targeted investment strategy using knowledge from a range of fields, including finance, statistics, and computational geometry. Humans are inherently emotional beings, and a variety of circumstances can affect any choice, especially in the case of investment work, where the surroundings of investor and the thoughts of others may have an impact on the investor and have an impact on the decision and result in a failed investment. The first benefit of quantitative investment is that it improves the objectivity and rationality of investment decisions, effectively solving the issue of how human psychology affects investment decisions.

Quantitative investment requires a comprehensive analysis of historical and current market data and information, based on modern statistical and mathematical methods, to construct a quantitative model, which is then used to give objective recommendations on trading behavior. This method of investment management prevents investors from making emotional investment decisions and avoids the influence of human variables, thus ensuring the rationalization and objectivity of the investment process. This is since there is no active investment and the investor simply needs to closely adhere to the mathematical model of the investment. Additionally, as well since quantitative investment can act quickly and make decisions, it can seize the market's fleeting opportunities, respond to market changes quickly to complete transactions, etc. These would help investors harness the power of the most reasonable purchasing and selling opportunities promptly and improve the overall standard of quantitative investment.

In China, the investigation of quantitative investment actually started in 2004, and several quantitative funds gradually began to appear on the market. Due to the U.S. financial crisis, several Wall Street quantitative investment experts returned to China one after another in 2009, bringing with them cutting-edge quantitative investment strategies from abroad, it was during this period, that the multi-factor stock selection strategies gradually emerged in China. The introduction of the financing and securities system in 2010 helped domestic quantitative investment grow even further. From 2013 to September 2015, before the new stock index policy, this time could be regarded as the most scenic period in the history of Chinese quantitative funds. Since September 2015, the market has started to shift from low-yield, low-risk arbitrage hedging strategies to long-short strategies and long equity strategies [1]. Due to the relatively late start and lack of competition, China offers a favorable environment for the growth of quantitative investing. In recent years, With China's economy expanding fatly, various financial industries have sprung up, particularly the stock industry is the financial choice of most investors, it is crucial to research how portfolio investment theory could be applied to stock investments in order to promote the healthy evolution of the investment market [2].

An investor's portfolio is made up of the various securities they are simultaneously investing in, such as stocks, bonds, certificates of deposit, etc. A portfolio is more than just a random collection of assets; it represents the investor's objectives and the limitations placed on him or her, such as the importance of investment returns, how to distribute investment proportions, risk preferences, etc[3].

In this regard, the western modern portfolio theory Markowitz portfolio theory guides research, The American economist Markowitz was the one who initially methodically proposed this hypothesis in 1952, he clarified the essential concepts and techniques of securities return and risk analysis and developed the mean-variance portfolio model's fundamental framework., Since then, the Markowitz portfolio theory has undergone rapid enrichment, expansion, and improvement [4]. Portfolio theory's practical applicability was considerably aided by William Sharp's 1964 proposal of a single-factor model that could be used to optimize the estimation of the covariance matrix [5]. In the 1960s, Sharp, Lint, and Merchant put forth a model named the Capital Asset Pricing Model in 1964, 1965, and 1966, respectively [6]. In 1976, Ross put forth a replaceable capital asset pricing model, the Arbitrage pricing theory model, to address the shortcomings of the Capital Asset Pricing Model in terms of untestability [7]. This approach is completely responsible for the widespread adoption of multi-index portfolio analysis techniques in investment practices. [8].

However, the CAPM and APT models have been the subject of numerous empirical researches by Chinese academics. For instance, Shi Donghui conducted the first empirical study on China's stock market using the CAPM model and indicated that there was an essential adverse correlation between β and stock profits [9]; Ruan Tao and Lin Shaogong conducted an empirical test using a sample of 40 stocks in Shanghai from 1996 to 1998 and pointed out that the CAPM model was not suitable for the Shanghai stock market [10]. Li Chia-rui exposed the shortcomings of the APT model by empirically testing it on the Shenzhen stock market in China [11].

In conclusion, the empirical findings suggest that neither model is appropriate for China's stock market. Therefore, based on previous studies, this paper employs Markowitz's Mean-Variance (MV) model to explore the application of quantitative investment in China's stock market. Starting from the Chinese stock market, several stocks that constitute the Hang Seng Index are analyzed by applying Markowitz's model to find the ideal portfolio of assets, which is empirically demonstrated and tested, and compared with the returns of equally weighted portfolios, providing helpful guidelines for practitioners to deal with portfolio selection issues.

The remaining parts of this paper are organized as follows: Part 2 introduces Markowiz Mean-variance model; Part 3 shows empirical results; Part 4 discusses limitation of Mean-Variance model; Part 5 is a summary of this paper.

