Application of Monte Carlo model in financial field

Yitong Zou *

Northeast Yucai school, Shenyang, China

* Corresponding Author Email: Narumi2022@163.com

Abstract. MC (Monte Carlo) model is also called statistical test method or statistical simulation method. According to the statistical sampling theory, it is a method of approximate solution through probability simulation and statistical test of random variable function. Compared with the traditional mathematical statistics method, MC method alleviates the difficulty of realization with the help of computer, and is easy to operate and understand. Other mathematical methods can find an appropriate way to deal with complex problems that are difficult or even impossible to deal with. This paper studies the application of MC model in the financial field: the application in the pricing of financial options and the application in the guaranteed price of multi-period income. Using MC simulation model, we can not only analyze the risk of investment projects, but also play a great role in management, financial analysis and marketing.

Keywords: Monte Carlo, Option, Guaranteed price.

1. Introduction

Financial derivatives are important hedging tools in the secondary market, and accurate pricing of derivatives is necessary to avoid risks smoothly. Some simple options have analytical expressions under strict assumptions. Generally speaking, numerical methods should be used to price options [1]. MC (Monte Carlo) model is also called statistical test method or statistical simulation method. According to the statistical sampling theory, it is a method of approximate solution through probability simulation and statistical test of random variable function. The application of MC method to the prediction of resources is a process of simulating the distribution of resources according to the random sampling theory, and the calculated result is a probability distribution curve of reserves [2-3]. MC simulation has diversity and practicability in risk analysis, and can be used in various business decisions. Its main application fields are: business management, financial analysis and marketing. This paper mainly introduces its application in the financial field.

2. Overview of MC model

In the MC model, its basic mathematical thought is very simple and intuitive. Its basic idea is: firstly, establish a probability model of the problem to be solved, express the required solution value or its function as the mathematical expectation of the model, then observe the model with a large number of samples, and finally take the arithmetic average of the random variables generated by the above samples as the approximate estimation value [4]. For the pricing problem of financial derivatives, the main requirement is the price at $t=0$ moment, that is, the unconditional expectation of discounted payment, which can be taken as a numerical solution by the parameter estimator of sample mean. In the pricing problem of financial derivatives, only the first moment of discounted payment is generally required.

General parameter estimation (such as moment estimation and maximum likelihood estimation) regards the parameter as a constant, while Bayesian estimation regards the parameter as a random variable, so the parameter will also have a random distribution, which is subjectively determined. The advantage of this choice is that it is convenient to calculate and the estimator of parameters can be well explained. Of course, because the prior distribution is selected subjectively, the meaningless prior distribution can also be selected, but this choice may bring some unnecessary troubles to the subsequent calculation and estimation [5-6].

The basic idea of MC model is: in order to solve geological problems, firstly, a probability model is established according to the existing data [7]. The key steps of applying MC method to solve practical problems are as follows: firstly, establish a probability model, then generate sampling random numbers that meet the requirements, and finally sample geological random parameters with known distribution with random numbers to calculate the solution to be solved.

Compared with the traditional mathematical statistics, the MC model can reduce the implementation difficulty of the algorithm by computer, and the algorithm is easier to use and understand. Using other mathematical methods, we can find appropriate solutions for some difficult or unsolvable problems. In general, to get the probability distribution of an uncertain object, it is necessary to carry out multiple simulations. At this time, the MC model can quickly get better approximation results, so it has high application value.

Specifically, suppose there is a random variable X with a distribution function $D(x)$ to generate a set of independent random variable sequences $X_1, \cdots, X_n$ under the distribution function $D(x)$. According to the theorem of large numbers, it can be known that for the integrable function f of D, there must be:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{1\le n\le N} f(X_n) = E[f(x)] = \int f(x)D(x)dx \tag{1}$$

MC model is a numerical technique based on probability and statistics, with random sampling as the main means.

When the inverse function of the distribution function $f(x)$ of the random variable X is difficult to obtain, other methods can be used. One of them is the accept-reject method. This method is suitable for the simulation of random numbers whose density function has a compact support set, but it is not easy to sample directly from it.

