The fixed-time consensus of leader-following nonlinear multi-agent systems under event-triggered impulsive control

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Abstract. This paper studies the leader-following fixed-time consensus problem for a class of first-order nonlinear multi-agent systems via event-triggered impulsive control where the communication topologies are general undirected. For each agent, the controller is updated only when some state-dependent errors exceed a tolerable bound, and the impulsive control time sequence is produced by the event triggered mechanism. This original control protocol can substantially improve the convergence speed, obtain the lower energy consumption and reduce the update times of the controller. Also, it is shown that continuous communication of neighboring agents can be avoided and there is no Zeno-behavior. The results are verified by a numerical simulation.

Keywords: Fixed time, event-triggered, multi-agent, impulsive control, consensus.

1. Introduction (Heading 1)

Over the past several years, the consensus of multi-agent systems (MASs) has drawn widely attention due to their important applications, such as formation control [1], flocking[2] [3], rendezvous[4] and data fusion[5]. Owing to the basic issue about this topic, the cooperative control has attracted particular attention, how to design a control protocol to achieve consensus become an important problem.

Consensus problem is an extensively investigated topic of cooperative control during the past few decades, which aims to make all the agents achieve an agreement [6]–[10] with relative local information. Usually, consensus problem is composed of several aspects: dynamics, controller design, and communication topology [10]. A significant index of convergence performance is the convergence rate. As shown in [8] and [9], undirected network topology and strongly connected topologies are important factor for considering convergence rate. Although under there constraints the MASs can achieve consensus, it can't be applied in practical issues. Then the finite-time consensus problems have been investigated. However, the default of finite-time consensus is the guaranteed convergence speed usually rely on the initial states, which are unknown in reality. Actually, a lot of real tasks need to be completed in a predefined time, such as gathering at a certain place, intercepting a target. So the fixed-time consensus was discussed.

For fixed-time consensus problem, the settling time has an upper bound which only relies on the parameters of the system and can be calculated previously. Then, many control strategies were designed to achieve fixed-time consensus[11] [12] [13] [14] [15] [16]. in [11](among followers), [13], and [14], the communication topology was assumed undirected. Ning et al. [15] considered the fixed-time tracking problem under directed communication topologies. However, the topology was required to be detailed balanced. There results can't be expended to the general directed communication topologies. In [12], the fixed-time coordinated tracking problem with bounded input uncertainties was studied. Reference [16] provides another research aspect for fixed-time consensus: specified-time consensus. In [16], motion planning method was introduced to make a group of linear MASs to reach consensus at a specific time with directed communication topology.

Thereafter, the distributed event-triggered control strategies have been proposed in [17]-[19]. In [17], a threshold function has been presented which the controller is updated only when the measurement error exceed the predefined function. Moreover, in [18] and [19], an agent's measurement error depends on the neighboring agents' state between current and last event triggered time, and the threshold function depends on state of the agent's neighbor at last event triggered time.

Reference [20] studies the distributed consensus tracking problem of networked Lure system based on event triggered information interaction under fixed directed topology in most existing research results, the triggered event of each agent is related not only to its own trigger time, but also to the triggered time of the agent's neighbors. In order to further save communication resources, Reference [21] proposed an event trigger controller with joint measurement literature. From another point of view, on the basis of static event triggering control, document [22] introduces internal dynamic variables and proposes a dynamic event triggered mechanism, which can further reduce the triggered frequency In order to save the communication resources and improve the convergence time of the system, the finite time consensus based on event triggered is developed.

In order to further raise the convergence rate, Jiang et al. [23] studied the multi-agent systems with directed switching graph, mainly from the perspective of impulse control theory, constructed the impulse protocol by using Dirac function, and constructed the corresponding consensus sufficient conditions. In the research of Guan et al.[24], the protocol with impulsive control is used to improve the uniform convergence speed of multi-agent system compared with the traditional protocol. At the same time, the consensus problem under external interference is also studied. Wang et al [25] further simplified the impulsive protocol algorithm. While ensuring the consensus, the information exchange of second-order multi-agent can achieve the consistency only by relying on the displacement information, which reduces the difficulty of control, and gives the sufficient condition for the consistency of time-delay system.

Motivated by the aforementioned discussion, this paper presents the event-triggered impulsive control algorithm for the fixed-time consensus problem of MASs. The impulsive control is exerted when the trigger condition is satisfied. Compared with existing results, our control strategy can not only guarantee that the MASs achieve the consensus within fixed time, but also has the advantages of high convergence rate, small communication consumption and low frequency of the controller updates.

