

# Tension Control in Unwinding System Based on Nonlinear Parameter Estimation

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**Abstract.** In this paper, two representative nonlinearities in the unwinding system are modelled. One is the rotation-disturbance caused by the rotational change of unwinding roll position; another is the time-varying radius of unwinding roll. Based on the nonlinear mechanism model, the parameters of rotation-disturbance are obtained by the Gauss-Newton algorithm, and the time-varying radius is estimated online by the Kalman filter. Then the feedforward and time-varying feedback controller are designed based on the estimated parameters to compensate for nonlinear effect in unwinding system. The experiment results show that the parameter estimation algorithm can accurately estimate parameters. And the use of estimated parameters to control can effectively solve the problem of tension fluctuation and tension deviation which caused by the nonlinearity of unwinding system.

**Keywords:** Unwinding System, Nonlinear Parameter, Tension Control.

## 1. Introduction

Tension control performance is directly related to the processing effect of flexible materials [1-4]. It is very important to maintain accurate and stable tension during the whole operation. The fluctuation of the tension will cause the printing pattern to be misaligned, the cutting offset, and even the material may be pulled off, which seriously affects the work efficiency [5-7].

The unwinding system is vital for generating web tension in the tension control system for the machine. However, unwinding tension control is time-varying and nonlinear [6, 8]. During the operation of the unwinding system, automatic reloading process will require rotating the unwinder roll which will bring rotation-disturbance to tension. The radius of unwinding roll will keep decreasing resulting in the tension error constantly increase [9, 10].

The modeling of unwinding tension control system is the first step of tension control. The establishment of tension mechanism model between two consecutive rolls is generally based on three laws: Hooke's law, Coulomb's law and Mass conservation law [5, 11]. Experimental identifications of the system for a web processing and the overlapping decomposition technique of the web transport system are discussed based on the identified transfer functions [12]. The model of coiling tension and dynamic model of unwinding system is deduced to achieve higher tension control accuracy [13, 14].

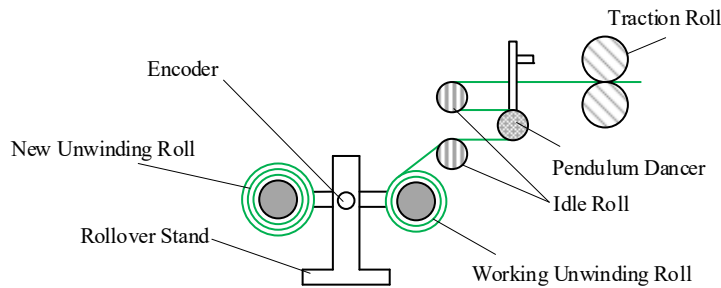
One method to solve the rotation-disturbance is to shorten the unwinding material turning stroke by changing the mechanical design of the rollover stand [2]; the other is to establish the functional relationship between the unwinding line speed and the rotation angle, then feedforward compensation is used to suppress tension fluctuations [1]. However the compensation parameters of this method are manually measured, and there are certain difficulties in engineering implementation.

Aiming at the tension control problem under the time-varying radius, Fuzzy-PID is used to calculate the fuzzy PID parameters through the time-varying coil diameter estimated online, and the final simulation shows that better control is achieved [10]. But in this article the method of estimating the roll diameter is relatively simple, it is obtained directly by dividing the traction speed by the unwinding angular velocity. A  $H^\infty$  robust control strategy with varying gains and a linear parameter varying (LPV) control strategy are used respectively and combinedly and improve the performance and robustness compared to classical PID control [5]. Similarly, operating-point-dependent controller is used for tension control which is realized by the gain scheduling method [15]. A time-varying

feedforward filter for the reference input is introduced to attenuate the tension error that brought by time-varying radius [16, 17]. A decoupling controller based on active disturbance rejection control (ADRC) method is used to improve the stability and robustness of the tension in unwinding system of gravure printing machines, which can estimate and compensate both internal and external disturbances in real time [18, 19]. Kalman filter is used to estimate the time-varying winding diameter, the input voltage of the unwinding motor and the traction motor is used as the input of the estimator, and the motor output speed and swing arm deflection angle are used as the filter output to estimate Unwinding radius. The experimental results show that the radius measured by the sensor and the estimated radius have a fitness of 80.9% [20]. A Nonlinear Dynamic Matrix Control (NDMC) is proposed which is based on a math model of dynamic unwinding tension and the simulations shows it has a better performance than the PID algorithm [21].

