The Formulation of An Optimal Forest Management Scheme Based on Carbon Sequestration Models and Decision-Making Models

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Abstract. Carbon sequestration and scientific management of forests are critical to human life. The study determined the factors affecting forest carbon sequestration through analysis, and a model of carbon sequestration with time, annual deforestation rate, and annual survival rate was established. The carbon sequestration of forests and their products at any specific time in the short term was predicted. The optimal annual deforestation rate for maximizing forest carbon sequestration was identified. Based on the consideration of the six values of carbon sequestration created by forests, carbon sequestration, annual deforestation benefits, biodiversity, cultural values, water conservation, and climate regulation values, by determining the relationship between each evaluation factor and its parameters and the annual deforestation rate, combined with the actual weights of each factor. This decision-making model balances forest estimation was developed using a TOPSIS solution space model. The optimal annual deforestation rate when the forest was fully utilized was determined, and it provides forest managers with the best use of forest management schemes. This research is of great significance for actively and effectively dealing with climate change and creating higher benefits.

Keywords: carbon sequestration; deforestation rate; forest management; decision-making model.

1. Introduction

Global warming gradually increases the threat to human life and the ecological environment. Increasing carbon sequestration is an effective measure to combat climate change and reduce greenhouse gas emissions, of which forests are an essential part [1]. The use of living plants and products produced by trees to sequester carbon dioxide is the two main ways forests carry out carbon sequestration [2]. Over time, forests that are partially felled to make products may allow for more carbon sequestration than forests that are not at all [3]. Therefore, forest managers must balance the values of forest products obtained from logging and the values of allowing the forest to continue growing and absorbing carbon as living trees [4]. In addition, they need to make forest management decisions based on multiple values that may include but are not limited to factors such as carbon sequestration, forest products, conservation and biodiversity, recreation, and culture [5].

This study aimed to develop a carbon sequestration model by identifying factors that influence forest carbon sequestration to determine carbon sequestration in forests and their products over time. On this basis, other factors that create forest value, such as biodiversity, entertainment, and culture, are considered. A decision-making model for balancing forest evaluation is further developed, which provides the basis for forest managers to maximize the values of forests and manage forests more reasonably.

2. Establishment and solution of Models

2.1. A carbon sequestration model

2.1.1. Composition of carbon sequestration in forest trees

By considering the composition of carbon sequestration in forest trees, the establishment of a carbon sequestration model is divided into the following three aspects: Determination of carbon sequestration of uncut trees; Determination of carbon sequestration of forest products from felled trees; Determination of carbon sequestration of the newly planted trees in the fell area.

The carbon density of trees must be determined before calculating their carbon sequestration, both for uncut trees, for products produced from felled trees, and for newly planted trees in felled areas. The carbon density of trees is considered to satisfy the age-related logical growth equation (1):

$$C = \frac{s}{1+ke^{-ut}} \tag{1}$$

Where C is the carbon density (Mg/hm^2); s, k, u are the fitting parameters, respectively, which fitted using non-nonlinear regression in the statistical software SPSS.

The amount of carbon sequestered by different species of trees varies due to their different carbon densities and proportions of the entire forest footprint. Equation (2) [6] gives the calculation method of the carbon sequestration per unit area of the i-th class trees.

$$f(t) = \sum_{i=1}^{p}(1+\alpha+\beta)C_i\mu_i \tag{2}$$

Where C_i is the carbon density of i-class tree, μ_i is the footprint ratio in the entire forest, α is 0.195, β is 1.244.

2.1.2. Establishment of a carbon sequestration model

(1) Carbon sequestration of uncut trees

When the total area of a forest, the average annual survival rate, the average age of the included trees, and the annual deforestation rate are known, the total carbon sequestration of the uncut trees in the forest in the t-th year can be calculated by combining formula (2). The formula for the quantity is as follows:

$$f(t) = \sum_{i=1}^{p}(1+\alpha+\beta)C_i\mu_i \tag{3}$$

Where S is the total area of a forest, δ is the average annual survival rate of a forest, t_0 is the average age of the trees in a forest, and k is the annual deforestation rate of a forest.