2. Preparation and problem description

2.1 Graph theory

Consider an undirected graph $\mathcal{G}$ consisting of N agents, which can be expressed as $\mathcal{G}=(\mathcal{V},\varepsilon,A)$, and the N agents can be modeled as N nodes. $\mathcal{V}$=1,2,…,N is the set of nodes, and $\varepsilon\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges. If (j, i)$\in\varepsilon$, agent i and agent j can obtain information from each other, so they called neighbors. Let adjacency matrix $A=\left(a_{ij}\right)_{N\times N}$, an undirected graph means that $a_{ij}=a_{ji}$. If a_{ij}>0, so (j, i)$\in\varepsilon$, if not, a_{ij}=0. The degree matrix $D=diag\{d_1,d_2,...,d_N\}$, in which $d_i=\sum_{j=1,j\neq i}^{N}a_{ij}$. Define the Laplacian matrix $L=\left[l_{ij}\right]$, and L=D-A. Moreover, if there exists a path between any two agents, that is called the graph is connected. For a directed graph with a leader, this paper assumes that the leader is labeled as vertex 0. The directed graph $\hat{\mathcal{G}}$ with N+1 agents. $\hat{\mathcal{G}}$ is described by the weight matrix $\hat{L}=L+B$, $B=diag\left[a_{10},a_{20},...,a_{N0}\right]$. If the agent I can obtain the information from the leader, a_{i0}>0, otherwise, a_{i0}=0.

2.2 Fixed-time stability theory of impulsive dynamic system

For a given impulsive sequence of agent i: $\zeta=\{t_k^i\},k\in N$, if there is a positive function V(X(t),t) satisfying:

(A1): $d_1 \| X(t) \|^c \le V(X(t),t) \le d_2 \| X(t) \|^c$

(A2): $when\ t \ne t_k^i$:

$$\dot{V}(X(t),t) \le \begin{cases} -\mu V^p(X(t),t), & 0 < V(X(t),t) \le 1, \\ -\nu V^q(X(t),t), & V(X(t),t) > 1, \\ 0 & V(X(t),t) = 0; \end{cases}$$

(A3):when $t = t_k^i, V(X(t_k^+), t_k^+) \le \eta V(X(t),t)$

Then the system is globally fixed-time stable, the bound of settling time T satisfying:

(1) $T = \frac{\theta_1}{(1-q)\ln\eta}\ln\left(1 - \frac{\eta^{1-q}\ln\eta}{\nu\theta_1}\right) + \frac{\theta_1}{(1-p)\ln\eta}\ln\left(1 + \frac{\ln\eta}{\mu\theta_2\eta^{2(1-q)} - \ln\eta}\right), when\ 0 < \eta < 1$

(2) $T = \frac{1}{\mu(1-p)} + \frac{1}{\nu(q-1)}, when\ \eta = 1$

Where $d_1 > 0, 0 < \eta \le 1, d_2 > 0, c > 0, \mu > 0,$

$\nu > 0, 0 < p < 1, q > 1, \theta_1 and\ \theta_2$ are the upper and lower bounds of impulsive interval, respectively.

2.3 Problem description

Consider a multi-agent system as follows:

$$\begin{aligned} \dot{x}_0(t) &= f(x_0(t), t) \\ \dot{x}_i(t) &= u_i(t) + f(x_i(t), t),\ i = 1, \dots, N \end{aligned} \tag{1}$$

The MASs contain N followers that are labeled as agent 0 to N, and the leader is marked as agent 0, with $X(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T \in R^N, u(t) = [u_1(t), u_2(t), \dots, u_N(t)]^T \in R^N. x_i(t)$ is the state of agent i, $u_i(t)$ is the control input. In this paper, as the leader-following consensus, the agent 0 is leader. Conversely, the leaderless consensus, there is no agent 0.

Definition 1: The problem of leader-following fixed time consensus will be solved under the distributed fixed-time consensus protocol $u_i(t)$ for each agent, there exists a settling time T such that $\lim_{t \to T} \| x_i - x_0 \| = 0$ and $x_i(t) = x_0(t)$ when t>T.

Remark 1: For the finite-time consensus, if the initial states are large enough, the bound of settling time will be very large. Therefore, the original fixed-time consensus algorithms via event-triggered impulsive control are proposed in this paper.

3. Leader-following distributed event-triggered fixed-time consensus tracking algorithms via sign function

Assumption 1 : The undirected graph $\mathcal{G}$ is connected, and there exists at least one follower can receive information from leader, which means $a_{i0} > 0$.

