

Study on the robustness of Washington Metro Network

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Abstract. The Washington subway networks is taken as the research object. The space-L network topology model of the Washington subway network is constructed, and the complex network characteristics and robustness of the Washington subway network are studied by complex network theory. Firstly, by calculating different topological network statistics, the complex network characteristics of the Washington subway network are analyzed. Secondly, through the two simulations of random attack and deliberate attack on the Washington subway network, the index values of the two attack modes are counted and their changing trends are analyzed, and the robustness of the Washington subway network under attack is analyzed. The results show that the Washington subway network has strong robustness. Deliberate attacks are more destructive to the Washington subway network, while random attacks have less impact.

Keywords: Complex network, subway network, robustness, random attack, deliberate attack.

1. Introduction

The subway, as a means of transportation, has the advantages of convenience, low pollution and large passenger capacity by the rapid development of economy and increasing congestion of traffic roads.

The analysis of subway network with complex network system can directly reflect the stability and structural characteristics of subway structure. Pan Yilei et al analyzed the Wuhan subway network and pointed out that it has the characteristics of high clustering coefficient and short path length, showing the characteristics of small world network[1]. Yin Xiaohong et al used Space L method to construct the topology of Hefei subway, and simulated the robustness of Hefei subway network in different scenarios. The results show that the Hefei subway network is different from the scale-free network and the small-world network [2]. When the key nodes are attacked under the cascading failure scenario, the structural damage is the most serious. Elisa Frutos Bernal studied the main structural characteristics, topological characteristics and robustness characteristics of the Madrid metro network [3]. Hailong Wang et al's research shows Shenzhen Metro has serious traffic connectivity [4]. After deleting the important seven nodes, the average node degree decreases by 4, the average shortest path increases by 0.3, and the network diameter or clustering coefficient changes relatively little, indicating that these seven important nodes play an important role in transfer travel and can effectively reduce the average transfer times. Taking Shanghai metro network as an example, Yuanyuan Wang simulates five attack scenarios to study cascading failure process and network vulnerability [5]. The results show that cascading failure has caused serious consequences in the subway network. The vulnerability of subway network without considering cascading failure is underestimated. A bus-subway interdependent network model based on passenger transfer relationship is constructed by Chen Chen [6]. The vulnerability process of interdependent network to different disturbances from the perspective of structure and function is studied. Experiments show that the increase of coupling distance cannot effectively alleviate the vulnerability of interdependent network. Shaojie Wu studied the subway network in Boston and verified that the subway network has small-world characteristics, and proposed the concepts of network efficiency and connectivity index [7].

In the study of complex networks, the average path length, clustering coefficient and degree distribution can reflect complexity of the network. This paper analyzes complexity of the Washington subway network by using complex network theory and topology.

Washington subway network studied in this paper has the characteristics of large number of nodes (stations) and complex network structure. Therefore, based on the complex network theory, this paper constructs the Space-L network topology model of Washington subway network, calculates degree distribution, average degree, standardized average degree, average path length, diameter, robustness, efficiency and other related topological parameters of subway stations, and analyzes the complex characteristics of it. This paper also studies the robustness of Washington metro network, and formulates two strategies of deliberate attack and random attack on Washington metro network. The relative size, efficiency, connectivity, number of cycles and average path length of the largest connected subgraph under the two attack strategies are recorded.

2. Network construction

2.1. Modeling method

In this paper, we use a network graph $G = (V, E)$ to represent a specific complex network. The number of nodes is $N = |V|$. And the number of edges is $M = |E|$.

2.1.1. Adjacency matrix

The adjacency matrix is a network used to analyze the representation of practical problems in computer. The adjacency matrix $A = (a_{ij})_{N \times N}$ is a square matrix of order N . For the element a_{ij} on the j column of the i line of the unweighted undirected graph, the definition is as follows:

$$a_{ij} = \begin{cases} 1, & (vi, vj) \in E \\ 0, & other \end{cases} \quad (1)$$

2.1.2. Space L and Space P

A subway network is abstracted as a graph, and the subway station can be defined as a node. The method of using the subway network to describe the node connection relationship can be summarized into two kinds: one is the Space L method, that is, the station is regarded as node. If two stations are adjacent on a subway line, they are connected by edges; the other is the Space P method, that is, the subway station is regarded as a node. If two stations have direct subway lines, they are connected by edges. This paper selects the Washington subway network for analysis, and will use the Space L method to study the Washington subway network. Statistics show that there are 115 subway stations and 6 subway lines in the Washington subway network as shown in Figure 1.