(2) Carbon sequestration of forest products from felled trees

According to the relevant data provided by the forest for production in 2020, there are three main types of forest products produced from the forest's fallen trees: paper and cardboard, wood pulp, and wood. They accounted for 55%, 16%, and 29% of the total felled trees, respectively. Their average lifespans were 20 years, 500 years, and 40 years, and the annual wear rates were 20.5%, 0.9%, and 10.8%, respectively. Then formula (2) is combined to obtain the total carbon sequestration of trees felled for production in the forest in year t.

$$\sum_{i=1}^{3}\sum_{j=1}^{t}\left[SkH_i\left(1-\alpha_i\right)^{t-j}(1-k)^{j-1}f(j+t_0)\right] \tag{4}$$

Where j is a specific year in t year, H_i is the proportion of the i-th type of forest products to the total felled trees and α_i is the annual loss rate of the i-th type of forest products.

(3) Carbon sequestration of the newly planted trees in the fell area

Since the areas in the forest where trees have been felled need to be used to plant new trees to maintain the balance of the forest ecosystem, it is easy to calculate the total carbon sequestration of new trees planted in these areas as:

$$\sum_{m=1}^{t} Sk\delta^{m}(1-k)^{t-m} f(m) \tag{5}$$

Where m is a specific year within t.

(4) Total carbon sequestration of the forest

when $0 \le t \le 30$ the total carbon sequestration of the forest in year t is:

$$\begin{aligned} F_0(t,k,\delta) &= S\delta^{t+t_0}(1-k)^{t} f(t+t_0) \\ &+\sum_{i=1}^{3}\sum_{j=1}^{t}\left[SkH_i(1-\alpha_i)^{t-j}(1-k)^{j-1} f(j+t_0)\right] \\ &+\sum_{m=1}^{t} Sk\delta^{m}(1-k)^{t-m} f(m) \end{aligned} \tag{6}$$

When $30 \le t \le 60$, the average age of trees after 30 years:

$$t_1 = \delta^{t_0+30}(1-k)^{30}(t_0+30)+\sum_{i=1}^{30} ik\delta^{i}(1-k)^{30-i} \tag{7}$$

When $30n \le t \le 30(n+1)$ (that is, after n times of 30 years), we can get the average age of trees after n times of 30 years and carbon sequestration model:

$$t_n = \delta^{t_{n-1}+30}(1-k)^{30}(t_{n-1}+30)+\sum_{i=1}^{30} ik\delta^{i}(1-k)^{30-i} \tag{8}$$

$$\begin{aligned} F_n(t,k,\delta) &= S\delta^{t+t_n-30n}(1-k)^{t} f(t+t_n-30n) \\ &+\sum_{i=1}^{3}\sum_{j=1}^{t-30n}\left[SkH_i(1-\alpha_i)^{t-30n-j}(1-k)^{j-1} f(j+t_n)\right] \\ &+\sum_{m=1}^{t-30n} Sk\delta^{m}(1-k)^{t-30n-m} f(m) \end{aligned} \tag{9}$$

2.1.3. Solution of the carbon sequestration model

The parameter values $F(t,k,\delta)$ are also changed in different forests. Take a mixed coniferous forest in Heilongjiang Province as an example. The average carbon density of its trees is $141.463/\left(1+5.777e^{-0.034t}\right)$ Mg/hm^2, the area of the mixed coniferous forest is 3.924×10^{6} m^2. A function of $F(t,k,\delta)$ with t, k was obtained by bringing the parameters into formula (6). The initial value of t is taken for the next 30 years, and δ is a constant, then $F(t,k,\delta)$ can be expressed as $F(k)$ which only related to the deforestation rate ***k*.**

The image of the function is shown in Figure 1 (The area of the forest affects the maximum of $F(k)$, but not k, which is the stationary point, so we omit it in the function). $F(k)$ peaks at k=0.045 after measuring the highest point of its image. Therefore, the best forest management scheme should account for 4.5% of the annual felled forest area in carbon sequestration.

