The principle and state-of-art applications of permutation tests

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Abstract. Contemporarily, permutation test is widely used in plenty of types of verification in state-of-art bigdata analysis. In this paper, the principle of permutation tests will be demonstrated with corresponding applications in various fields. Generally speaking, the permutation test can be regarded as an exact exam. On this basis, it can be also treated as a type of statistical significance level verifications where the value of sample distributed among the null hypothesis and alternative hypothesis are derived according to resampling and calculating of all possible combinations of the element. In other word, permutation test can be also defined as a procedure of resampling. For the sake of explicating the applications and meaning of the test clearly, this paper will first introduce the basic descriptions and formulae of the test and illustrate the applications subsequently. Specifically, the result of principle of permutation test is constructing null hypothesis and significance level, then equation will be presented to calculate p-value to compare with the primary data to determine one whether rejects the null hypothesis or not. Besides, permutation tests' extensive functions on biological, medical, and economical areas will be discussed. Based on the analysis, these results offer a better understanding on principle and applications of permutation tests.

Keywords: Permutation tests; Statistic tests; Statistic applications.

1. Introduction

In retrospect, the proposal of the permutation tests can be traced back date to the 1930's, which was first designed and proved by Fisher, Pitman and other statistical figures [1-3]. However, without the help of computers, these exams could only be utilized for cases with small values in the selected samples in the era of its birth, i.e., restricted the applications in most of approaches [4-6]. Even with today's computing power, the large number of permutations associated with even modest sample size is still a challenge. Initially, the permutation tests are implemented for two sample problems the single variable linear fitting/regression and block designs [1, 3, 7]. Moreover, the randomization tests introduced by Fisher is also considered as a special type of permutation tests [7]. Contemporarily, permutation tests are great because their non-parametric properties and only require the assumption of exchangeability [8]. Permutation tests develop relatively data to estimate the population. Scholars and researchers usually assigned p-value to observed data as well.

Permutation tests have widely applications at present. To be specific, the term permutation needs a weak distributional hypothesis that ensures the interchangeability. In addition, the permute value will be varied in terms of the alternative assumptions, while maintaining other correlated relationship in the sample [9]. Some examples of permutation tests are permutation with the presence of nuisance, permutation with the existence of the inherit relationships in samples, and comparison between two models. Overall, the permutation test scenario is a widely utilized instrument to evaluate effects in experiments and other data-analysis based study. In detail, it is rather simple to carry out without the limitation of other assumptions required by other tests [10, 11].

In this paper, the principle as well as the applications of permutation tests will be demonstrated based on information retrieval as well as literature analysis. Specifically, the applications in the field of human disease and animal research will be presented. The rest part of the paper is organized as follows. The Sec. 2 will introduce the basic principle of permutation test. The Sec. 3 will present the applications of it in the research area of animal research and human disease. Eventually, a brief summary is given in Sec. 4.

2. Principle of permutation tests

Permutation test is an approach of statistical hypothesis testing to verify the validity of a certain statement about a parameter associated with a well-defined underlying population. It is always used for a sample with abundant cases which cannot be tested as usual. Besides, permutation tests are non-parametric tests that require very few assumptions. Therefore, when there is less data generating the population, permutation tests are an effective way to determine statistical significance [1].

To use the permutation test, one first needs to state clearly what situation will prevail if the hypothesis to be tested is not true as "alternative hypothesis". Subsequently, a carefully constructed sample from the population of interest ought to be taken, which all sample values are mutually independent of each other. Afterward, the researcher combines the sample values into a single statistic, and this would be a statistic that had already been found to be a good "good" estimator of the parameter under test which these estimators would have to be transformed to make it "pivotal" (sampling distribution that does not depend on any other unknown parameters). Subsequently, the researcher tests whether the null hypothesis were true or not by T-test and P value. In other words, the result of test statistic is "surprising" or not. If it is surprising, then one rejects the null hypothesis. Conversely, the corresponding null hypothesis would not be rejected accordingly [2]. To apply the T-test and calculate the P value, constructing significance level and setting up the alpha will be the primary step, and the p-value with the value of alpha is compared. In this case, the null hypothesis will be rejected if the p-value is smaller than alpha but failing to reject the null hypothesis if the p-value is greater than alpha. The formula of p-value is

$$p(t \geq t_0) = \frac{\sum_{j=1 \times I(t_j \geq t_0)} (n+m+j)!}{(n+m)!} \tag{1}$$

Here, the t_0 is the measured point of the test statistic, and t represents for the obtained t-value obtained from the resampling objects. The construction of significance value is also very important in the whole process under the condition for checking the null hypothesis, where one could make the recorded differences in order. Subsequently, it is necessary to observe if T is covered in the middle (1-å) ×100% for a certain significance degree å. Otherwise, one ought to refuse the assumption of identical probability curves at the å×100% significance extent. However, permutation tests have some limitations as well. As mentioned earlier, a critical hypothesis is that the value measured from resamples of permutation test are swappable in the case of the null hypothesis, but sometimes the observations are not exchangeable which cannot form two-sided randomization test or paired two-sample randomization test. With regarding to some special applications, permutation tests based on statistics of appropriate study can be asymptotically accurate, even though the commutability hypothesis is violated.

