Research on the Queuing Theory based on M/M/1 Queuing Model

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Abstract. A collection of mathematical theories and models make up queueing theory. The theory, which is derived from several models, makes an effort to recreate numerous typical queue waiting models. The purpose of this study is to do research on the M/M/1 queuing model-based queueing theory. After a brief introduction to the queuing theory, the independent indexes and the dependent variables will be shown. This paper contains some fundamental queuing theory formulas, some of which are specifically derived for the M/M/1 queuing model. This study then presents a simulation of the M/M/1 queuing model. Since some results are attained, this paper revealed some probability distributions and distribution functions of several indexes. The data characteristics displayed by these figures are very consistent with the daily life situation, which also proved the rationality and M/M/1 queuing model's applicability. By investigating this queuing theory, people shall be able to understand, analyze, and predict the phenomenon of waiting in a line.

Keywords: Queuing Theory, The Model of M/M/1 Queuing, Distribution Figures.

1. Introduction

A subfield of statistics is queueing theory, which has a long history. Back in 1909, Agner Krarup Erlang, an engineer working at the Telephone Exchange in Copenhagen, published the first paper about the queuing theory. Now in our daily life, queuing is quite a common phenomenon. We don't want to wait in line since queuing means time consumption and inefficiency. However, it is inevitable that we can avoid queuing, relating to many factors. The three most important parts are the arrival process, service mechanism, and queue discipline. The paper will simulate a model for M/M/1 queuing, and examine the outcomes.

Up to now, many scholars have studied the queuing theory. They have applied queuing theory well, combining theoretical research with realistic scene simulation. Not only that, they also built some optimization models related to queuing theory, which provide important guidance and reference value for people's research today.

First and foremost, a large number of researchers used the queuing theory to study phenomena and processes similar to queuing in life. Previously, in 2003, in terms of medical work, some researchers have analyzed the model of hospital process on the basis of queuing theory, and studied the main factors and main service parts of the whole process in detail [1]. In 2005, there are some researchers who have also used queuing theory to study random service systems in traffic engineering. From this, they derived the main indicators that measure the efficiency of the service system, illustrating the connection of these indicators to the method the way is served [2]. In 2011, some researchers have used queuing models to develop a model of electric vehicle charging serving system. They analyzed the probability characteristics of charging serving system in queuing theory and the operation efficiency of the charging devices [3]. In 2012, some researchers took a specific amusement park as the research scenario, calculated the reasonable daily capacity of each project in the park, the reasonable daily capacity and the reasonable annual capacity of the whole park, then compared and analyzed the results with the original set capacity of the park [4]. These studies are based on queuing theory, and researchers have applied it to build new models.

Additionally, lots of researchers established a new optimization model based on queuing theory. Using queuing theory and probability, some academics proposed an original model of dynamic load balancing in 1996. They quickly introduced the dynamic load balancing technology before modeling the optimization method and used simulation technologies to validate the analysis results [5]. In 2009, according to the characteristics of the supermarket cash register queuing system and the queuing theory, some researchers have converted it into a mathematical model and established an optimization model of the supermarket cash register queuing system [6]. In 2010, some researchers describe the subway passage and passenger flow during peak hours as a queuing system. Based on analyzing the characteristics of parameters, a queuing optimization model is constructed. The concept of pedestrian service level is introduced into the calculation of the capacity of subway pedestrian passage, and a new method for determining the width of subway passage is well developed [7]. In 2011, some researchers, based on the queuing theory, set the bus arrival rate, the number of service stations, and the average boarding and alighting time of passengers as the model parameters. They built a calculation model for the line capacity of bus stops at densely spaced points, and optimized the capacity of bus stops by changing parameters [8]. In 2013, some researchers regard the storage and transportation vehicle queuing system as composed of two independent queuing systems, and establish the corresponding mathematical optimization model. They also discussed the model's solution algorithm, which aims to reduce the system's overall user time cost [9]. In 2015, some researchers have observed and investigated the problem of university cafeteria crowding, and found out the key reasons for it. Then they use the queuing theory to build the model and arrange the service window reasonably. This not only reduces the congestion and time waste of students during meals, but also improves the service efficiency of the canteen and alleviates the problem of canteen congestion during the peak period of meals [10]. These studies above all reflected the researchers' deep understanding of queuing theory. On this basis, they also established optimization models and made suggestions for specific scenarios in life to improve efficiency.