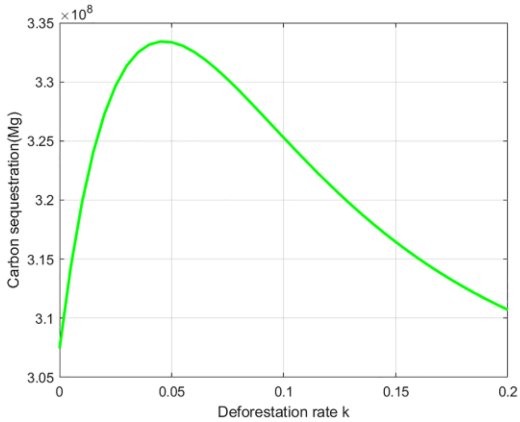

Fig. 1 Variation of carbon sequestration with a deforestation rate

2.2. A decision-making model for balanced forest valuation

2.2.1. Identification of constructed elements of forest management schemes

The values of a forest include carbon sequestration and the direct values of the benefits from felled trees, and the combined net present value of ecosystem regulation services expected to produce in the future, including biodiversity conservation and water conservation, climate regulation, and cultural value. Among them, we define the deforestation rate *k* corresponding to the optimal allocation of six valuation factors for a specific forest, which is called a transition point. Therefore, the constructed elements of forest management schemes can be set as the following six attributes:

(1) Carbon sequestration

According to the carbon sequestration model, the total carbon sequestration of the forest was obtained as follows:

$$\begin{aligned} Z_1 &= F_n(t,k,\delta) = S\delta^{t+t_n-30n}(1-k)^t f(t+t_n-30n) \\ &+\sum_{i=1}^{3}\sum_{j=1}^{t-30n}\left[SkH_i(1-\alpha_i)^{t-30n-j}(1-k)^{j-1}f(j+t_n)\right] \\ &+\sum_{m=1}^{t-30n} Sk\delta^m(1-k)^{t-30n-m}f(m) \end{aligned} \tag{10}$$

(2) Annual deforestation benefit

$$W=\sum_{i=1}^{3}H_i\omega_i$$
$$Z_2=\frac{\sum_{j=1}^{t}Sk(1-k)^{j-1}f(j+t_n)W}{\gamma} \tag{11}$$

Where W is the total income of forest products per unit forest area, ω_i is average proceeds per unit volume of the i-class forest product, γ is the carbon sequestration of trees.

(3) Biodiversity

Since the new and long trees have very little impact on diversity compared to the old ones in the short term, biodiversity is mainly related to the old tree floor area:

$$Z_3 = B(1-k)^t \tag{12}$$

Where B is the biodiversity coefficient of the old tree region, which is related to the specific forest.

(4) Cultural value

Since the cultural value-related data of a forest are difficult to count, we assume the following function [7] with the biodiversity within a forest:

$$Z_4 = \rho Z_3^{\sigma} \tag{13}$$

Where ρ and σ are the conversion coefficients, $0<\sigma<1$. Since ρ was irrelevant after the normalization treatment, so generally $\rho=1$.

(5) Water conservation

We used to replace the cost of reservoir construction with the value of water conservation [8]:

$$Z_5 = V_s = 10SC\left(P-R-ET\right)\cdot \left[g(t_0+\tau)(1-k)^{\tau} + \sum_{i=1}^{\tau} g(i)k(1-k)^{\tau-i}\right] \tag{14}$$

Where V_s is the value of water conservation (\$), C is the project cost of the unit reservoir storage capacity (\$/m^3), P is the rainfall (mm), R is the rainstorm runoff (mm), ET is the amount of steam (mm), $g(t)=1/\left(1+5.777e^{-0.17t}\right)-1$ obtained through the Logistic growth formula is the volume ratio of forest t-age trees to t'-age trees τ is tree maturation.

(6) Climate regulation value

The shadow engineering method [9] can also be used to replace the electricity cost of air conditioning with the climate regulation value:

$$Z_6 = Q'\times C'\times\left[F(t)-F(0)\right] \tag{15}$$

Where Q' is the energy stored within the per-unit net volume of trees C' is transferring the cost of the unit energy. Taking the air conditioner as an example, the calculated Q' is 3.91×10^7 *j/g* and C' is 1.58×10^{-7} *yuan/j*.