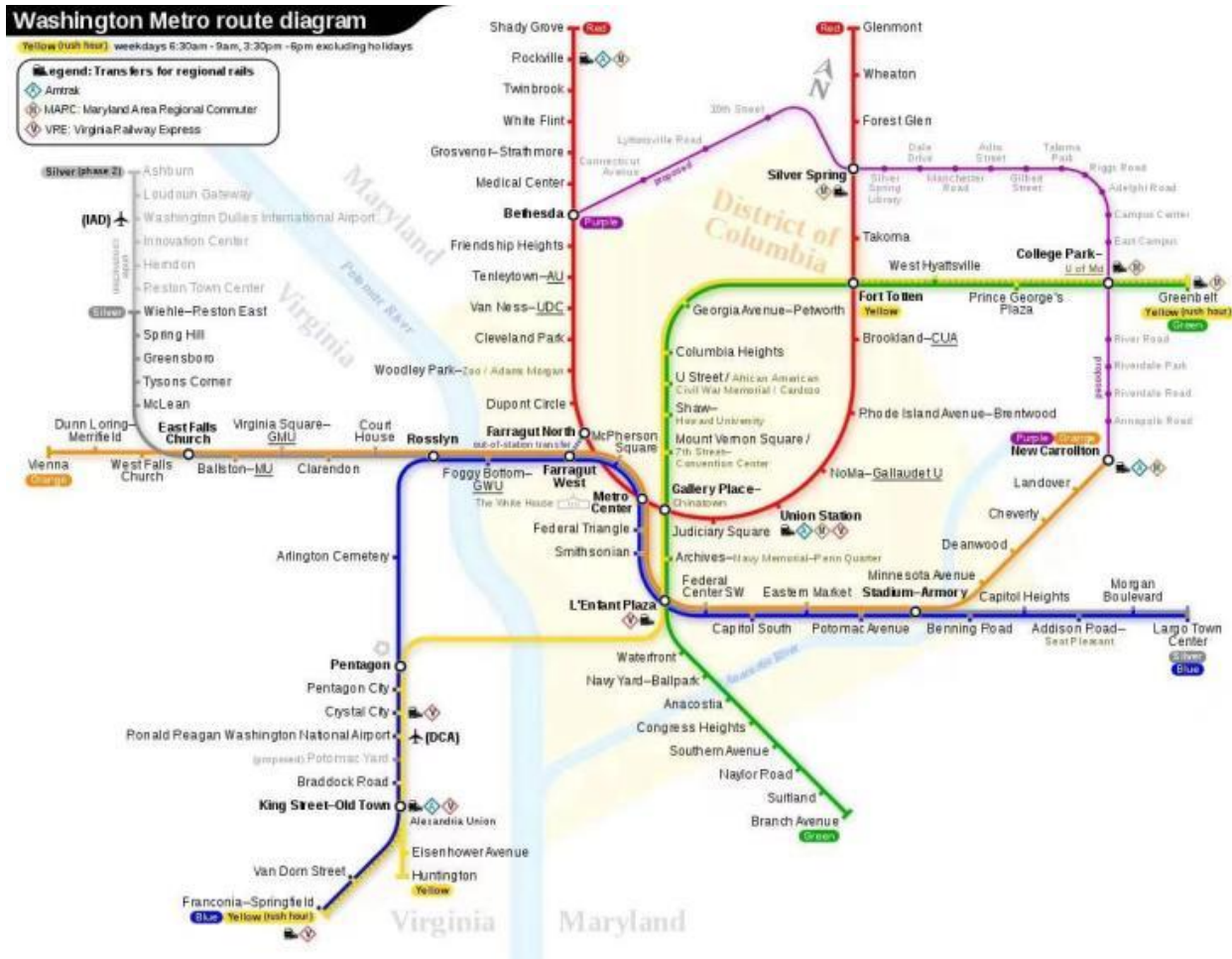


Figure 1. Washington Metro Map.

2.2. Statistical parameters

2.2.1. Degree

The average degree reflects the importance of nodes in the network. The degree k_i of node i is defined as the number of edges connected to the node. For a simple graph without self-loops and multiple edges, the degree k_i of node i is the number of other nodes that node i has direct links to.

$$\langle k \rangle = \frac{2M}{N} \quad (2)$$

The average degree of all nodes in the network is the average degree of the network. The normalized mean degree is $2M / [N(N+1)]$.