All in all, the similar phenomenon of queuing is indeed common, and people are unwilling to spend a lot of time on queuing. So far, many scholars have carried out research, application and modeling on the basis of queuing theory. They built corresponding optimization models and put forward corresponding suggestions for some phenomena in life. The M/M/1 queuing model will be examined in more detail in this paper using queuing theory and related research. The M/M/1 queuing model will be simulated using Rstudio for this work, and the simulation results will be presented as tables. The following step involves drawing and analyzing a number of probability distribution and function distribution figures for various variables.

2. Methods

2.1. Data Source

This paper uses simulation-based data. The M/M/1 queuing model simulation is used to implement all data. A unique kind of queuing model is an M/M/1 model. It can also be expressed as an M/M/1/∞/∞ queuing model with one server, an infinite number of customers a system can hold, and an infinite number of potential customers. The Poisson Distribution for the arrival process (which is equivalent to an exponential distribution of time between arrivals) and an Exponential Distribution of service hours are also supported. Here is the use process of the queuing model, as Figure 1 shows.

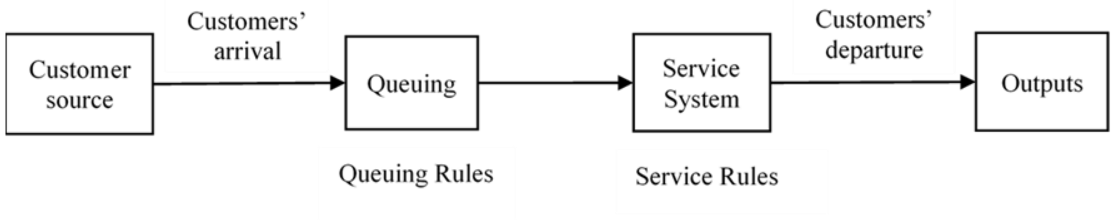

Figure 1. The Use Process of the Queuing Model

2.2. Illustration of Indexes

In the queuing theories, there are the following variables, as Table 1 and 2 showing.

Table 1. Some independent variables and their meanings

variables	meanings
λ	the mean rate of service completions
μ	the mean rate of arrival
n	how many servers there are in the system

Table 2. Some dependent variables and their meanings

variables	meanings
$1/\lambda$	the customer arrival interval
$1/\mu$	the service time
W (or S)	the typical duration of use of the system
Wq	the average length of time in line
ρ	the traffic intensity (the utilization factor)
γ	the mean rate of arrival
P_n	the likelihood of having n customers in the system
Lq	the average number of customers waiting in line
L	the typical client count in the system

Table 2 and 3 show the independent and dependent variables of Queuing model used in this paper, all the variables are simulated in the model process.

2.3. Some Basic Formulas

For the (1) formula, customers' mean time spent in the system is made up of their mean time spent in the queue and their predicted wait time. For the (2) formula, γ is called the “load” of the system, representing the “estimated number of arrivals during the typical service completion time”. For the (3) formula, ρ is referred to as the utilization factor with λ being the arrival rate and sμ being the service capacity. If ρ>1, the rate of arrival is greater than service capacity, the queue will approach infinity without additional mechanism. If ρ<1, the service capacity is smaller than the rate of customer arrivals, the probability of any particular system size no longer changes over time. For the (4) and the (5) formulas, these are two fundamental formulas called the “Little’s formulas”, which can be combined together to get the (6) formula. These are some specific formulas for calculating dependent variables, particularly for the model of M/M/1 queuing.

$$\mathrm{S} = W_q + \frac{1}{\mu} \tag{1}$$

$$\gamma = \frac{\lambda}{\mu} \tag{2}$$

$$\rho = \frac{\lambda}{n\mu} \tag{3}$$

$$L_q = \lambda W_q \tag{4}$$

$$\mathrm{L} = \lambda \mathrm{W} \tag{5}$$

$$\mathrm{L} = L_q + \gamma \tag{6}$$

For the (7) formula, with P_n known in the (6) formula, since there are no customers in the queue when 0 or 1 customer exists in the system with 1 server, L_q could be calculated in the (8) formula. After that, other three variables could be derived in the (9), (10), and (11) formulas.

