Elaborative Research between Markowitz Model and the Index Model

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Abstract. In today's world, most people are not satisfied with only one income stream, people are actively seeking more income other than salary. Thus, many of those choose to invest, as the most popular way for additional income. This paper is aiming to dig deeply into the method of optimizing the Asset portfolios. The result of the analysis is that the Markowitz model and Index model are returning a similar result on the analysis of 10 selected stocks, and both model favors the Travelers Companies Inc and SPX in most cases. It turns out that investors would love to put more weight on Insurance company since insurance business return would not be affected by market. This feature grants insurance business a relatively steady returns and dividend due to its low variance. Plus, Insurance companies can deliver solid long-term returns.

Keywords: Asset Portfolios; Markowitz model; Index model.

1. Introduction

An asset portfolio is a combination of multiple investments including stocks, real-estate, cryptocurrency, bonds, etc. A wisely built invest portfolio should have risk mitigation, diversification, systematic approach, higher immunity to external crisis, and great for capital building or passive income [1]. All those features would earn a higher possibility to gain more profit and loss less money from investing.

In recent years, many scholars have committed themselves into portfolio and Markowitz model researching. Some of the practical problems that will be encountered during the optimization process under the Markowitz model have been pointed out in those papers, which are (i) its lower out-of-sample performance than the naive rule, (ii) the resulting asset weights with extreme values, and (iii) the high sensitivity of those asset weights to small changes in the data [2-5]. Nevertheless, Markowitz Model is still widely applied when analyzing the optimal portfolio since the value of Markowitz model could not be overshadowed by its problem. Dedicating themselves into Markowitz model research, scholars are seeking for a method to improve the volatility of the optimal weights [2,6-8].

In 1990, A well-known economist, Harry Markowitz won the Nobel Prize for his theoretical contributions in the fields of financial economics [9]. His work laid the foundation for modern portfolio theory, which is that portfolio selection and construction should depend on the concept of maximizing expected returns while minimizing risk [9]. In this paper, A real case is presented to elaborate the Markowitz Model. For more information and comparison, we also apply the Index Model to complete the paper research. This comprehensive presentation of the complex mathematical models is linked with Markowitz Mean-Variance theory and using the solver table to figure out these intricate calculations.

The construction of this paper is as follows: Section 2 is the data of the 10 stocks that are chosen for calculation. Additionally, some necessary aspects of data are also shown in section 2. Section 3 articulates the method of modeling. Section 4 presents the detailed results of Markowitz Model and Index model under five disparate constraints. The last section draws the conclusion of this whole experiment.

2. Data

The data that will be analyzed in this article is derived from Yahoo Finance [10], a well-known media property that provides financial information, news, data, etc. 10 representative firms' stocks along with SPX500 are selected for this article. The daily data for these 11 stocks from past 20 years is collected and filtered into monthly data for future analyze. The information that are needed for the models are the annualized average return (AAR), annualized standard deviation (AS), beta(B), annualized alpha (AA), and residual standard deviation (RS). By calculation, the results are shown in the Table 1.

Table 1. Descriptive statistics of the selected stocks

	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC
AAR	7.33%	21.22%	11.30%	4.40%	-7.32%	11.03%	3.03%	5.35%	9.43%	8.32%	7.31%
AS	14.6%	30.4%	36.4%	39.4%	42.3%	61.9%	22.8%	31.2%	33.3%	19.7%	28.067%
B	1	1.39	1.07	1.64	2.02	1.55	0.99	1.06	1.46	0.81	1.08
AA	0.00%	11.81%	10.60%	0.24%	-11.21%	16.32%	-1.44%	0.15%	2.62%	3.67%	5.15%
RS	0.00%	20.34%	28.37%	30.38%	26.97%	54.28%	16.99%	32.39%	24.53%	9.09%	20.18%

Based on the analyzed data from the Table 1, it can be easily concluded that stock ADBE(Adobe) has the highest annualized average return for the past 20 years. Surprisingly, HA (Hawaiian Holdings) has the highest annualized standard deviation of almost 62%. Which indicates that there is a high risk for holding the stocks from HA. In addition, the correlation coefficient matrix of each stock is also calculated for Markowitz model, as shown on Table 2.