3. Application of MC model in financial field

3.1. Application in financial option pricing

Option is a derivative based on futures. In essence, option is a separation of rights and responsibilities in the financial field, which enables the transferee of rights to exercise their rights within a certain period of time, while the debtor needs to fulfill his rights. The holder of the call option has the option, so there is no need to pay the deposit.

Short positions of options need to pay an option deposit to ensure the smooth implementation of long rights of options. Long positions of options have rights conferred by options and have no obligations, so they don't need to pay a deposit. Short positions of options need to pay an option deposit to ensure the smooth implementation of long rights of options [8-9]. Generally speaking, other things being equal, the longer the maturity, the greater the fluctuation of the underlying asset price, and the greater the time value of the option. For call options, the connotative value is the difference between the exercise price and the underlying asset price.

If a variable changes at any time in some uncertain way, it is said that it follows some random process. Random processes can be divided into discrete time and continuous time. Discrete time random process is a process in which variables can only change at certain time points, while a continuous time random process is a process in which the value of variables can change at any time [10]. For European options, the execution time is fixed, and it will not be executed in advance. Therefore, it is only necessary to calculate the option income of various paths on the maturity date, and then take the average of the option income of each path as the option price.

The subject matter of the option is a non-dividend stock, the stock price is 30 yuan, the annual expected rate of return is 0.18, the volatility is 0.26, and the risk-free interest rate is 0.09. If there is a call option with the stock as the subject matter and the exercise price is 30 yuan, the option is priced by MC model.

In the Matlab command window, we can compare the option value under different simulation times according to the program. Figure 1 shows the fluctuation chart of option value.

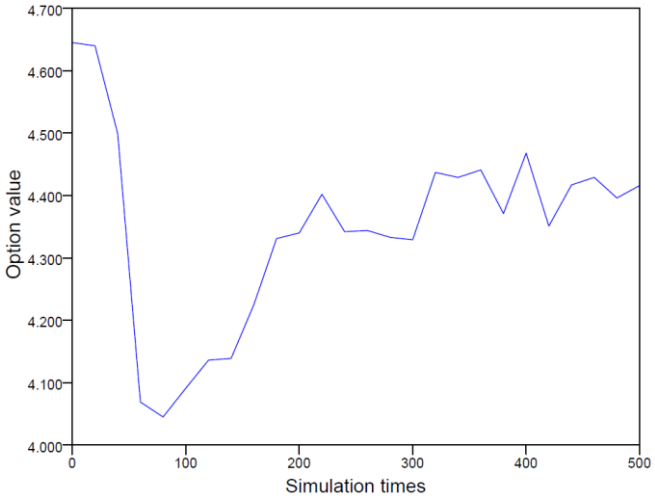


Figure 1. Option value fluctuation chart

It can be seen that when the number of simulations is less, the volatility of the option value is greater, but with the gradual increase of the number of simulations, the volatility of the option value becomes smaller and smaller. After 300 simulations, the volatility of the option value is basically stable at around 4.3, which shows that it has approached the value of the option itself well, and the result of the option value obtained is relatively stable. It is very simple to use MC model to price European options. As long as we understand the idea of pricing European options by MC model, we can program it on Matlab.

3.2. Application in multi-period income guaranteed price

Compared with tactical asset allocation, constant mixed asset allocation is not based on the change of investment return rate or investor's expected utility function, but on the assumption that investment return rate and investor's expected utility function are relatively constant.

When the capital market declines, their risk tolerance is different from that of mass investors, and they remain basically constant, so the proportion of equity assets will increase and the risk premium will also increase. Because this kind of income guarantee has embedded options, we can establish an arbitrage-free pricing model of income guarantee price by random analysis method. The quantitative financial problem selected in this paper is the pricing of multi-period income guarantee price with constant mixed asset allocation strategy, which is embedded with path-dependent options. MC simulation method is good at dealing with this high-dimensional quantitative financial problem.