$$P_n = \rho^n(1-\rho) \tag{7}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \tag{8}$$

$$\mathrm{L} = L_q + \gamma = \frac{\rho^2+\rho-\rho^2}{(1-\rho)} = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \tag{9}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)} \tag{10}$$

$$\mathrm{W} = \frac{L}{\lambda} = \frac{1}{\mu-\lambda} \tag{11}$$

2.4. Reaearch Method

Using Rstudio, this paper developed the M/M/1 queuing model in this study. The values of the following three variables were set when we imported the M/M/1 queuing model package: λ, μ, and n. The value of n in the model of M/M/1 queuing is 1. By simulation, the impact of the model is examined in this research.

3. Results and discussion

3.1. Results

This paper set up a model of M/M/1 queuing by using Rstudio, which is the most basic queuing situation in life. We assumed this queuing situation with the value of λ is 3, the value of μ is 4, and the value of n is 1. More specifically, there are three customers coming per unit time, four customers being served per unit time, and one waiter. After setting the values of these three arguments, we used Rstudio to operate. Then we got some relevant data and results, as shown in the following table.

Table 3. The results of dependent variables

variables	ρ	γ	P_0	W	W_q	L	L_q
value	0.75	0.75	0.25	1	0.75	3	2.25

In simple terms, the results in the Table 3 of these dependent variables are explained as follows. The traffic intensity is 0.75. The mean rate of arrival is 0.75. The possibility (P_0, P_1, …, P_n) of the n=0 customers in the queuing system is 0.25. One minute on average is spent in this system. The average amount spent in line is 0.75. There are typically 3 clients in the system. 2.25 clients on average are waiting in the queue.

3.2. Probability Distribution and Discussion

Since the result were shown, a Possion Distribution for arrival procedure, an Interarrival Time Exponential Distribution, and for a service process, an Exponential Distribution could be drawn.

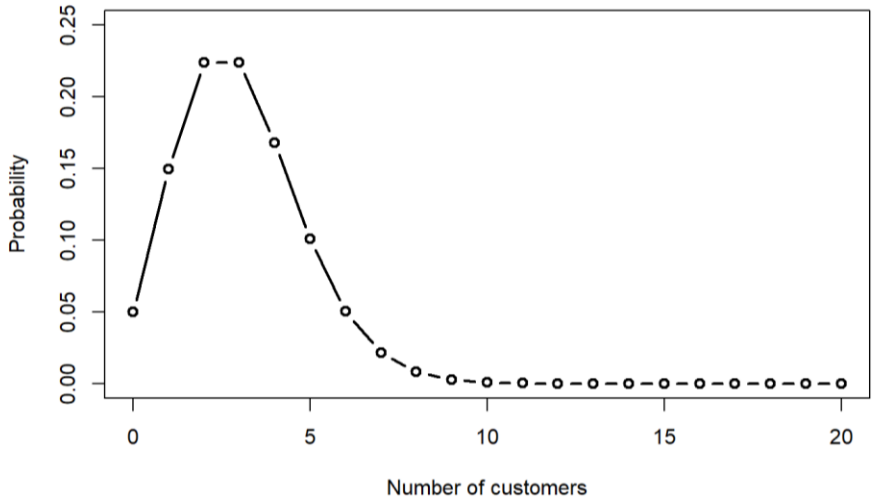

Figure 2. Possion Distribution for Arrival Process

In the Figure 2, the x-axis is the number of customers and the y-axis is the probability. When the number of costumers is 3, the probability reached the highest, and then the probability became lower. In the model of M/M/1 queuing, the greater the volume of clients, the less likely they are to receive service.