In this study, we propose a tension control method which takes into account both the rotation of unwinding roll and the time-varying radius. Firstly, we establish the tension model, the rotation-disturbance model, and the time-varying radius model; secondly, parameters of the rotation-disturbance model and the time-varying radius are estimated; then, the nonlinear controller is designed according to the estimated parameters; Finally, the function of the proposed algorithm is verified through experiments.

## 2. Mathematical modelling of unwinding system



**Figure 1.** Unwinding system structure

Fig. 1. shows the general unwinding system. Rollover stand is a device that realizes automatic reloading. Encoder is equipped for recording the rotation angle of the rollover stand. One side of the rollover stand is hung with the working unwinding roll, and the other side is hung with the new roll to be run next time. The web from working roll will first pass through the idle roller, then through the pendulum dancer which measures the tension error, and finally enter the traction roller. The traction roll can provide a constant linear speed because of its fixed radius. The control objective is to keep the web tension between unwinding and traction stable at the set value. Next, we will establish the model of unwinding tension, rotation-disturbance and the unwinding roll radius.

### 2.1 Unwinding tension model

Unwinding tension model is as follows [12]

$$\frac{dT_u}{dt} = \frac{ES}{L}(V_T - R_u\omega_u) \quad (1)$$

where  $E$ ,  $S$ ,  $L$ ,  $T_u$ ,  $V_T$ ,  $R_u$  and  $\omega_u$  are elastic modulus, cross-sectional area, web length, unwinding tension, traction speed, unwinding roll radius, and unwinding roll angular speed, respectively. As shown in Eqs. (1), the dynamics for the unwinding system are time-dependent, that is,  $R_u$  is time-varying.

## 2.2 Rotation-disturbance model

For the feedforward control, the disturbance caused by rotation of rollover stand needs to be modelled. This disturbance is intuitively caused by the change in the length of the web between the unwinding roll and the idle roll, so it is a length disturbance  $L_d$ . Take  $L_d$  as a derivative of time to obtain a speed disturbance to facilitate the introduction of feedforward control. Fig.2 shows the geometric relationship of  $L_d$  with respect to  $\theta$ , the origin of coordinate system is the center of the rotating circle of rollover stand. The dynamic equations describing the relationship between  $L_d$ ,  $V_d$  and  $\theta$  are given by

$$L_d = \sqrt{(x_t - R_r \cos \theta)^2 + (y_t - R_r \sin \theta)^2 - (R_u - R_t)^2} \quad (2)$$

$$V_d = \frac{dL_d}{d\theta} \frac{d\theta}{dt} = \frac{\omega_r (R_r x_t \sin \theta - R_r y_t \cos \theta)}{\sqrt{(x_t - R_r \cos \theta)^2 + (y_t - R_r \sin \theta)^2 - (R_u - R_t)^2}} \quad (3)$$

where  $x_t, y_t, R_u, R_t, R_r$ , and  $\omega_r$  are horizontal coordinate of the idle roller, vertical coordinate of the idle roller, radius of unwinding roll, radius of idle roll, radius of rollover stand, angular velocity of rollover stand rotation.