The degree distribution describes the probability that the degree of a randomly selected node is exactly k . The degree distribution of nodes in the undirected network is represented by the distribution function $P(k)$. When the degree distribution of network nodes obeys the power-law function, the network exhibits scale-free network characteristics.

$$p(k) = Ck^{-\alpha} \quad (3)$$

Where C is the proportional constant and α is the power exponent.

2.2.2. Average path length

The distance d_{ij} is defined as the number of edges on the shortest path connecting the two nodes, and the average path length L of the network is defined as the average distance between all nodes.

$$L = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} d_{ij} \quad (4)$$

2.2.3. Diameter

The maximum distance between any two nodes in the network is diameter D.

$$D = \max\{d_{ij}\} \quad (5)$$

2.2.4. Clustering coefficient

Clustering coefficient reflects the closeness between sites around the site. Suppose that the degree of a node i in the network is k_i , if the k_i adjacent nodes of the node i are neighbors to each other, and the clustering coefficient C_i is the ratio of the actual number of edges e_i between all adjacent nodes to the total number of possible edges $k_i(k_i-1)/2$.

$$C_i = \frac{2e_i}{k_i(k_i-1)} = \frac{\sum_{j,m} a_{ij} a_{im} a_{mi}}{k_i(k_i-1)} \quad (6)$$

In the above formula, a_{ij} is the adjacency matrix. Clustering coefficient of the network is the average clustering coefficient of all nodes, that is,

$$C = \frac{1}{N} \sum_{i=1}^N C_i \quad (7)$$

2.2.5. Robustness

Robustness reflects the stability of the system. When the degree of node connection is damaged, system transmission capacity is affected.

$$X = \frac{\ln(M - N + 2)}{N} \quad (8)$$

2.3. Reliability evaluation index of subway traffic network connectivity

2.2.1. The relative size of the largest connected subgraph

The relative size of the largest connected subgraph S represents the network structure after it is destroyed. Where N' is the number of nodes in the largest connected branch contained in damaged network, and N is the sum of the number of nodes, that is,

$$S = \frac{N'}{N} \quad (9)$$

2.2.2. Connectedness

Connectivity reflects the mutual connectivity of nodes. It reflects the ratio of the actual number of network edges to the theoretical maximum number of edges.

$$\gamma = \frac{|M|}{3|N| - 6} \quad (10)$$

2.2.3. Cycle

The cycle number μ defined by Berge is used to represent the alternative path of the network after the attack.

$$\mu = |M| - |N| + 1 \quad (11)$$

The above formula is normalized to obtain the cycle rate μ^T

$$\mu^T = \frac{\mu}{N} \quad (12)$$

2.2.4. Efficiency

Efficiency X is the evaluation of network connectivity, which represents the reliability of the network. When the distance between two sites w_{ij} is larger, the global efficiency of the network is lower.

$$E = \frac{2}{N(N-1)} \sum_{1 \leq i \leq j \leq N} \frac{1}{a_{ij}} \quad (13)$$

3. Washington metro network structure analysis

3.1. Static index analysis

This paper analyzes the complexity of Washington's subway network by selecting Washington's subway route map. Combined with the actual network of Washington subway, the adjacency matrix of subway network is constructed by using graph theory, and network topology model is constructed by using Space L modeling method. Static parameter statistics of Washington metro network are obtained by Matlab, including the average degree, standardized average degree, average path length, diameter, clustering coefficient, efficiency X, robustness r, in Table 1.

According to the calculation results of Table 1. Average degree of the subway network is 2.0870, that is, the average number of destinations can be reached directly without transfer. The average path length is 12.6946, the network diameter is 35, indicating that when the travel volume of each station is equal, each person takes the subway to reach the destination through an average of 12 stations.

Table 1. Washington metro network static index value.

Static index	average degree	Standardized average	Average path length	diameter	robustness	efficiency
numerical	2.0870	0.0180	12.6946	35	0.0169	0.1264

3.1.1. Degree and degree distribution of nodes

Figure 2 is degree distribution map of Washington subway network, describing its node distribution.