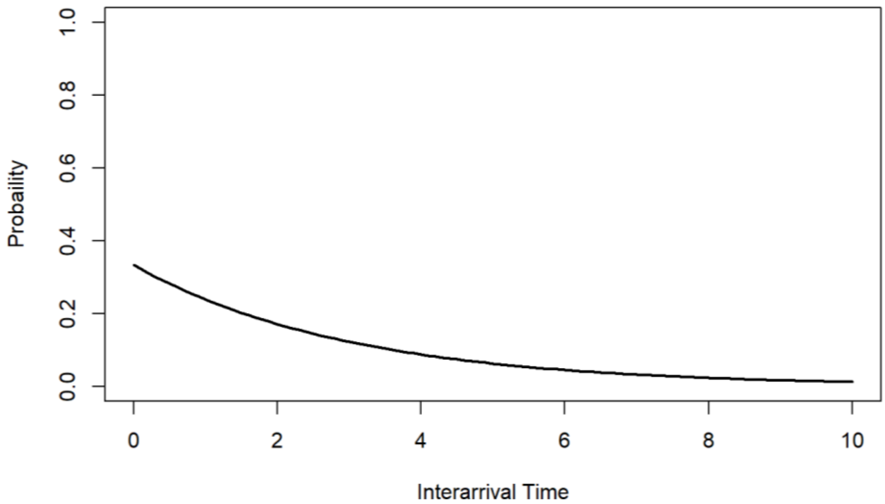

Figure 3. Time Interval Exponential Distribution

Figure 4. For a Service Process, Exponential Distribution

In the Figure 3, it shows that as the interarrival time went longer, the probability went lower. Also in the Figure 4, in the first minute, the probability reduced drastically. As the service waiting time got longer, the probability was getting closer to zero.

All the results show that the longer the customer waits, the lower probability he gets the service. This is the same as in our daily lives. When customers were at the store ready to line up to check out the items they want to buy, the longer the wait time will be if the more people in front of the queue. When people wait for a certain amount of time, people's patience will also wear down. This can lead to some people abandoning the checkout and choosing not to buy the item and leaving the store.

3.3. Figures of Distribution Function of Different Variables and Discussion

Concerning about the different variables, the distribution function of L and L_q, a distribution function of W and W_q- due to the model of M/M/1 queuing and the model of M/M/c queuing could be drawn.

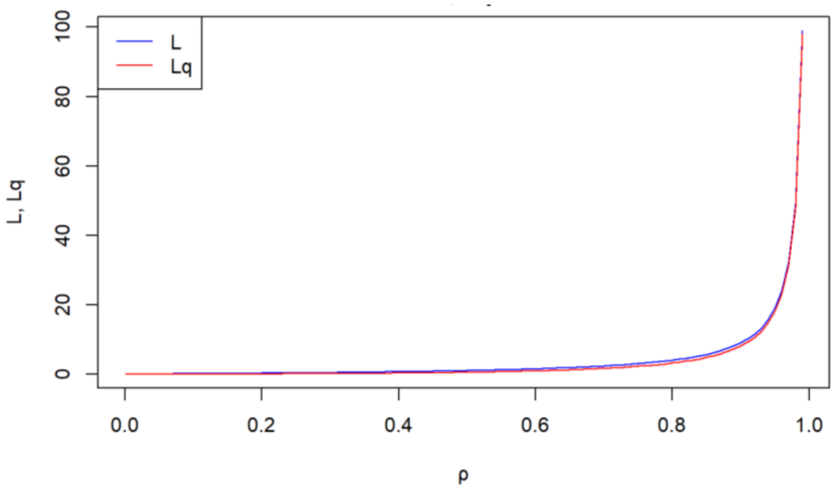

Figure 5. A Distribution Function of L and L_q

In the Figure 5, the input parameter λ in the model of M/M/1 queuing was changed. From this figure, it tells that when ρ approaches 1 infinitely, the values of L and L_q grow faster and faster. In general, however, the value of L is always greater than L_q. This implies that there must be more clients in the system than there are clients waiting in line. It is because just one person is ever served in the model of M/M/1 queuing, this is the case.