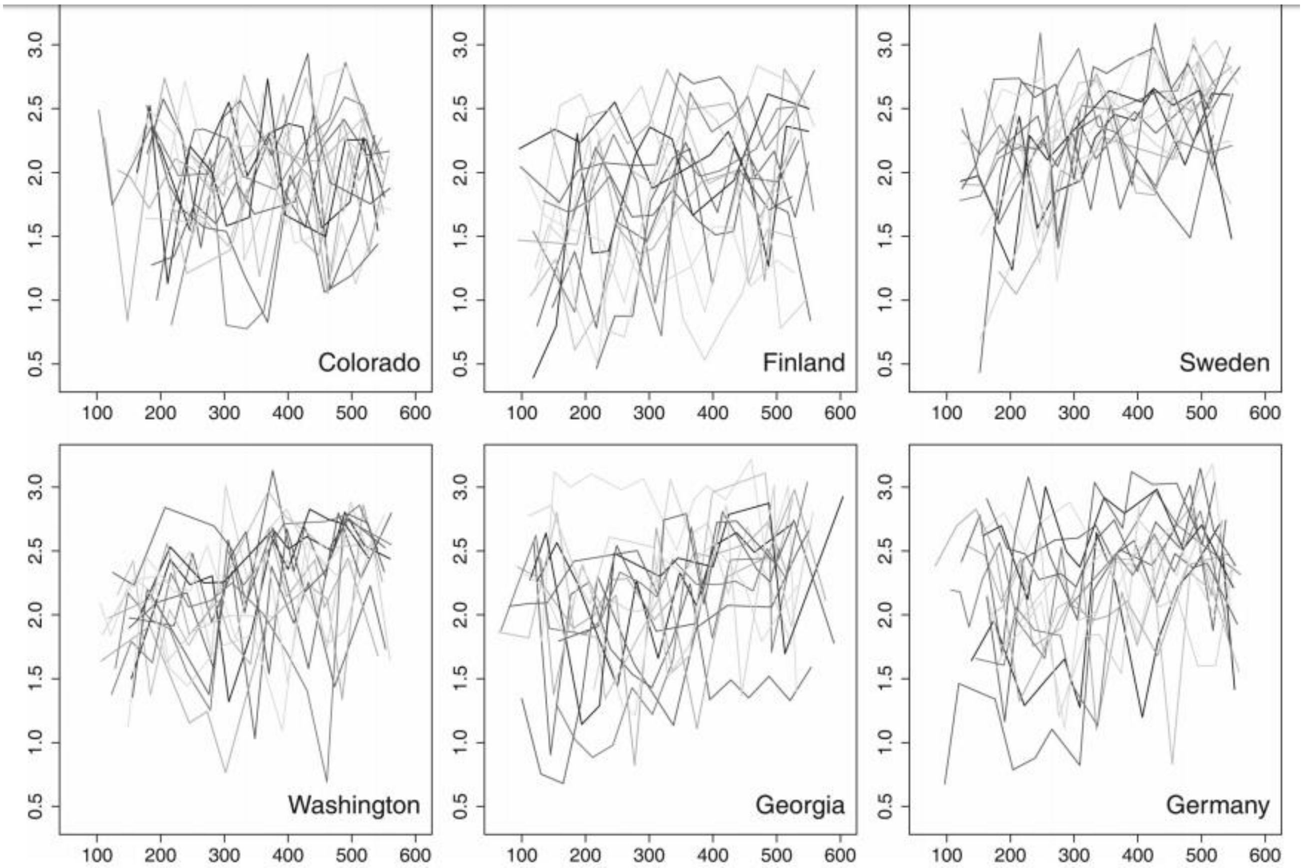

Figure 1. The results for the six different selected countries [12].