In order to study leader-following fixed time consensus, we propose the following control input for system (1):

$$\begin{aligned} u_i(t) = &-c_1 sign\left[\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right] \\ &-c_2 sig^r\left[\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right] \\ &-c_3\left[\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right] \\ &-\delta\left(t - t_k^i\right) m\left[\sum_{j=0}^{N} a_{ij}\left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right)\right] \end{aligned} \tag{2}$$

Assumption 2: There is a known and positive constant γ satisfying:

$$\left|f\left(x_i\left(t\right),t\right)-f\left(x_j\left(t\right),t\right)\right|\le\gamma\left|x_i\left(t\right)-x_j\left(t\right)\right| \tag{3}$$

Remark 2: Note that Assumption 1 in the literatures, which is the common assumption of undirected graph. In assumption 2 the control input is designed as bounded, which is inspired by the [26] .

Remark 3: In this paper, presume the followers cannot send messages to leader, the leader's dynamic can be designed in advance. The impulsive signal cannot affect the leader's state too. In a word, the leader's behavior is independent of followers. $\delta\left(t\right)$ represents the Dirac impulsive function, which means when t=0, $\delta\left(t\right)$=1, when t≠0, $\delta\left(t\right)$=0. C1, C2 are positive constants, m denotes the impulsive strength and r>1. Some particular agents can receive the leader's information, and the other followers' states are only influenced by the states of its neighbors. For each $t\in\left[t_k^i,t_{k+1}^i\right]$ is the latest triggered time of agent j, in this paper, the agent i is updated both at its own and neighbors event instant. On account of the leader does not have the event instant when j = 0, $x_0\left(t_{k'}^0\right)=x_0\left(t\right)$. So the agent i will not update its state at this moment.

3.1 Proof

Define $y_i\left(t\right)=\sum_{j=0}^{N}a_{ij}\left(x_i\left(t\right)-x_j\left(t\right)\right)$,

Hence, one has

$y\left(t\right)=\left[y_1\left(t\right),y_2\left(t\right),...,y_N\left(t\right)\right]^T=\hat{L}\hat{X}\left(t\right)\in R^N$. Define

$$e_i\left(t\right)=x_i\left(t_k^i\right)-x_i\left(t\right) \tag{4}$$

And

$$\begin{aligned}\varepsilon_i(t)&=\sum_{j=0}^{N}a_{ij}\left(x_i\left(t_k^i\right)-x_j\left(t_{k'}^j\right)\right)\\&=\sum_{j=0}^{N}a_{ij}\left(x_i\left(t\right)+e_i\left(t\right)-x_j\left(t\right)-e_j\left(t\right)\right)\end{aligned} \tag{5}$$

So the $u_i\left(t\right)$ can be rewritten as:

$$u_i(t)=-\begin{pmatrix}c_1sign\left(y_i\left(t\right)\right)+c_2sig^r\left(y_i\left(t\right)\right)+c_3y_i\left(t\right)\\+\delta\left(t-t_k^i\right)my_i\left(t\right)+\xi_i\left(t\right)\end{pmatrix} \tag{6}$$

Here, $\xi_i\left(t\right)$ denotes the measurement error, it can be expressed as:

$$\begin{aligned}\xi_i\left(t\right)=&c_1sign\left(\varepsilon_i(t)\right)+c_2sig^r\left(\varepsilon_i(t)\right)+c_3\varepsilon_i(t)+\delta\left(t-t_k^i\right)m\varepsilon_i(t)\\&-c_1sign\left(y_i\left(t\right)\right)-c_2sig^r\left(y_i\left(t\right)\right)-c_3y_i\left(t\right)-\delta\left(t-t_k^i\right)my_i\left(t\right)\end{aligned} \tag{7}$$

From (2) and (5), the derivative of $\hat{x}_i(t)$ can be obtained:

$$\begin{aligned}\dot{\hat{x}}_i(t)&=\dot{x}_i(t)-\dot{x}_0(t)\\&=u_i(t)+f\left(x_i\left(t\right),t\right)-f\left(x_0\left(t\right),t\right)\\&=-\left(c_1sign\left(y_i\left(t\right)\right)+c_2sig^r\left(y_i\left(t\right)\right)+c_3y_i\left(t\right)+\delta\left(t-t_k^i\right)my_i\left(t\right)+\xi_i\left(t\right)\right)\\&\quad+f\left(x_i\left(t\right),t\right)-f\left(x_0\left(t\right),t\right)\end{aligned} \tag{8}$$

Equation (7) is written in a compact form, becomes

$$\begin{aligned}\dot{\hat{X}} = &-\xi(t) - c_1 sign\left(\hat{L}\hat{X}(t)\right) - c_2 sig^r\left(\hat{L}\hat{X}(t)\right) \\ &- c_3\left(\hat{L}\hat{X}(t)\right) - \delta\left(t-t_k^i\right)m\left(\hat{L}\hat{X}(t)\right) + F\left(\hat{X}(t),t\right)\end{aligned} \tag{9}$$

For the reason of reducing communication budget among agents and decreasing the updates of controller, the event-triggered mechanism is introduced. Consider the MASs (1) which is composed of the controller (2), the distributed event-triggered function for each agent has the following form:

$$f_i(t) = |\xi_i(t)| - \beta c_2 |y_i^r(t)| - \beta m \tag{10}$$

From the event-triggered function, we can see that every agent has its own triggered function. At the kth event instant of agent i, the agent i will sample its state, furthermore the agent i will use the newly state to update its controller and send the state to its neighbors. Then the neighbors will update the controllers with the received state, but the neighbors will not sample their states at this instant unless their triggered function exceeds zero.