Therefore, the following formula is obtained:

$$Z_6 = 6.1778\left[F(t,k,\delta)-F(0,k,\delta)\right] \tag{16}$$

2.2.2. Development of a decision-making model

The best allocation of the forest management scheme can be determined according to a TOPSIS solution space model [10]. We can take a mixed coniferous forest in Heilongjiang Province as an example:

Step1: The above matrices were normalized by the vector planning method

$A=\left(a_{ij}\right)_{m\times n}$ is the original data matrix; $B=\left(b_{ij}\right)_{m\times n}$ is the normative decision matrix.

$$b_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m}(a_{ij})^2}},\ \ i=1,2,\cdots,m;\ \ j=1,2,\cdots,n \tag{17}$$

Step2: We can derive the weighted gauge matrix $C=\left(c_{ij}\right)_{m\times n}$ from the perspective of forest managers

$$\begin{gathered} Weight\ vector\ \vec{\omega}=[\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6] \\ c_{ij}=\omega_j b_{ij},\ \ i=1,2,\cdots,m;\ \ j=1,2,\cdots,n \end{gathered} \tag{18}$$

Step3: Determination of the positive ideal solution C^+ and the negative ideal solution C^-

Set up: The No. j attribute value of C^+ is c_j^+, the No. j attribute value of C^- is c_j^-.

$$\begin{gathered} c_j^+ = \begin{Bmatrix} max_i c_{ij},\ j\ is\ the\ benificial\ attribute \\ min_i c_{ij},\ j\ is\ the\ intrinsic\ attribute \end{Bmatrix}\ j=1,2,\cdots,n \\ c_j^- = \begin{Bmatrix} min_i c_{ij},\ j\ is\ the\ benificial\ attribute \\ max_i c_{ij},\ j\ is\ the\ intrinsic\ attribute \end{Bmatrix}\ j=1,2,\cdots,n \end{gathered} \tag{19}$$

Step4: Calculation of the distance between each scheme and C^+ and C^-

The distance from the alternative (d_i) to C^+ is:

$$s_i^+ = \sqrt{\sum_{j=1}^{n}(c_{ij} - c_j^+)^2}, \quad i = 1, 2, \cdots, m \tag{20}$$

The distance from the alternative (d_i) to C^- is:

$$s_i^- = \sqrt{\sum_{j=1}^{n}(c_{ij} - c_j^-)^2}, \quad i = 1, 2, \cdots, m \tag{21}$$

Step5: Calculation of an evaluation index value (comprehensive evaluation index)

$$f_i^* = \frac{s_i^-}{s_i^+ + s_i^-}, \quad i = 1, 2, \cdots, m \tag{22}$$

Step6: Plot a figure of the comprehensive evaluation index (f_i^*) as a function of the deforestation rate (k)

2.2.3. Solution of the decision-making model

We still chose a mixed coniferous forest in Heilongjiang Province as an object and counted all the six factors. The weight relationship is $\vec{\omega} = [0.28, 0.34, 0.21, 0.05, 0.05, 0.07]$. The values of some parameters are shown in Table I.

Table. 1 values of some parameters

Parameters	Values	Parameters	Values
α	0.195	R	360
β	1.244	ET	238
γ	0.500	ρ	1
q	0.570	δ	0.995
S	3.924×10^6	B	1
C	12	τ	30
P	753		

We normalized the statistics (eliminating the effect of the magnitude) and obtained the relationship between six considerations of the management method and different deforestation rates k. We finally obtained the relationship between the comprehensive evaluation index used to describe the forest management scheme and the deforestation rate, as shown in Figure 2. In addition, the optimal deforestation rate (indicated by the solid red square in Figure 2) was determined to obtain the best forest management scheme.