Assuming that the market is arbitrage-free, there is a risk-neutral probability measure $\tilde{P}$, which makes the risk asset price $S(t)$ obey the following stochastic differential equation:

$$dS(t) = rS(t)dt + \sigma_S S(t)d\tilde{W}(t) + S(t-)d\left(Q(t) - \tilde{\lambda}\tilde{\beta}t\right) \tag{2}$$

Where $t \in [0,T]$, T is the termination time, $S(t-)$ represents the left limit of the price of risky assets at time t which is left continuous, r is the risk-free interest rate, σ_S is the volatility of risky assets price $S(t)$, $\tilde{W}(t)$ is the Brownian motion at time t, and $Q(t) - \tilde{\lambda}\tilde{\beta}t$ is the compensated compound Poisson process at time t.

The "multi-period" of the multi-period income guarantee price is mathematically embodied as path dependence, that is, it is required to simulate the whole path of $\tilde{\eta}(t)$, and the incremental independence of $\tilde{W}(t), N(t)$ should be considered. The following formula is used to generate the estimate of $d\tilde{W}(j)$:

$$d\tilde{W}(j)=\sqrt{dt}Z \tag{3}$$

Among them, $Z \sim N(0,1)$.

The key to the application of MC in quantitative finance is to simulate the sample path of the underlying asset price. Ordinary MC model, variance reduction technology and quasi-MC model are used to price the multi-period income guarantee price of constant mixed asset allocation strategy respectively.

Fig. 2 shows the confidence interval length of various methods at the significance level of $\alpha = 0.05$. Because quasi-MC is a deterministic method, its confidence interval cannot be calculated, so there is no statistics.

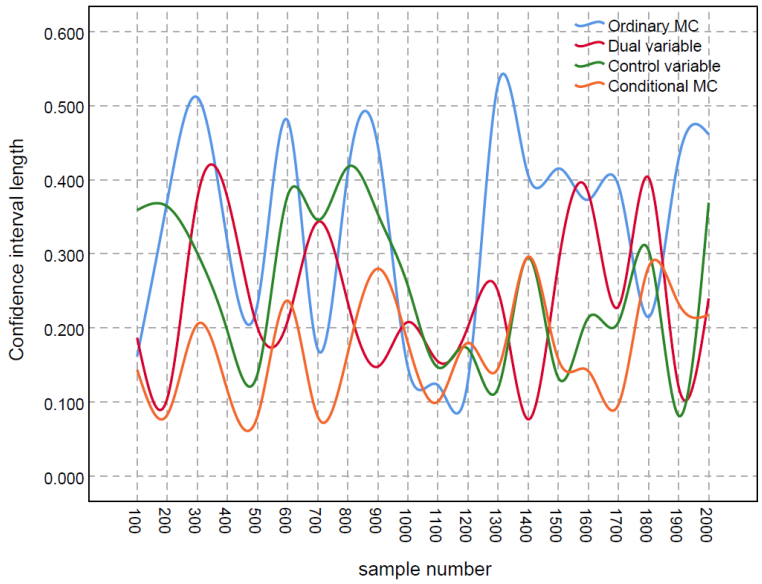


Figure 2. Confidence interval length

It can be seen that the accuracy of ordinary MC simulation is the worst, the accuracy of dual variable method is similar to that of control variable method, and the control variable method is slightly better, and the conditional MC method has the highest accuracy. This is not difficult to understand, because the conditional MC method uses the analytical solution of conditional expectation, and the numerical solution is not as accurate as the analytical solution.

The difficulty of the strip control variable method lies in the solution of the analytical formula of the mean value of the control variable, which is not as complicated as the conditional MC method. The dual variable method is the simplest, and its accuracy is only slightly worse than that of the control variable method, which is improved compared with the ordinary MC method.

4. Conclusions

Applying MC model to predict resources is a process of simulating the distribution of resources according to random sampling theory, and the calculated result is a probability distribution curve of reserves. MC simulation has diversity and practicability in risk analysis, and can be used in various business decisions. Its main application fields are: business management, financial analysis and marketing. This paper mainly introduces its application in the financial field. Using MC simulation model, we can not only analyze the risk of investment projects, but also play a great role in management, financial analysis and marketing. Any method is constantly improving and progressing. In view of the simulation idea of MC model, it can be combined with other methods to make the simulation value more accurate. According to the law of large numbers, the simulation accuracy is determined by the simulation times and variance, so controlling variance can reduce the simulation times and reduce the calculation amount.

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