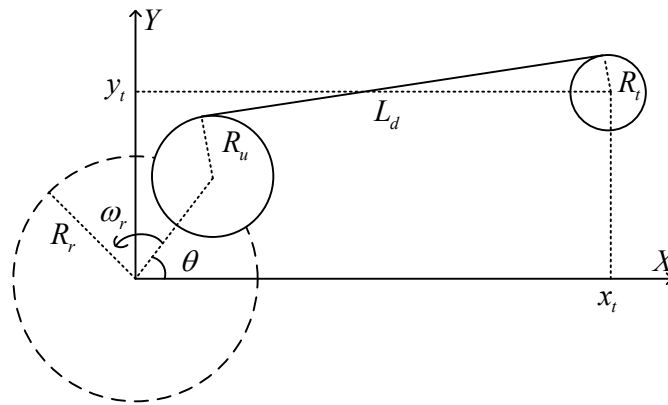


Figure 2. Rotation-disturbance model

## 2.3 Unwinding roll radius model

The unwinding radius decreases with time, and the estimation of the unwinding radius is based on the unwinding radius change model

$$R_u = R_0 - \int_0^{t_0} \frac{\delta \omega_u(t)}{2\pi} dt \quad (4)$$

where  $R_0$  and  $\delta$  are the initial radius of unwinding roll and material thickness.

## 3. Nonlinear parameter estimation

In this section we first simplify the rotation-disturbance model and use Gauss-Newton algorithm to estimate fixed parameters  $x_t, y_t, R_t, R_r$  in Eqs. (3). Then we use Kalman filter to estimate the time-varying radius  $R_u$  online.

### 3.1 Rotation-disturbance parameters estimator

Expand the denominator of Eqs. (3)

$$V_d = \frac{\omega_r (R_r x_t \sin \theta - R_r y_t \cos \theta)}{\sqrt{x_t^2 + y_t^2 + R_r^2 + 2R_u R_t - R_u^2 - R_t^2 - 2y_t R_r \sin \theta - 2x_t R_r \cos \theta}} \quad (5)$$

let  $A = -2y_t R_r$ ,  $B = -2x_t R_r$ ,  $C = x_t^2 + y_t^2 + R_r^2 + 2R_u R_t - R_u^2 - R_t^2$ . Then Eqs. (5) is simplified to

$$V_d = \frac{\omega_r (B \sin \theta - A \cos \theta)}{\sqrt{A \sin \theta + B \cos \theta + C}} \quad (6)$$

The problem of estimating parameters  $R_u$ ,  $x_t$ ,  $y_t$ ,  $R_t$ ,  $R_r$  is simplified to the problem of estimating nonlinear parameters  $A$ ,  $B$ ,  $C$ . Convert the problem of estimating parameters to a nonlinear least squares problem

$$\underset{\hat{A}, \hat{B}, \hat{C}}{\operatorname{argmin}} f(\hat{A}, \hat{B}, \hat{C}, \theta) = \frac{1}{2} \sum_{i=1}^N \left[ V_d(\theta_i) - \frac{\omega_r (\hat{B} \sin \theta_i - \hat{A} \cos \theta_i)}{\sqrt{\hat{A} \sin \theta_i + \hat{B} \cos \theta_i + \hat{C}}} \right]^2 \quad (7)$$

In order to solve Eqs. (7), it is necessary to obtain the rotation-disturbance observation signal  $V_d(\theta_i)$ . The method is to rotate rollover stand very slowly to ensure that the frequency of disturbance  $V_d(\theta_i)$  is very low. Then the output of feedback tension controller is approximately equal to  $-V_d(\theta_i)$ . We call this process as calibration experiment.

After calibration, we apply Gauss-Newton algorithm to solve Eqs. (7), the solution is as follows[22]

$$X_{GN}(k+1) = X_{GN}(k) + S(k) \quad (8)$$

$$S(k) = - \left[ J(X_{GN}(k))^T J(X_{GN}(k)) \right]^{-1} J(X_{GN}(k))^T f(X_{GN}(k)) \quad (9)$$

Where

$$J(X_{GN}(k)) = \left( \frac{\partial f(\theta)}{\partial \hat{A}} \quad \frac{\partial f(\theta)}{\partial \hat{B}} \quad \frac{\partial f(\theta)}{\partial \hat{C}} \right) \Big|_{\hat{A}=\hat{A}(k-1), \hat{B}=\hat{B}(k-1), \hat{C}=\hat{C}(k-1)}^T \quad (10)$$

$$X_{GN}(k) = [\hat{A}(k) \quad \hat{B}(k) \quad \hat{C}(k)], \theta = [\theta_1 \quad \theta_2 \quad \dots \quad \theta_N]^T \quad (11)$$

It should be noted that under the general mechanical structure,  $x_t^2 + y_t^2$  is obviously larger than  $2R_u R_T - R_u^2$ , therefore, even if the unwinding radius of the calibration process is different from the unwinding radius during working process, it will not bring obvious changes in the disturbance model.