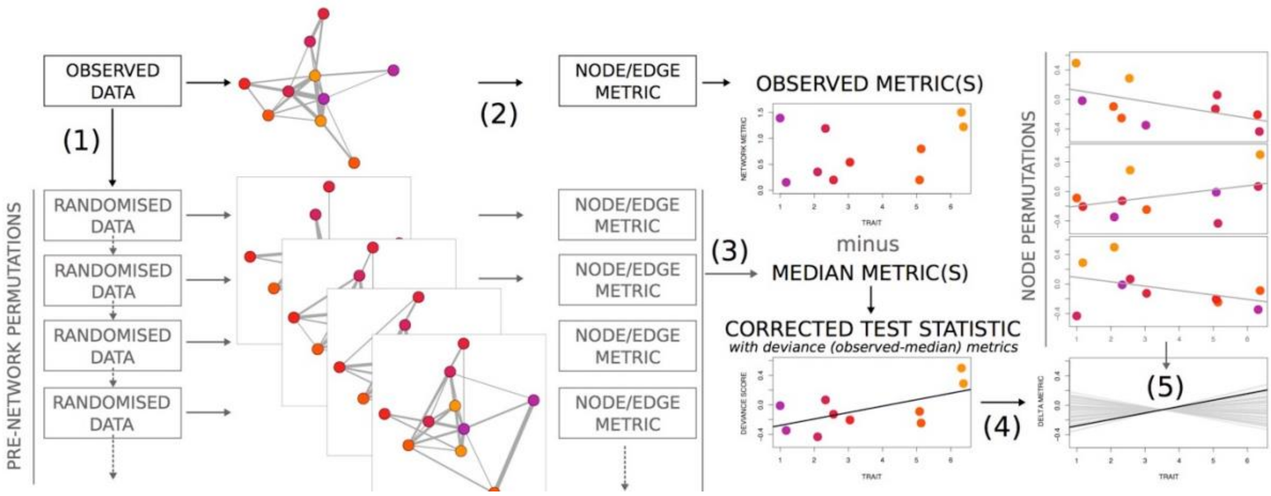

Figure 2. The approach and framework of permutations for the biology case [13].

3. Applications

The application of permutation test is very diversity, which is widely used in biological, mathematical, medical, economical, and other different areas. The first example is permutation test's function in biostatistics area. As a matter of fact, scholars suspect whether the gut microbiome plays a vital role in the development of T1D. To address the issue, the composition of the gut microbiome of children in six different locations was analyzed across time accordingly. To be specific, the selecting observables are children with high dangerous diagnosed from gene for T1D while are free of islet autoantibodies or disease. TEDDY's team collected intestinal microbiome data from children at six different sites. The sample units were children at high risk of type 1 diabetes, but with no current islet autoantibodies or disease. According to the analysis, one draws the conclusion that the microbiome diversity across time varies a lot in the focusing six choosing sites.

Permutation test also has extensive application in animal action researching area. To study group of animals, one needs to splits samples into two groups: the controlling and an experimental group with corresponding assumptions that the two subsets are homogenous. The step of researching is approximately as same as the normal permutation test. One applies an operation to adjust the conditions of the experimental group and measures the feedback of the measurement. Subsequently, researchers should take consideration of all possible configuration and value of the experimental group as well as the recorded retroaction. Finally, researchers the p-value will be obtained based on the replying correlated to all the potential experimental groups. Take mice as an example, the goal is to investigate the duration of the mice accomplish the novel maze. One wishes to offer proof to support an experimental treatment, demonstrating the experimental group will address the issue faster than the controlling group. Assume there are six mice A, B, C, D, E, F, and research randomly pick three mice in treatment group and other three in observation group. They pick A, C, E in treatment group and treat them with specific medical. Through the testing, one observes that A runs the maze in 10 seconds, C runs the maze in 9 seconds, and E runs the maze in 11 seconds. Respectively, B runs the maze in 12 seconds, D runs the maze in 11 seconds, and F runs the maze in 13 seconds. Therefore, the question is the treatment accelerate the finishing time of maze or the result just appear as lucky phenomenon? Subsequently, tester build null hypothesis as there is no difference between treatment groups, and alternative hypothesis is the average time for all mice with the adjustment will be quicker. There are totally C (6,3) =20 numbers of permutation among all choice of A, C, E in six mice. Therefore, the specific one with A, C, E in the experimental group has a probability of 5% occurring. One lists all the twenty choice and calculate the mean time of each group. Subsequently, one continually ranks the difference among the average level. Additionally, one tabulates the ratio of the different groups varying in average. Comparing these results, research set up p-value of 10%. Finally, research can hold the null hypothesis or reject null hypothesis after the simple calculation. It is a simple application of permutation test in animal researching area.

In Ref. [13], it is shown that the rate of false positives and false positives around 5% in a range of conditions. The proposed scenario (seen from Fig. 2) applies pre-network permutations in order to measure nuisance effects based on restricting exchange of observations within time and space blocks. Afterwards, node permutations arde implemented for the sake of nonparametric exam of the parameters impacts. This "two-permutation" approach tests the null hypothesis that there is no relationship between predictors and random social structure biases (under specific temporal and spatial constraints).

4. Conclusion

In summary, this paper introduces the principle of permutation tests as well as illustrate the corresponding various state-of-art applications including the field of human disease and animal research. Based on the analysis, the paper figures out the using step of permutation tests and the relationship between the principle of permutation tests and the application of it in different areas. According to the results, permutation tests are not only applied in various fields, but also improves the efficiency and effectiveness of statistical data. Nevertheless, there are some drawbacks for nowadays approaches. Permutation tests are inefficient compared with asymptotic tests that it might be impossible to control alpha at the appropriate level and still use permutation test when people have only a few observations. In the future, people should use other tests if research have no data comparisons and a few observations. Moreover, people should continually study deeply about the permutation tests. Overall, these results offer a guideline for employing and applications of permutation tests.

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