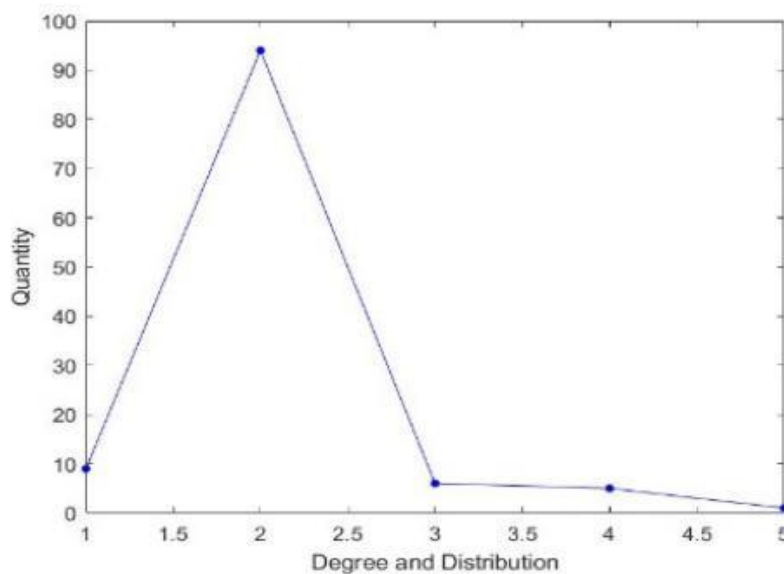


Figure 2. Degree and degree distribution of nodes in the Washington Metro network.

The results of figure 2 show that the node with degree 2 accounts for the highest proportion of the total nodes, which is 82 %, indicating that the Washington subway mainly relies on the station with degree 2 to achieve, so that the convenience of taking the subway is higher. At the same time, higher the degree of the node, higher the influence of the node in the subway network.

3.1.2. Clustering coefficient

Clustering coefficient $C = 0$ of the Washington metro network is calculated, so according to definition of C , it can be found that each node in the network does not have a 'triangle' structure, and the lower the clustering coefficient and the greater the possibility of transfer provided by the network, but in the event of failure, the damage to the entire network will be greater, so the Washington network has higher transferability and low damage resistance.

3.2. Connectivity Reliability Analysis of Washington Metro Network

3.2.1. Evaluation index of subway network robustness

Complex networks's robustness is to study the impact on the transmission behavior of the system when the degree of node connection in the network is impaired [8]. The subway network robustness studied in this paper refers to whether the subway network can maintain connectivity when some stations fail or are attacked. Lai Liping [9] pointed out that the evaluation indexes of subway network robustness mainly include: connectivity, network efficiency, relative size of maximum connected subgraph and cycle rate. Cheng Yi [10] measured the robustness of Guangzhou rail transit by studying the changing trend of global probability and betweenness after node attack. In this paper, the maximum connected subgraph node ratio, global efficiency, connectivity and cycle rate of the network are used as measurement indicators to simulate the attack on the Washington subway network to analyze the robustness of it.

3.2.2. Subway network random attack and deliberate attack

Breakdown of subway may be caused by natural environment disasters, machine failures or signal problems, and the attacks may include malicious attacks or terrorist attacks. There is uncertainty in the occurrence of faults. The attack is targeted and has a strong purpose. In order to cause greater attack effect, people generally choose to attack crucial nodes in the network. This paper will simulate the Washington metro network after random failure and deliberate attack. In the simulation process, nodes with the highest degree value are removed to simulate the deliberate attack, and nodes with the same degree value will be selected by generating random numbers. In order to make the simulation results more credible, this simulation will carry out 5 simulation experiments, and take the average value of node ratio, global efficiency, connectivity and cycle rate of the largest connected subgraph after each experiment. Because the Washington subway network has 115 stations, in order to make the simulation results more comprehensive, this article will select 40 stations to attack.

Before the simulation experiment, two hypotheses are made. Each attack only targets one station, that is, each attack only destroys one node. When the network crashes, the robustness index is 0. In reality, after the local subway network suffers a certain degree of attack, its global efficiency will drop to a very low level, and the role of the subway network is almost 0. Therefore, it can be considered that when the global efficiency is 0, the subway network is equivalent to a crash state.

3.3. Simulation experiment under node failure strategy

When the Washington subway network is randomly faulty, the global efficiency decreases more evenly, and there is no obvious fluctuation during the decline. When the Washington subway network is deliberately attacked, the global efficiency decreases greatly at the beginning, and the decline tends to be gentle when the number of nodes attacked is large. Global efficiency curve of subway network under random attacks and deliberate attacks is shown in Fig.3.