2. The fundamentals of Markowitz Mean-Variance Model

In his 1952 article "Portfolio Selection," Harry Markowitz introduced probability theory and a quadratic programming method to the problem of portfolio selection. [4]. He created the basis for contemporary portfolio theory as a statistical problem, which was the beginning of contemporary portfolio management [12]. Markowitz's portfolio theory, which incorporates both the efficient frontier theory and the MV model, offers a way to assess the performance of a particular portfolio under the presumption that investors are risk averse and want to maximise expected returns.

The primary assumption of this theory is to treat the returns of single assets as random variables, and to quantify the investment return and investment risk by using the mean and variance, respectively. Finding the portfolio with the lowest risk at the equivalent level of return, the maximum return at the identical degree of risk, or maximizing the projected return at a specific level of risk is the objectives of investment decision-making. The portfolio situated on the efficient frontier satisfies both requirements.

2.1. Mean-Variance Model

Think about a portfolio with n various asset classes, where asset number I equals 1, we

2 ...n, will give the returnR_i. And ωi is a weighting factor for asset i. To preserve the budget constraint, it needs that the sum of the weights to1:$\sum_{i=1}^{n} \omega_i = 1, \omega_i \geq 0$

$$\mu = E(R) = \sum_{i=1}^{n} \mu_i \omega_i \sum_{i=1}^{n} R_i \omega_i \quad (1)$$

$$\sigma^2 = Var(R) = \sum_{i=1}^{n} \sum_{j=1}^{n} Cov(R_i, R_j) \omega_i \omega_j \quad (2)$$

$$Cov(R_i, R_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} \quad (3)$$

M is the mean of the portfolio composed of n assets, R is the expected return of the portfolio, σ^2 represents the variance of the portfolio composed of n assets, i represents different assets, $Cov(R_i, R_j)$ reflects the covariance between asset I and asset j, an indicator used to determine how closely two assets' returns are linked; ρ_{ij} represents the correlation coefficient between asset j and asset j, which can be used to compare the magnitude of correlation between two assets; σ_i andσ_j subscales represent the standard deviation of two assets.

The risk of the portfolio, according to this model, is primarily influenced by the investment weight of each stock, the correlation between various securities, and the standard deviation of each asset. Therefore, when building a portfolio to lessen investment risk, we should choose to select assets with smaller variance and lower correlation coefficients between two assets. The authors use the return and sample variation of historical return data to estimate future return and risk in the empirical evidence that follows.

2.2. Efficient Frontier

For different choices ofω_1 $\omega_2 \ldots$, ω_n the investor will get different collections of μ andσ^2. The set of all possible (σ^2, μ) pairings is called the attainable set. And the efficient set (or efficient frontier) is made up with (σ^2, μ) with minimumσ2 for a given μ or more and maximum μ for a given σ^2 or less. Its shape is similar to the following figure 1. Where the points on the left boundary of the effective frontier are called global minimum variance combinations.

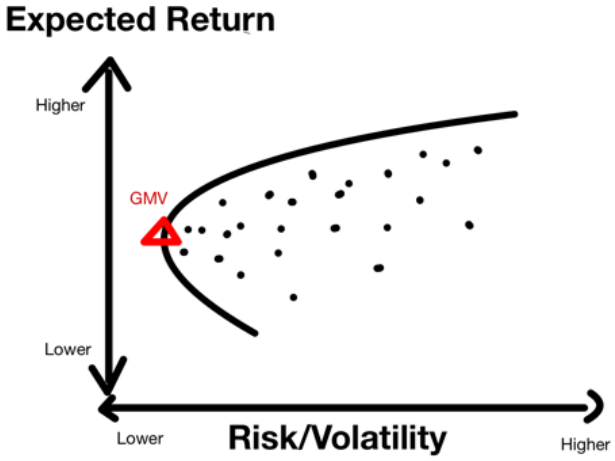


Figure 1. Efficient Frontier.

Each dot represents a portfolio, the investor should select a portfolio that provides a (σ^2, μ) combination in the efficient frontier if they aim to get a high return (profit) and a low risk and seek to maximize μ and minimize σ^2 .

2.3. Sharpe Ratio

Sharpe ratio, commonly referred to as Sharpe index, is a standardized metric for assessing the performance of investment funds. The Sharpe ratio has been studied by modern investment theory to show that a portfolio's success is greatly influenced by the level of risk. To eliminate the detrimental effects of risk variables on performance evaluation, taking into account both return and risk, the risk-adjusted return is a composite measurement. The Sharpe ratio is one of the three conventional metrics that accounts for both return and risk [13].