Theorem 1: Suppose that Assumption 1 and Assumption 2 hold. Let $\mu = (c_1 - \beta m)(2\lambda_{min}(\hat{L}))^{\frac{1}{2}}$, p = $\frac{1}{2}$,
$\nu = (c_2 - \beta c_2)N^{\frac{1-r}{2}}(2\lambda_{min}(\hat{L}))^{\frac{r+1}{2}}, q = \frac{r+1}{2}$ and $\eta_k = 1 + m^2\lambda_{max}^2(\hat{L}) - 2m\lambda_{min}(\hat{L})$. Assume that η is the upper bound of η_k. If $0 < m \le \frac{2\lambda_{min}(\hat{L})}{\lambda_{max}^2(\hat{L})}$, $\mu > 0$, $0 < \beta < 1$ and $\gamma \le c_3\lambda_{\min}(\hat{L})$ are satisfied, the leader-following fixed-time consensus can be achieved under the control input (2). The bound of settling time is $T = \frac{\theta_1}{(1-q)\ln\eta}\ln\left(1 - \frac{\eta^{1-q}\ln\eta}{\nu\theta_1}\right) + \frac{\theta_1}{(1-p)\ln\eta}\ln\left(1 + \frac{\ln\eta}{\mu\theta_2\eta^{2(1-q)} - \ln\eta}\right), when\ 0<\eta<1$

$And\ T = \frac{1}{\mu(1-p)} + \frac{1}{\nu(q-1)}, when\ \eta = 1$.

Proof. Define the Lypunov candidate function

$$V = \frac{1}{2}\hat{X}^T\hat{L}\hat{X}$$

It can be deduced that

$$\sum_{i=1}^N y_i^2(t) = \left(\hat{L}\hat{X}(t)\right)^T\left(\hat{L}\hat{X}(t)\right) \ge \lambda_{min}\left(\hat{L}\right)\hat{X}^T(t)\hat{L}\hat{X}(t)$$

$\sum_{i=1}^N y_i^2(t) = \left(\hat{L}\hat{X}(t)\right)^T\left(\hat{L}\hat{X}(t)\right) \le \lambda_{max}\left(\hat{L}\right)\hat{X}^T(t)\hat{L}\hat{X}(t)$ Differentiating (16), when $t \ne t_k^i$

$$\begin{aligned}\dot{V} = &\hat{X}^T\hat{L}\left(-\xi(t) - c_1 sign\left(\hat{L}\hat{x}(t)\right) - c_2 sig^r\left(\hat{L}\hat{x}(t)\right) - c_3\left(\hat{L}\hat{x}(t)\right) + F(\hat{x}(t),t)\right) \\ \le &-\sum_{i=1}^N y_i(t)\xi_i(t) - c_1\sum_{i=1}^N y_i(t) sign\left(y_i(t)\right) - c_2\sum_{i=1}^N \left|y_i(t)\right|^r y_i(t) sign\left(y_i(t)\right) \\ &- c_3\sum_{i=1}^N y_i^2(t) + \sum_{i=1}^N \left|y_i(t)\right|\left|\left(f\left(x_i(t),t\right) - f\left(x_0(t),t\right)\right)\right| \\ \le &\sum_{i=1}^N |y_i(t)||\xi_i(t)| - c_1\sum_{i=1}^N |y_i(t)| - c_2\sum_{i=1}^N |y_i(t)|^{r+1} - c_3\sum_{i=1}^N y_i^2(t) + \gamma\sum_{i=1}^N |y_i(t)|\left|\hat{x}_i(t)\right|\end{aligned}$$