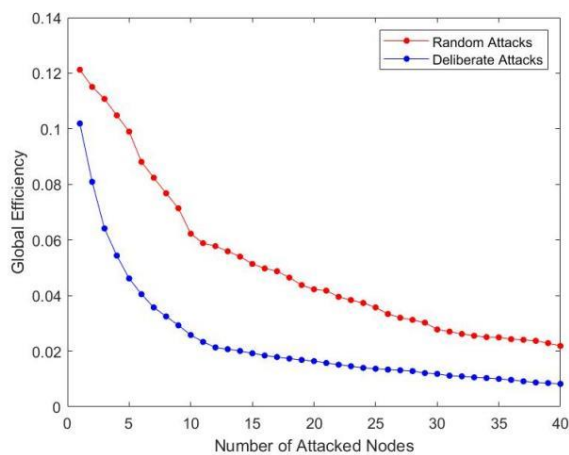


Figure 3. Global efficiency.

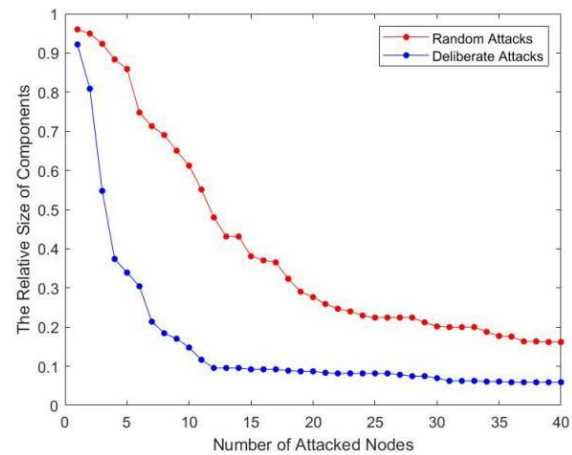


Figure 4. Relative size of components.

When the Washington metro network station is subject to random failure, relative size of the largest connected subgraph drops twice. The first time when 5-6 nodes fail, the second time when 10-15 nodes fail. When the number of nodes with obstacles is large, the relative size of the largest connected subgraph decreases more evenly. When the Washington subway network is deliberately attacked, it suffers a greater degree of damage even only a few nodes are attacked. When 2 and 3 stations are deliberately attacked, the relative size of the maximum connected subgraph of the network decreases greatly. If a large number of attack nodes exist, the relative size of components decreases slowly. The relative size change curve of the maximum connected subgraph of the network under random failure and deliberate attack is shown in Fig.4.

It can be seen that the response of the Washington subway network to deliberate attacks is stronger than that of random failures when few nodes are attacked. When the number of attacked nodes is greater than a certain value, the response to deliberate attacks is weakened. At this time, the response to random failures is stronger than deliberate attacks.

For connectivity, the connectivity of Washington metro network stations will decrease uniformly when they are subjected to random faults and deliberate attacks. Compared with random faults, the connectivity of Washington metro network stations decreases significantly after deliberate attacks. This shows that when the Washington subway network suffers from external damage, the deliberate attack on the subway network is faster. The curve of connectivity under random failure and deliberate attack is shown in Fig.5. The network cycle rate curve of the Washington subway network under random failure and deliberate attacks is shown in Figure 6.

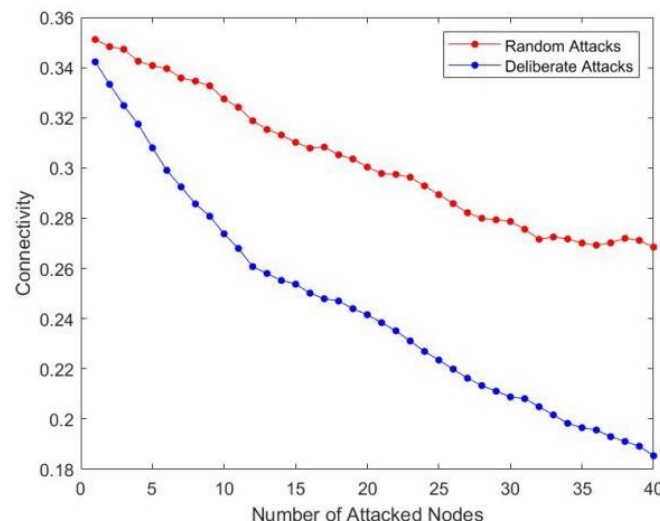


Figure 5. Change curve of network connectivity.