An established principle of investing is that the investor can carry a greater risk of volatility when the underlying investment's projected return is larger, and vice versa;the lower the expected return, the lesser the risk of volatility. To pursue the greatest benefit at the most fixed risk they are willing to tolerate, or to pursue the least amount of risk at the maximum fixed anticipated reward, is the main goal of rational investors when selecting investment targets and portfolios. The excess expected return of a portfolio divided by the total standard deviation is known as the Sharpe ratio.

$$\text{Sharpe Ratio} = \frac{E(R_i) - Rf_i}{\sigma_i} \tag{4}$$

σ_i Is the standard deviation of the portfolio; Rf_i is the risk-free rate; $E(R_i)$is the expected return; The Sharpe ratio represents the potential excess return on the asset portfolio for each additional unit of estimated risk.

3. Empirical Results and Analysis

3.1. Data Processing

The research selects five stocks at random from the Hang Seng Index, a key gauge of prices on the Hong Kong stock market, after filtering out the stocks with relatively low risk. They are HSBC, China Construction Bank, Petro China, China Resources Beer Holdings, and Country Garden Holdings. , Firstly, using December 2009 to December 2021 Then, by taking the expected return, risk, and Sharpe ratio as input variables, the data are brought into Python software to obtain the return, risk, and Sharpe ratio of different weighted portfolios under maximization of Sharpe ratio and minimization of variance, the two optimization techniques, respectively, and compare them with the return of equally weighted portfolios. Finally, the effective frontier is visualized.

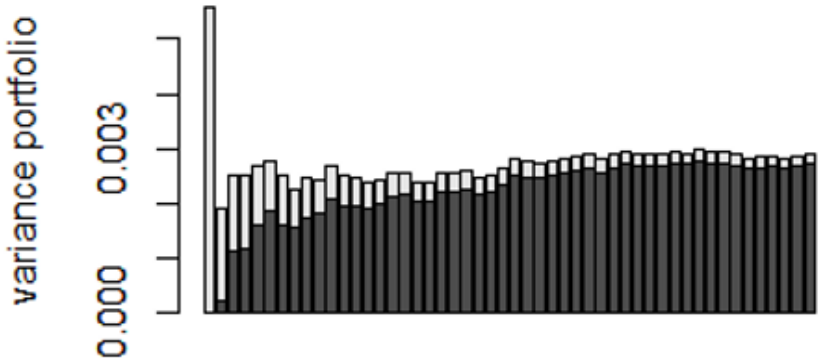


Figure 2. Risk Bar.

Start by removing all except 50 of the stocks with complete time series from the excel table 1. Then ascertain the return of these 50 equities, use the Fama-test on the return series, and obtain the subsequent number.

The light grey bar in this figure 2 represents the idiosyncratic risk while the dark grey part indicates systematic risk. And the figure 2 above shows that from the 26th bar, the dark grey part of that bar which represents the systematic risk gets very small with respect to before and stays almost unchangeable since then. So, it could indicate that a reasonable number of stocks in order to well diversify the idiosyncratic risk from a portfolio in our case is 26. Then randomly select 5 stocks from the 26 stocks; HSBC, China Construction Bank, Petro China, China Resources Beer Holdings, and Country Garden Holdings

Price of the 5 stocks is as follows:

Table 1. Price of five stock.

Date	HSBC	Petro China	China Construction Bank	China Resources Beer Holdings	Country Garden Holdings
Dec-21	46.9	3.47	5.4	63.85	6.92
Nov-21	42.95	3.39	5.08	63.8	6.88
Oct-21	47.15	3.76	5.29	64.5	7.31
Sep-21	40.95	3.69	5.58	57.5	8.05
Aug-21	41.45	3.41	5.62	64	8.6
Jul-21	43.05	3.24	5.42	58.15	7.6

The return line chart for the 5 stocks is as follows:



Figure 3. Return Line Chart.

Figure 3 indicates that China Resources beer and Country Garden Holding have negative returns during the period.

Table 2. Mean value of return.

HSBC	Petro China	China Construction Bank	China Resources Beer Holdings	Country Garden Holdings
0.0044	0.0067	0.0012	-0.0108	-0.0068

Table 2 demonstrates the mean value of return of five stocks. It could be seen in the table that China Resources beer and Country Garden Holding have negative returns during the period, which is matched with the result of Figure 3.