$$
\begin{aligned}
&\le \sum_{i=1}^{N} | y_i(t) | \left(\beta c_2 | y_i^r(t) | + \beta m\right) - c_1 \sum_{i=1}^{N} | y_i(t) | \\
&-c_2 \sum_{i=1}^{N} | y_i(t) |^{r+1} - c_3 \sum_{i=1}^{N} y_i^2(t) + \frac{\gamma}{\lambda_{\min}\left(\hat{L}\right)} \sum_{i=1}^{N} y_i^2(t) \\
&= \beta c_2 \sum_{i=1}^{N} | y_i(t) |^{r+1} + \beta m \sum_{i=1}^{N} | y_i(t) | - c_1 \sum_{i=1}^{N} | y_i(t) | \\
&- c_2 \sum_{i=1}^{N} | y_i(t) |^{r+1} + \left(\frac{\gamma}{\lambda_{\min}\left(\hat{L}\right)} - c_3 \right) \sum_{i=1}^{N} y_i^2(t) \\
&\le \left(\beta c_2 - c_2\right) \sum_{i=1}^{N} | y_i(t) |^{r+1} + \left(\beta m - c_1\right) \sum_{i=1}^{N} | y_i(t) | \\
&\le -\left(c_2 - \beta c_2\right) N^{\frac{1-r}{2}} \left(\sum_{i=1}^{N} | y_i(t) |^2 \right)^{\frac{r+1}{2}} - \left(c_1 - \beta m\right) \left(\sum_{i=1}^{N} | y_i(t) |^2 \right)^{\frac{1}{2}} \\
&\le -\left(c_2 - \beta c_2\right) N^{\frac{1-r}{2}} \left(2\lambda_{min}\left(\hat{L}\right) \right)^{\frac{r+1}{2}} V^{\frac{r+1}{2}} - \left(c_1 - \beta m\right) \left(2\lambda_{min}\left(\hat{L}\right) \right)^{\frac{1}{2}} V^{\frac{1}{2}}
\end{aligned}
\tag{11}
$$

Let $\begin{aligned} \mu &= (c_1 - \beta m)(2\lambda_{min}(\hat{L}))^{\frac{1}{2}}, p = \frac{1}{2}, \\ \nu &= (c_2 - \beta c_2) N^{\frac{1-r}{2}} (2\lambda_{min}(\hat{L}))^{\frac{r+1}{2}}, q = \frac{r+1}{2} \end{aligned}$, we have $\dot{V} \le -\mu V^p - \nu V^q$.

According to (19), the $\dot{V} < 0$ is satisfied and we have $\sum_{i=1}^{N} y_i^2(t) \le 2\lambda_{max}\left(\hat{L}\right) V(t) \le 2\lambda_{max}\left(\hat{L}\right) V(0)$. Then one has $| y_i(t) | < \| y(t) \| \le \sqrt{2\lambda_{max}\left(\hat{L}\right) V(0)}$.

When $t = t_k^i$, it follows from the assumption of Theorem 1 that $0 < \eta_k < 1$ and $\eta_k = 1 + m^2 \lambda_{\max^2}(\hat{L}) - 2m\lambda_{\min}(\hat{L}), \left(I_N - m\hat{L}\right)^2 \le \eta_k I_N$. Then we obtain

$$
\begin{aligned}
V\left(t_k^+\right) &= \frac{1}{2} \hat{X}^T \left(t_k^+\right) \hat{L} \hat{X}\left(t_k^+\right) \\
&= \frac{1}{2} \hat{X}^T \left(t_k^-\right) \left(\left(I_N - m\hat{L}\right)^2 \hat{L} \right) \hat{X}\left(t_k^-\right) \\
&\le \frac{\eta_k}{2} \hat{X}^T \left(t_k^-\right) \hat{L} \hat{X}\left(t_k^-\right) \\
&= \eta_k V\left(t_k^-\right)
\end{aligned}
\tag{12}
$$

From Lemma 1, the MASs discussed above can achieve fixed-time consensus. Furthermore, if η_k has an upper bound η, when 0< η < 1 is satisfied, the MASs achieve consensus within $T = \frac{\theta_1}{(1-q)\ln\eta} \ln\left(1 - \frac{\eta^{1-q}\ln\eta}{\nu\theta_1}\right) + \frac{\theta_1}{(1-p)\ln\eta} \ln\left(1 + \frac{\ln\eta}{\mu\theta_2\eta^{2(1-q)} - \ln\eta}\right)$,when η =1, $T = \frac{1}{\mu(1-p)} + \frac{1}{\nu(q-1)}$. The proof is completed.