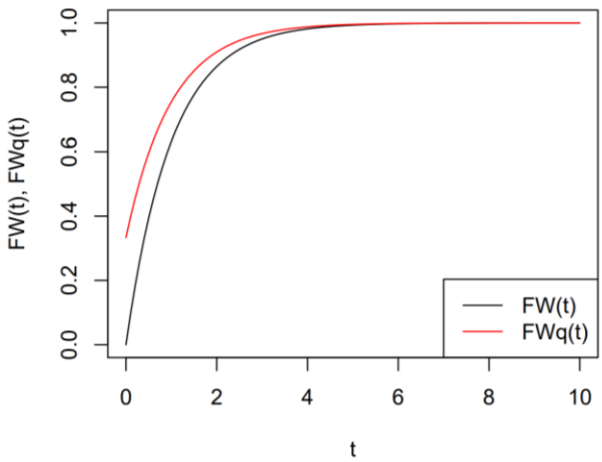

Figure 6. A Distribution Function of W and W_q-Based on the Model of M/M/1 Queuing

Within Figure 6, F_w means that the likelihood that a client may need to use the system, and F_{Wq} means that the probability of a customer having to wait in the queue. It is clear that when the customers have waited at least 4 units of time is 1, the value of F_w and F_{Wq} are equal. Both have the same probability of 1.

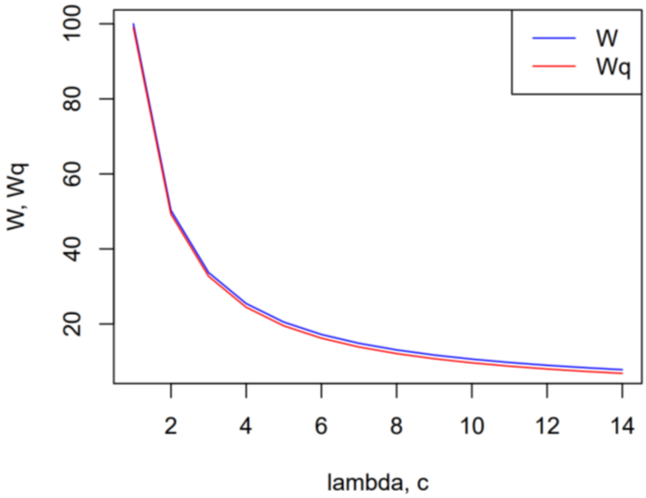

Figure 7. A Distribution Function of W and W_q-Based on the M/M/c Queuing Model

Based on the model of M/M/1 queuing, a study of the M/M/c queuing model could be shown. The difference between the models of M/M/1 and M/M/c queuing is that whether there is one or more than 1 server in the system. In the Figure 7, the arrival rate changes in line with how many servers there are. This figure shows that as the values of λ and c increase, the values of W and W_q decrease. In other words, the mean time spent in the system and the mean time spent in the queue rapidly decrease as the mean rate of service completions and the number of servers in the system increase. Obviously, this will enable people to improve the efficiency since the more servers there are, the less time people have to wait to receive service.

4. Conclusion

All in all, this paper gives an introduction of the queuing theory at first. The initial research into telephone communication queuing wiring is where the queuing theory got its start. Next, this paper briefly summarizes the research results of some scholars on queuing theory. Some scholars have further studied and applied queuing theory, and some scholars have used queuing theory to construct new mathematical models.

After that, this paper introduces some variables and basic formulas, particularly into the model of M/M/1. Finishing the introduction about the formula, this paper simulates an M/M/1 queuing situation by using Rstudio. In this scenario, there is one server, an exponential service time of four clients per minute, and a Poisson arrival rate of three consumers per minute. Then, the results are calculated and shown in the form of tables.

A Possion Distribution of Arrival Process, a Time Interval Exponential Distribution, and an Exponential Service Process Distribution are some of the probability distributions that are provided in this work using Rstudio since the results have been obtained. Also, this paper shows some figures of distribution functions, including a distribution function of L and L_q, a distribution of W and W_q that according to the model of M/M/1 queuing, and a distribution function of W and W_q that based on the model of M/M/c queuing. Then, using these figures, this paper gives some discussions and indicates that the data characteristics shown in these figures are similar to the reality of everyday life.

Regarding the model of M/M/1 queuing constructed in this paper, the author believes that it can be further studied. This model is the basis of the M/M/c queuing model, and through the in-depth study and analysis of the model of M/M/1 queuing, the conclusions reached could be further generalized to more complex model of M/M/c queuing to deal with the more common multi-server situation in reality. As future works, the author will continue to study the content of queuing theory and conduct in-depth research of the M/M/C queuing model.

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