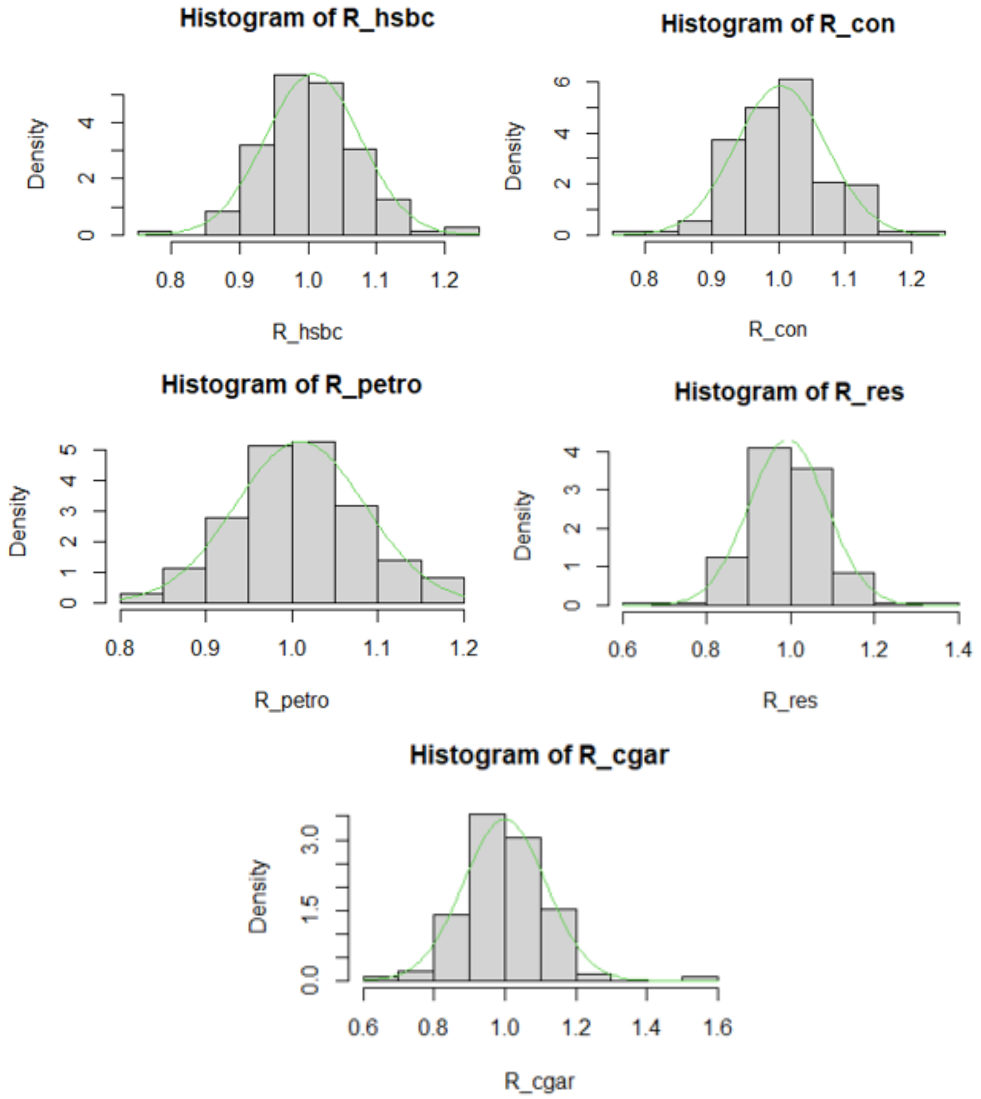


Figure 4. Normality Test.

Figure 4 shows that the returns of the above-mentioned stocks obey the normal distribution.

3.2. Optimization of portfolio

The steps of optimization are as follows: (1) Find minimum variance portfolio: Use optimization to minimize variance. The result indicates that weight for minimal variance portfolio is 34.4% HSBC, 29.7% China Construction Bank, 20.8% Petro China and 15.2% China Resources Beer. The return is 2%, standard deviation is 19.8%, and the Sharpe ratio is 10.2%; (2) Find Maximum Sharpe ratio portfolio: The result indicates that weight for the maximum Sharpe ratio portfolio is 24.5% HSBC and 75.5% PetroChina. The return is 7.5%, standard deviation is 23.1%, and the Sharpe ratio is 32.4%; (3) Equal-weighted Portfolio: The return is 0.28%, standard deviation is 26.65%.

3.3. Efficient Frontier

The return data are given in, and then use Python to solve the optimization issue and obtain the combination of return and standard deviation of the model utilizing the return and variance data of each stock. Run the efficient frontier charting function to construct the efficient frontier of the ideal portfolio using the model volatility as the horizontal axis and the expected return as the vertical axis (Figure 5).