3.2 The proof of avoiding Zeno-behavior

With regard to the multi-agent systems (1), there is no Zeno-behavior with the control protocols (2).

$$\begin{aligned}
\left|\dot{\xi}_i(t)\right| &= \left|\begin{pmatrix} -c_1 sign\left(y_i(t)\right) - c_2 sig^r\left(y_i(t)\right) \end{pmatrix}^{\dot{}} \\ -c_3 y_i(t) - m\delta\left(t-t_k^i\right) y_i(t) \right| \\
&\le \left(\left|c_2 r y_i^{r-1}(t)\right| + c_3 + m\right)\left|\dot{y}_i(t)\right| \\
&\le \left(\rho_1 + c_3 + m\right)\left|\sum_{j=0}^{N} a_{ij}\left(\dot{x}_i(t) - \dot{x}_j(t)\right)\right| \\
&= \left(\rho_1 + c_3 + m\right)\left|\sum_{j=0}^{N} a_{ij}\left(u_i(t) + f\left(x_i(t),t\right) - u_j(t)\right) - f\left(x_j(t),t\right)\right| \\
&\le \left(\rho_1 + c_3 + m\right)\left(\left|\sum_{j=0}^{N} l_{ij} u_j\left(t_{k'}^j\right)\right| + \left|\sum_{j=0}^{N} a_{ij}\gamma x_i(t) - x_j(t)\right|\right) \\
&= \left(\rho_1 + c_3 + m\right)\left(\rho_2\left(t_{k'}^j\right) + \rho_3\right)
\end{aligned} \tag{13}$$

Where

$$\begin{aligned}
\rho_1 &= \left|c_2 r\left(2\lambda_{max}\left(\hat{L}\right)V(0)\right)^{\frac{r-1}{2}}\right|, \\
\rho_2 &= \left|\sum_{j=0}^{N} l_{ij}\left(c_1 sign\left(y_j\left(t_{k'}^j\right)\right) + c_2 sig^r\left(y_j\left(t_{k'}^j\right)\right) + m y_j\left(t_{k'}^j\right)\right)\right|, \\
\rho_3 &= \left|\gamma\left(2\lambda_{max}\left(\hat{L}\right)V(0)\right)^{\frac{1}{2}}\right|
\end{aligned} \tag{14}$$

Thanks to $\xi_i\left(t_k^i\right) = 0$, it follows

$$\begin{aligned}
|\xi_i(t)| &\le \int_{t_k^i}^{t}\left(\rho_1 + c_3 + m\right)\left(\rho_2\left(t_{k'}^j\right) + \rho_3\right)ds + \left|\xi_i\left(t_k^i\right)\right| \\
&= \int_{t_k^i}^{t}\left(\rho_1 + c_3 + m\right)\left(\rho_2\left(t_{k'}^j\right) + \rho_3\right)ds
\end{aligned} \tag{15}$$

The next event of agent i will not be triggered until $f_i(t)$=0, from (10), we can get

$$\begin{aligned}
|\xi_i\left(t_{k+1}^i\right)| &= \beta c_2 \,|\, y_i^r(t_{k+1}^i)\,| + \beta m \\
&\le \int_{t_k^i}^{t_{k+1}^i}\left(\rho_1 + c_3 + m\right)\left(\rho_2\left(t_{k'}^j\right) + \rho_3\right)ds \\
&\le \int_{t_k^i}^{t_{k+1}^i}\left(\rho_1 + \rho_2 + m\right)\left(\rho_2(0) + \rho_3\right)ds
\end{aligned} \tag{16}$$

Then,

$$t_{k+1}^i - t_k^i \ge \beta m / \left(\rho_1 + \rho_2 + m\right)\left(\rho_2(0) + \rho_3\right) \tag{17}$$

3.3 Simulations

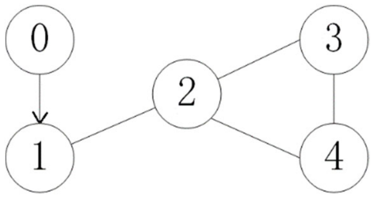

Figure 1. the topology of MASs

In this section, a numerical simulations are presented to demonstrate the effectiveness of the theoretical results. In Figure 1, the leader is labeled as 0.

It is assumed that the MASs contain of four followers and one virtual leader. The relationship of all agents is represented by the communication graph, which is shown in Figure 1. Suppose that the state dimension of all agents is 2. According to the undirected graph, we can get the adjacency matrix

A and the diagonal matrix B as: $A=\begin{bmatrix}0&1&0&0\\1&0&1&1\\0&1&0&1\\0&1&1&0\end{bmatrix}$, $B=diag\{1\quad 0\quad 0\quad 0\}$.Set the parameters $c_1=3.5, c_2=0.2, c_3=1.4, \beta=0.3, r=2$. From the condition of Theorem 1, the impulsive strength must satisfy m < 0.0164 , in this paper we choose m= 0.0145. The nonlinear function is given as $f(x,t)=[-0.2\sin(x+10), 0.3\cos(x+10)+(t-10)]$. Give the initial state as [8,12;10,3; - 8, - 7; - 9, - 7]. From the parameters, the η can be calculated η = 0.9992. And the fixed-time T = 2.0201s. The simulation results are shown in Figures 1-3.