### 3.2 Radius estimator

Use Eqs. (1) and (4) to establish a predictive model of the unwinding radius, and use the tension error as the observation. Therefore, the Kalman filter for online estimation of the unwinding radius can be established as follows

$$X_{KF}(k+1) = F(k)X_{KF}(k) + G(k)u(k) + w(k), z(k) = H(k)X_{KF}(k) + v(k) \quad (12)$$

$$X_{KF}(k) = [R_u(k) \quad \delta(k) \quad T(k)]^T, u(k) = V_T(k) \quad (13)$$

$$F(k) = \begin{pmatrix} 1 & -\omega_u T_s & 0 \\ 0 & 1 & 0 \\ \frac{EST_s}{L} & 0 & 1 \end{pmatrix}, G(k) = \begin{bmatrix} 0 & 0 & \frac{EST_s}{L} \end{bmatrix}^T, H(k) = [0 \quad 0 \quad 1] \quad (14)$$

where  $T_s$ ,  $w(k)$  and  $v(k)$  are sampling interval, process noise and observation noise. Other variables are defined before.

The implementation of Kalman filter is given by [23]

Step1 Error covariance prediction:

$$\hat{P}(k) = F(k-1)P(k-1)F^T(k-1) + Q(k) \quad (15)$$

Step2 Updating Kalman gain matrix:

$$K(k) = \hat{P}(k)H^T(k)(R(k) + H(k)\hat{P}(k)H^T(k))^{-1} \quad (16)$$

Step 3 State prediction:

$$\hat{X}_{KF}(k) = F(k-1)X_{KF}(k-1) + G(k-1)V_T(k-1) \quad (17)$$

Step 4 State update:

$$X_{KF}(k) = \hat{X}(k) + K(k)(z(k) - H(k)X_{KF}(k-1)) \quad (18)$$

Step 5 Prior covariance update:

$$P(k) = (I - K(k)H(k))\hat{P}(k) \quad (19)$$

## 4. Control method

Unwinding tension control method is shown in Fig. 3. below. The controller contains three parts: the first is rotation-disturbance feedforward controller, the second is time-varying PID controller, the third is radius feedforward controller. The first part is based on the estimated parameters of rotation-disturbance model and the last two parts is based on the estimated radius.

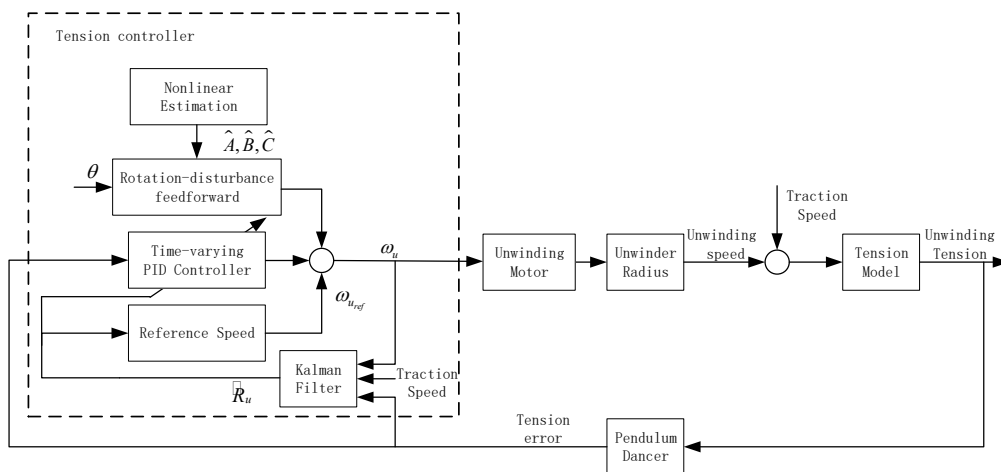


Figure 3. Unwinding tension control system

### 4.1 Rotation-disturbance feedforward controller

After the parameters  $\hat{A}, \hat{B}, \hat{C}$  are estimated, rotation-disturbance can be compensated according to equation Eqs. (6) during the rollover stand rotation.