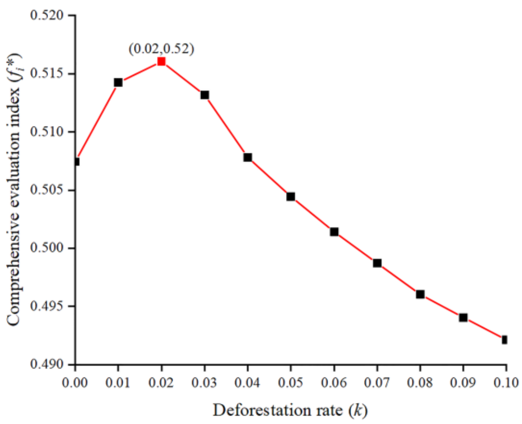

Fig. 2 Variation of the comprehensive evaluation index with a deforestation rate

2.2.4. Sensitivity analysis of the decision-making model

Due to the subjectivity in the weight values of the decision-making model, it is necessary to explore the impact of each weight change on decision-making, that is, the sensitivity of each valuation factor. By the control variable method, only one of the six factors was changed, and it was multiplied by weighted volatility to explore the sensitivity of various valuation factors to the decision-making model. According to the parameters in the decision-making model for a mixed coniferous forest in Heilongjiang, the following sensitivity analysis diagram was obtained after substituting the data (Figure 3). The horizontal axis is the weight fluctuation rate, and the vertical axis selects the optimal decision-making model.

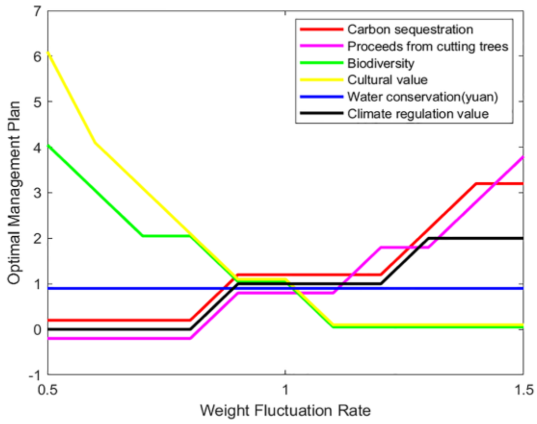

Fig. 3 Sensitivity analysis of the decision-making model

As can be seen from the figure, the slope of each line at different fluctuation rates represents the sensitivity of a valuation factor. The larger the slope, the higher the sensitivity. It can be found that the most sensitive factor is a cultural value, followed by biodiversity and the benefits of felling trees, and the least sensitive is water conservation. Therefore, in order to adapt the decision-making model to the needs of forest managers and forest users, the weight of income and cultural value of forest products should be appropriately adjusted, and the weight of water protection should be reduced.

3. Model assessment and promotion

The carbon sequestration model developed in this study is relatively novel. Attributes related to population, culture, topography, and climate were not selected when selecting attributes to characterize the carbon sequestration content of trees. Although they can describe carbon sequestration, their effects are complex and not easily expressed quantitatively. Instead, the parameter of carbon content (C) per unit area of a forest was used, which takes into account the influence of those factors on carbon sequestration and establishes a more accurate carbon sequestration model. In addition, the evaluation model of forest management scheme based on a TOPSIS solution space model considers many aspects of forest value, and the evaluation is comprehensive and reliable.

However, extensive field measurements need to be performed to obtain the parameter of carbon content per unit area of a forest, and it is not easy to accurately determine this parameter. In addition, the year-on-year decrease in tree deforestation of forests causes dynamic fluctuations in the average age of forest trees, and the developed carbon sequestration model can only accurately predict carbon sequestration in forests with a consistent average age of trees. At the beginning of the next phase, the average age of trees needs to be calculated to make accurate long-term predictions of carbon sequestration in forests and trees.

The parameter selection of the carbon sequestration model developed in this study is based on the real forest, and its unique carbon sequestration model can be established for a variety of forests. The model can accurately monitor the carbon sequestration of forests and their products at any specific time in the short term and has strong generality and generalizability. In addition, considering the multiple values of forests, the decision-making model for balanced forest assessment developed using the TOPSIS solution space model provides forest managers with management plans for optimal use of forests. By combining the differences in the various values of different forests, this decision-making model can be easily and quickly applied to the management of forests with different climates, regions, and other environmental conditions and has important economic, social and ecological benefits.

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