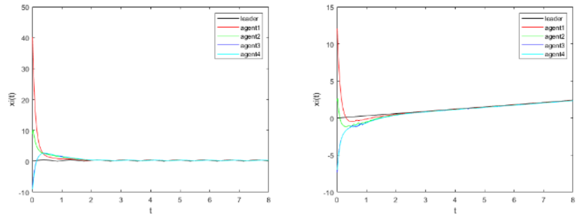

Figure 2. State trajectories of agent along, A-aixs, B-aixs

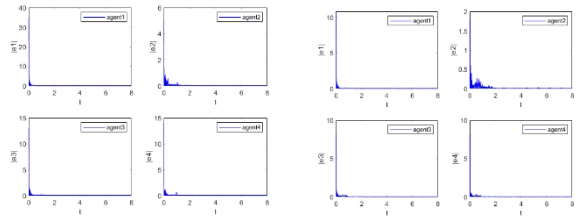

Figure 3. The fluttering of the measurement error of agent along, A-aixs, B-aixs

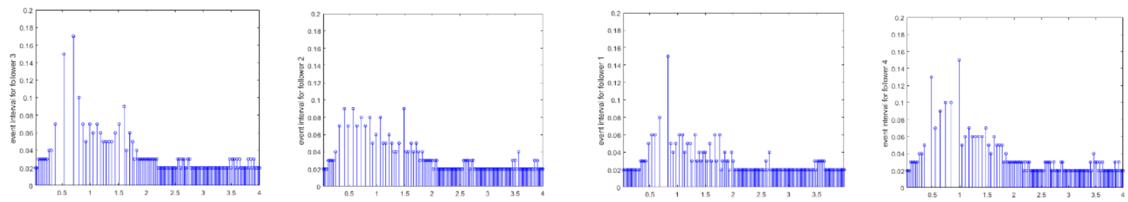

Figure 4. Event intervals of 4 followers under the first event-triggered control strategy

From figure 2, we can see in about 1.5-2 seconds the states of Example 1 can achieve consensus. Compared with [32], the convergence time in this paper is fixed and limited, which can be predicted by the preset parameters. It is 1.6-2 seconds faster than that of [32] and the measurement error converges to zero in 1.8s. From which it can be found that the lower bound for the event interval time is 0.02s, Based on above analysis, it is concluded that the convergence speed is faster and the energy is less than previous result, the MASs under fixed-time event-triggered impulsive control protocols have quicker convergence speed than those under normal fixed-time control protocol.

4. Conclusion

This article proposes an original fixed-time consensus tracking algorithms via event-triggered impulsive control. Some sufficient conditions are derived to achieve the LFFTC for MASs, novel control protocols can substantially reduce energy consumption and the frequency of controller updates. The impulsive event-triggered mechanism, as the combination of impulsive control and ETC, has the superiority of both two control mechanisms. By the Lyapunov stability and fixed-time impulsive theory, numerical simulations is employed to illustrate the feasibility of the deduction.

Considering the MASs may have different dynamics, the fixed-time consensus problem for the heterogeneous MASs with impulsive control and event-triggered control will be studied in the future.

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References

[1] Degroot M H. Reaching a consensus. Journal of the American Statistical Association,1974,69(345):118-121.

[2] Borkar V, Varaiya P P. Asymptotic agreement in distributed estimation[J]. IEEE Transactions on Automatic Control, 1982, 27(3):650-655.

[3] Tsitsiklis J N, Bertsekas D P, Athans M. Distributed asynchronous deterministic and stochastic gradient optimization algorithms[J]. IEEE Transactions on Automatic Control, 2003, 31(9):803-812.

[4] Dong X, Zhou Y, Ren Z, et al. Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying[J]. IEEE Transactions on Industrial Electronics, 2016, 64(6): 5014-5024.

[5] Hu Y, Zhan J, Li X. Self-triggered distributed model predictive control for flocking of multi-agent systems[J]. IET Control Theory \& Applications, 2018, 12(18): 2441-2448.

[6] J. Fu, G. Wen, T. Huang, and Z. Duan, “Consensus of multiagent systems with heterogeneous input saturation levels,” IEEE Trans. Circuits Syst. II, Exp. Briefs, to be published, doi: 10.1109/TCSII.2018.2867936.

[7] X. Wang, H. Su, X. Wang, and G. Chen, “Nonnegative edge quasiconsensus of networked dynamical systems,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 64, no. 3, pp. 304–308, Mar. 2017.

[8] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” IEEE Trans. Autom. Control, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[9] W. Yu, G. Chen, M. Cao, and J. Kurths, “Second-order consensus formultiagent systems with directed topologies and nonlinear dynamics,” IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 40, no. 3, pp. 881–891, Jun. 2010.