Table 2. Correlation Matrix

	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC
SPX	1.0000	0.1671	0.4350	0.4480	0.1431	0.4229	0.3520	0.5227	0.4271	0.3029	0.6684
ADBE	0.1671	1.0000	0.2727	0.2979	0.3988	0.3391	0.5126	0.2457	0.3608	0.3590	0.4288
ALK	0.4350	0.2727	1.0000	0.8273	0.3364	0.3096	0.4194	0.3339	0.4062	0.7649	0.6079
BAC	0.4480	0.2979	0.8273	1.0000	0.3391	0.4066	0.4251	0.4321	0.5116	0.7116	0.6977
C	0.1431	0.3988	0.3364	0.3391	1.0000	0.2428	0.4220	0.1245	0.2352	0.3577	0.3665
HA	0.4229	0.3391	0.3096	0.4066	0.2428	1.0000	0.3400	0.5822	0.3476	0.2737	0.6328
IBM	0.3520	0.5126	0.4194	0.4251	0.4220	0.3400	1.0000	0.2898	0.4037	0.4046	0.4970
LUV	0.5227	0.2457	0.3339	0.4321	0.1245	0.5822	0.2898	1.0000	0.3933	0.2970	0.6407
SAP	0.4271	0.3608	0.4062	0.5116	0.2352	0.3476	0.4037	0.3933	1.0000	0.3630	0.6007
TRV	0.3029	0.3590	0.7649	0.7116	0.3577	0.2737	0.4046	0.2970	0.3630	1.0000	0.5610
WFC	0.6684	0.4288	0.6079	0.6977	0.3665	0.6328	0.4970	0.6407	0.6007	0.5610	1.0000

Based on the correlation matrix, it is obvious that the correlation coefficient between C(Citigroup) and LUV (Southwest Airline) is minimum, and the correlation coefficient between ALK (Alaska Air Group) and BAC (Bank of America) is maximum.

3. Methods

In this paper, SPX 500 and ten representative stocks are selected for portfolio management. According to Modern Portfolio Theory, which holds the idea that a portfolio can reduce unsystematic risk. Securities and their weights together forms a portfolio, selection of uncorrelated securities should be the goal of portfolio construction [11]. And of course, a single security can also be considered as a special portfolio. Portfolio discussed in the article are limited to portfolios consisting of multiple stocks.

3.1. Markowitz Model

In 1952, Harry M. Markowitz, an American economist, first applied the mean and variance of asset portfolio returns in the academic paper "Portfolio Selection". The definition of investor preference and the explanation of the principle of investment diversification can be shown by two quantitative indicators, mean and variance [10]. This paper frame the problem of asset Portfolio and selection.

Stock price changes every day by market forces, which is stochastic, yet has a regular pattern at the same time. When dealing with the SPX500 and stock prices, it is necessary to apply the knowledge of probability theory to obtain correlations between eleven assets. In addition, correlation matrix is an essential feature of portfolio management. The correlation coefficient is calculated by the following equation,

$$r = \frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\overline{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i-\overline{Y})^2}} \tag{1}$$

where $\{X_i, Y_i\}$ are the annual returns of the eleven assets. The Markowitz model's portfolio return, and standard deviation is calculated as follows,

$$Expected\ return = \sum_{i=1}^{n} Weight_i * return_i \tag{2}$$

where $asset\,return_i$ is the annualized average return of $asset_i's$ monthly return,

$$Variance = \sqrt{\sum_{i=1}^{n} Weight_i^2 StDev_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} Weight_i Weight_j StDev_i\, StDev_j\, Cov(i,j)} \tag{3}$$

where $StDev_i$is the standard deviation of $asset_i$, and $cov(i,j)$ indicates the covariance between $asset_i$ and $asset_j$.