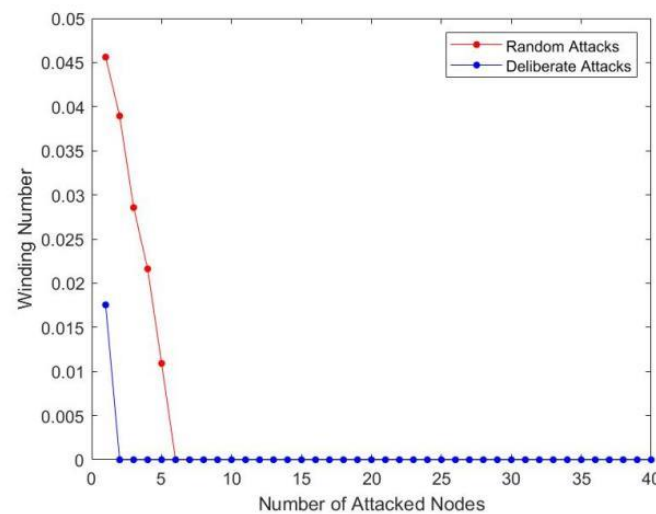


Figure 6. Network turn rate curve.

The curve of the average path length of Washington Metro network under random attacks and deliberate attacks is shown in Figure 7. As is revealed in Figure 7, in random attacks, when it is small in the number of attack nodes, the average path length value does not change significantly. When the number of attack nodes exceed a certain value, the average path length starts to fall gently. In contrast, the average path length of deliberate attacks shows a downward trend when they are attacked at the beginning. The average path length value gradually drops to a lower value with the increasing number of attack nodes.

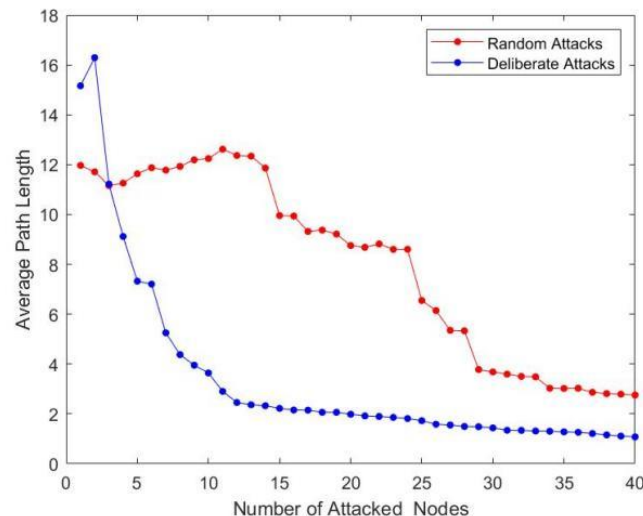


Figure 7. Average path length change curve.

It follows that the Washington Metro network responds more strongly to deliberate attacks than random ones.

4. Conclusions

The Space-L method is used to construct the Washington subway network. Then this paper analyzes the complex network characteristics of Washington metro network. Through MATLAB software, two simulations of random attack and deliberate attack on the Washington subway network are carried out to verify the robustness of the network. Draw the following conclusions:

(1) The degree values of each subway station are, for the most part, two, and the average degree is 2.087, indicating that there are relatively few large transfer hubs(stations with large degree values) in

the entire subway network. The average clustering coefficient of each station is 0, indicating that the whole subway network has no clustering characteristics. The diameter of the subway network is 35 and the average path length is 12.6946, indicating that the coverage of the Washington subway network is relatively broad, and the convenience of passengers taking the subway is high.

(2) In a bid to test the robustness of Washington subway network, random attack and deliberate attack is used to invalidate the stations in Washington metro network. When it is large in the number of attack nodes, the relative size of the largest connected subgraph is reduced to a relatively small value. For global efficiency and connectivity, the values of Washington metro network stations show a downward trend when they are subjected to random attacks and deliberate attacks, but the values decrease significantly when they are subjected to deliberate attacks. In summary, the Washington subway network has good robustness when it encounters attacks. Compared with random attacks, deliberate attacks are more destructive to the subway network.

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