[10] W. Ren, J. Wu, X. Zhang, R. Lai, and L. Chen, “A Stochastic model of cascading failure dynamics in communication networks,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 65, no. 5, pp. 632–636, May 2018.

[11] M. Defoort, A. Polyakov, G. Demesure, M. Djemai, and K. Veluvolu, Leader–follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics,” IET Control Theory Appl., vol. 9, no. 14, pp. 2165–2170, Sep. 2015.

[12] H. Wang, W. Yu, G. Wen, and G. Chen, “Fixed-time consensus tracking of multi-agent systems under a directed communication topology,” in Proc. 12th IEEE Int. Conf. Control Autom., 2016.

[13] H. Hong, W. Yu, G. Wen, and X. Yu, “Distributed robust fixedtime consensus for nonlinear and disturbed multiagent systems,” IEEETrans. Syst., Man, Cybern., Syst., vol. 47, no. 7, pp. 1464–1473.

[14] Y. Shang, “Fixed-time group consensus for multi-agent systems with non-linear dynamics and uncertainties,” IET Control Theory Appl.,vol. 12, no. 3, pp. 395–404, Feb. 2017.

[15] B. Ning, Z. Zuo, J. Jin, and J. Zheng, “Distributed fixed-time coordinated tracking for nonlinear multi-agent systems under directed graphs,” Asian J. Control, vol. 20, no. 4, pp. 1–13, 2018.

[16] Y. Zhao, Y. Liu, G. Wen, W. Ren, and G. Chen, “Designing distributed specified-time consensus protocols for linear multi-agent systems over directed graphs,” IEEE Trans. Autom. Control, to be published, doi: 10.1109/TAC.2018.2872534

[17] W. Xu, D. W. C. Ho, L. Li, and J. Cao, "Event-triggered schemes on leader-following consensus of general linear multiagent systems under different topologies," IEEE Trans. Cybern., vol. 47, no. 1, pp. 212–223, Jan. 2017.

[18] W. Zhu and Z.-P. Jiang, "Event-based leader-following consensus of multi-agent systems with input time delay," IEEE Trans. Autom. Control,vol. 60, no. 5, pp. 1362–1367, May 2015.Liu H W, Du S L, Wang X F, et al. Consensus of multiagent systems with time-varying delays: An observer-based distributed periodic event-triggered control approach[J]. Asian Journal of Control, 2021, 1-10.

[19] W. Zhu, H. Li, and Z.-P. Jiang, "Consensus of multi-agent systems with time-varying topology: An event-based dynamic feedback scheme," Int. J. Robust Nonlin. Control, vol. 27, no. 8, pp. 1339–1350, 2017, doi: 10.1002/rnc.3631.

[20] HUANG N, DUAN Z, WEN G, et al. Event-triggered consensus tracking of multi-agent systems with Lur'e nonlinear dynamics. International Journal of Control, 2016, 89(5): 1 – 23. Wang Y W, Lei Y, Bian T, et al. Distributed control of nonlinear multiagent systems with unknown and nonidentical control directions via event-triggered communication[J]. IEEE transactions on cybernetics, 2019, 50(5): 1820-1832.

[21] FAN Y, FENG G, WANG Y, et al. Distributed event-triggered control of multi-agent systems with combinational measurements. Automatica, 2013, 49(2): 671 – 675.Wang C, Wen C, Hu Q. Event-triggered adaptive control for a class of nonlinear systems with unknown control direction and sensor faults[J]. IEEE Transactions on Automatic Control, 2019, 65(2): 763-770.

[22] GIRARD A. Dynamic triggering mechanisms for event-triggeredcontrol. IEEE Transactions on Automatic Control, 2015, 60(7): 1992– 1997.

[23] J1ANG H B, B1 Q s. Impulsive synchronization of Networked Nonlinear Dynamical systems[J]. Physics Letters A, 2010, 374(27): 2723—2729.

[24] GUAN z H, wu Y H, FENG G. Consensus Analysis Based on Impulsive Systems in Multiagent Networks[J]. IEEE Transactions on Circuits and Systems I Regular Papers, 2012, 59(1): 170—178.

[25] WANG Y w, YI J w. Consensus in Second_0rder Multi—Agent Systems Via Impulsive Control Using Position-only Information with Heterogeneous Delays[J]. Control Theory and Applications Iet, 2014, 9(3): 336—345.

[26] Zhai X, Wen G, Peng Z, et al. Leaderless and leader-following fixed-time consensus for multiagent systems via impulsive control[J]. International Journal of Robust and Nonlinear Control, 2020, 30(13): 5253-5266.