3.2. Index Model

Just like Markowitz model, Index model is also an asset pricing model. Index model was invented by William Sharpe in 1963 for the purpose of reducing the calculation process in Markowitz model [12]. The reason for the advent of the index model is that the algorithm of the Markowitz model is too complicated. Markowitz model requires the data from covariance, which is usually a large amount of work of calculation. Sharpe has simplified this process by relating the return of an investment to a single Market index. Thus, the analysis of Index Model will not be using the correlation matrix, but Beta, annualized Alpha, and residual Standard Deviation. The Index model's portfolio returns, and standard deviation is calculated as follows,

$$Expected\ Return = \sum_{i=1}^{n} Weight_i * Beta_i * return_M + \sum_{i=1}^{n} Weight_i * Alpha_i \tag{4}$$

where $Weight_i$ is the weight of $asset_i$ in a portfolio, $return_M$ is the market return, which will be the return of SPX 500 in this case,

$$Variance = \sqrt{\sum_{i=1}^{n}(Weight_i * Beta_i * StDev_M)^2 + \sum_{i=1}^{n}(Weight_i * ResStDev_i)^2} \quad (5)$$

Where $StDev_M$ is the market standard deviation, which is the standard deviation of SPX 500 in this case. The prerequisite of both model's calculations is,

$$\sum_{i=1}^{n} Weight_i = 1 \quad (6)$$

The Sharpe ratio is also a measurement of an asset portfolio's quality and is one of the three classic measures that consider both return and risk. This method measures the excess return per unit of risk. It can be calculated as follows,

$$SharpeRatio = \frac{E(R_p) - R_f}{\sigma_p} \quad (7)$$

where $E(R_p)$ is the expected return of the portfolio, σ_p is the standard deviation of the portfolio, and R_f is the risk-free rate.

4. Results

In this paper, 5 cases with additional constraints are designed considering various situations that may occur in real transactions.

Constraint1: This optimization constraint is designed to emulate FINRA's Regulation T, which allows broker-dealers to allow their clients to hold positions 50% or more of which are funded by the client's account equity. The results are shown in Table 3 and 4.

$$|\sum_{i=1}^{n} Weight_i| \leq 2 \quad (8)$$

Table 3. Weights of Each Asset of Markowitz Model under Constraint 1

Constraint1	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
MinVariance	0.74	0.06	-0.14	-0.05	0.02	-0.02	0.09	0.04	0.02	0.47	-0.23	0.07	0.11	0.69
MaxSharpe	0.05	0.65	0.22	-0.01	-0.49	0.21	0.02	0.11	0.14	0.09	0.00	0.25	0.33	0.76

Table 4. Weights of Each Asset of Index Model under Constraint 1

Constraint 1	**SPX**	**ADBE**	**ALK**	**BAC**	**C**	**HA**	**IBM**	**LUV**	**SAP**	**TRV**	**WFC**	**Return**	**StDev**	**Sharpe**
Min Variance	1.20	-0.11	0.00	-0.08	-0.18	-0.02	0.01	0.00	-0.09	0.30	-0.01	0.05	0.11	0.49
Max Sharpe	-0.06	0.43	0.21	-0.02	-0.33	0.09	-0.09	0.00	0.01	0.60	0.17	0.21	0.20	1.08

Constraint2: This constraint is aiming to simulate some arbitrary "box" constraints on weights, which client may provide. The results are shown in Table 5 and 6.

$$|W_i| \leq 1 \quad (9)$$

Table 5. Weights of Each Asset of Markowitz Model under Constraint 2

Constraint2	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
MinVariance	0.74	0.06	-0.14	-0.05	0.02	-0.02	0.09	0.04	0.02	0.47	-0.23	0.07	0.11	0.69
MaxSharpe	1.00	1.00	1.00	-1.00	-1.00	1.00	-1.00	-1.00	1.00	1.00	-1.00	0.56	0.72	0.78

Table 6. Weights of Each Asset of Index Model under Constraint 2

Constraint 2	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
Min Variance	1.00	-0.10	0.00	-0.08	-0.17	-0.02	0.04	0.00	-0.08	0.41	0.00	0.06	0.11	0.53
Max Sharpe	-1.00	0.67	0.31	0.00	-0.38	0.13	-0.12	0.00	0.10	1.00	0.30	0.29	0.26	1.11

Constraint3: This constraint is a free situation, aiming to illustrate the area of possible portfolios in general for a comparison with other constraints in real life. The results are shown in Table 7 and 8.