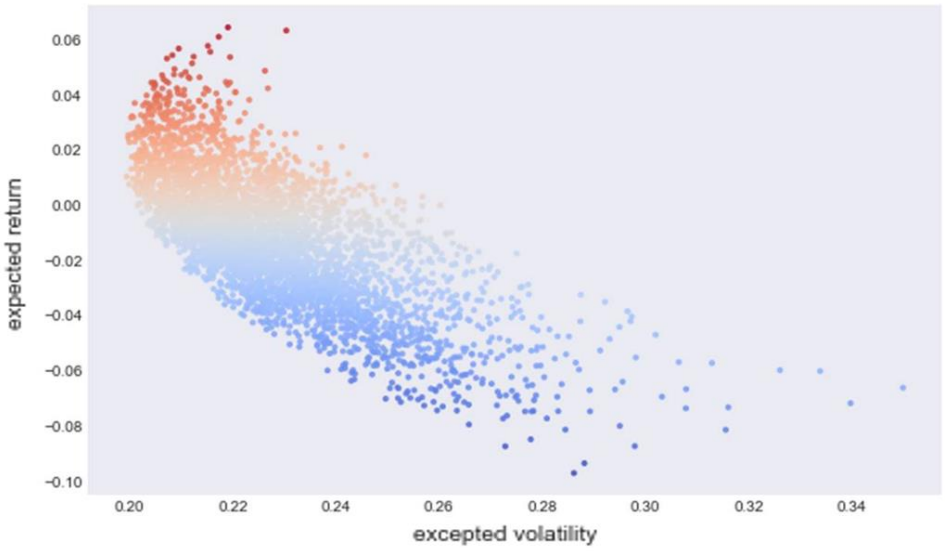


Figure 5. Efficient Frontier of Portfolio.

4. Discussion

The expected return of each security, the variance, and the covariance matrix between the stocks are used by the Markowitz portfolio model to determine the portfolio's efficient frontier. The investor's utility non-difference curve is then used to establish the perfect portfolio. The benefit of this model is that it is incredibly straightforward and only calls for the computation of the sample mean and covariance matrix in order to determine the ideal portfolio weights.

However, although the mean-variance model has good performance in the absence of parameter uncertainty, in practice, since the real model parameters can only be estimated from the data, the resulting estimation error problem greatly reduces the effect of MV model [14]. Especially in the Chinese market, the validity of the MV model is substantially impacted by the presence of parameter uncertainty, which can easily lead to poor model performance. Second, there are currently more stock suspensions in the Chinese stock market. Certain exclusions should be made when utilising the Mean-Variance model optimization for stocks with prolonged trade bans. [15].

Another drawback of the Markowitz model is that the variance of the portfolio is not a complete indicator of the risk taken by the investor. What is the value-at-risk of a given portfolio? This is impossible to answer if one only knows the variance and the mean but not the distribution. As a result, if an investor is ready to take a certain degree of risk of becoming bankrupt, the Markowitz model cannot tell them which portfolio they can afford [16].

This study has implications for guiding investment practice and further theoretical research in China. From the perspective of guiding investment practice, the mean-variance model can optimize the multi-objective optimization of return and risk to achieve the best equilibrium effect, and investment managers of Chinese fund companies and asset management companies can use the Markowitz mean-variance model for some reference in their investment management work, but should further consider whether to use the model when the parameters are uncertain.

5. Conclusion

From the comparison of the optimal portfolio, it is found that the maximum Sharpe ratio of the portfolio can reach 32.4%, standard deviation of 23.1%, for those investors who want to maximize the excess return per unit of risk taken, i.e., a portfolio consisting of 24.5% HSBC and 75.5% Petro China, and the expected return of the portfolio is 7.5%. For investors seeking to minimize investment risk, the optimal portfolio with the lowest variance should be selected, i.e., a portfolio consisting of 34.4% HSBC, 29.7% China Construction Bank, 20.8% Petro China and 15.2% China Resources Beer with the lowest standard deviation of 19.8%.

And from the comparison between the mean-variance model and the equally weighted portfolio, both the portfolio with the highest Sharpe ratio and the portfolio with the smallest variance have higher returns and perform considerably better than the equally weighted portfolio.

In conclusion, through the mean-variance analysis of assets, it could find an efficient frontier. The points on this efficient frontier are the optimal portfolios, which have the some feasures: The portfolio with the biggest return has the lowest risk based on level of return, and the least risk based on level of return. The optimal investment strategy determined by the mean-variance model is necessary and successful.

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