Table 7. Weights of Each Asset of Markowitz Model under Constraint 3

Constraint3	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
MinVariance	0.74	0.06	-0.14	-0.05	0.02	-0.02	0.09	0.04	0.02	0.47	-0.23	0.07	0.11	0.69
MaxSharpe	0.00	2.19	1.30	-1.18	-1.00	0.11	-1.24	-0.22	0.29	0.71	0.06	0.69	0.70	0.98

Table 8. Weights of Each Asset of Index Model under Constraint 3

Constraint 3	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
Min Variance	1.26	-0.12	-0.01	-0.09	-0.18	-0.02	0.01	-0.01	-0.10	0.29	-0.02	0.05	0.11	0.45
Max Sharpe	-1.21	0.71	0.33	0.01	-0.38	0.14	-0.12	0.00	0.11	1.11	0.32	0.31	0.28	1.11

Constraint4: This optimization constraint is aiming to model a typical constraint that the US mutual fund industry would face short positions are not allowed in US open-end mutual funds. The results are shown in Table 9 and 10.

$$W_i \geq 0 \tag{10}$$

Table 9. Weights of Each Asset of Markowitz Model under Constraint 4

Constraint4	**SPX**	**ADBE**	**ALK**	**BAC**	**C**	**HA**	**IBM**	**LUV**	**SAP**	**TRV**	**WFC**	**Return**	**StDev**	**Sharpe**
MinVariance	0.65	0.05	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.23	0.00	0.08	0.13	0.62
MaxSharpe	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.30	0.70

Table 10. Weights of Each Asset of Index Model under Constraint 4

Constraint 4	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
Min Variance	0.55	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.44	0.00	0.08	0.14	0.60
Max Sharpe	0.00	0.39	0.20	0.00	0.00	0.09	0.00	0.00	0.00	0.24	0.08	0.18	0.21	0.88

Constraint5: Finally, this constraint is designed to see whether including a broad index in the portfolio could result in a positive or negative change, so an additional optimization constraint shall be considered. The results are shown in Table 11 and 12.

$$W_1 = 0 \tag{11}$$

Table 11. Weights of Each Asset of Markowitz Model under Constraint 5

Constraint5	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
MinVariance	0.00	0.01	-0.14	-0.17	0.00	-0.05	0.29	0.19	0.07	0.77	0.03	0.06	0.15	0.43
MaxSharpe	0.00	2.19	1.30	-1.18	-1.00	0.11	-1.24	-0.22	0.29	0.71	0.06	0.69	0.70	0.98

Table 12. Weights of Each Asset of Index Model under Constraint 5

Constraint 5	SPX	ADBE	ALK	BAC	C	HA	IBM	LUV	SAP	TRV	WFC	Return	StDev	Sharpe
Min Variance	0.00	-0.02	0.05	-0.05	-0.13	-0.01	0.17	0.04	-0.03	0.89	0.09	0.10	0.14	0.72
Max Sharpe	0.00	0.46	0.22	-0.04	-0.36	0.09	-0.18	-0.02	0.03	0.61	0.18	0.22	0.21	1.09

5. Conclusions

By comparison of two models, the weights of each asset could be totally disparate even under the same constrain. And thus, leading to similar but different returns and standard deviations. By carefully looking at the optimal result in different constraints, it can be concluded that both model favors TRV, which is the Travelers Companies Inc and SPX in most cases. The reason is that these two stocks hold a relatively high return while remains relatively low variance. Plus, the correlation between these two stocks is around 0.3, which is not strong as it follows the purpose of portfolios which is to build diversification. On the opposite side, each model tends to put a small portion of asset weight in Citigroup and Alaska Air Group.

During the analyze process, some drawbacks of Markowitz model can be found. First, it requires many estimates of expected returns, variances, co-variances, which would enormously increase the difficulty for calculation if more assets were being considered. Moreover, it does not show the method on how to produce estimates, the only reasonable guess is that it uses historical data for future prediction. And that is why the Index model is needed as a complement. The Index model Significantly reduces number of estimates required, enhances the analysis of security expected returns and it could be used to identify optimal portfolios nearly as accurately as the much more data compared to the Markowitz model.

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