The rotation-disturbance feedforward compensation law  $\omega_{d_u}$  is

$$\omega_{d_u} = -\frac{\omega_r(\hat{B} \sin \theta - \hat{A} \cos \theta)}{\hat{R}_u \sqrt{\hat{A} \sin \theta + \hat{B} \cos \theta + \hat{C}}} \quad (20)$$

### 4.2 Time-varying PID controller

Time-varying PID control law  $\omega_{e_u}$  based on estimated unwinding radius  $\hat{R}_u$  is

$$\omega_{e_u} = \frac{1}{\hat{R}_u} \left( K_p e + K_i \int e + K_d \frac{de}{dt} \right) \quad (21)$$

where  $K_p, K_i, K_d$  are the P, I, D parameters respectively.

### 4.3 radius feedforward controller

The radius feedforward controller calculates the unwinding reference angular velocity  $\omega_{r_u}$  based on the estimated radius to compensate for the effect of the drastic decrease in unwinding radius. This compensation method requires the estimated radius to be very stable.

$$\omega_{r_u} = \frac{V_T}{\hat{R}_u} \tag{22}$$

### 4.4 Summery

The control output  $\omega_u$  of the whole controller is the summery of PID output and feedforward:

$$\omega_u = \frac{V_T}{\hat{R}_u} + \frac{1}{\hat{R}_u} \left( K_p e + K_i \int e + K_d \frac{de}{dt} \right) - \frac{\omega_r(\hat{B} \sin \theta - \hat{A} \cos \theta)}{\sqrt{\hat{A} \sin \theta + \hat{B} \cos \theta + \hat{C}}} \tag{23}$$

## 5. Result

### 5.1 Experiment settings

The experimental platform is the Unwinding of gravure Press, and the processing material is PET. The parameters of the experiment are set as follows

**Table 1.** Unwinding properties.

Parameter	Value	Dimension
Young's modulus of web	4.00	Gpa
Web thickness	12.00	μm
Width of web	1.05	m
Length of unwinding span	5.00	m
Radius of idle roller	0.06	m
Radius of rollover stand	0.57	m
Horizontal coordinate of idle roller	1.23	m
Vertical coordinate of idle roller	-0.06	m

The experiment is carried out as follows: firstly, we verify whether the algorithm can estimate the correct rotation-disturbance compensation parameters, and whether the Kalman filter can estimate a smooth and accurate radius. Then, we verify whether the proposed control algorithm is effective when rollover stand rotates or the unwinder radius decrease.

Therefore, we let the gravure printing machine work at constant speed, and then observe the influence of the rotation-disturbance and the change of radius on the tension when the feedforward compensation is added or not. The parameters  $[K_p, K_i, K_d]$  in Eqs. (23) are  $[0.378, 0.189, 0]$ .

### 5.2 Rotation-disturbance parameters estimation and radius estimation

Calibration is operated under  $R_u=0.22m$ , so according to Eqs. (7) and the measurement parameters in Table 1, the rotation-disturbance parameters are calculated as  $A=0.07, B=-1.4, C=1.82$ .

Fig. 4. shows the comparison between measured disturbance and the estimated disturbance calculated using the last generation estimated parameters. The result shows that the estimated disturbance can better fit the measured disturbance.

Fig. 5. shows the iteration process, After the seventh generation, the iteration parameters stabilized.

Table 2. shows the comparison between the measured parameters and the estimated parameters. The result shows that the proposed parameter estimation algorithm can still estimate accurate results even when the initial value deviates.

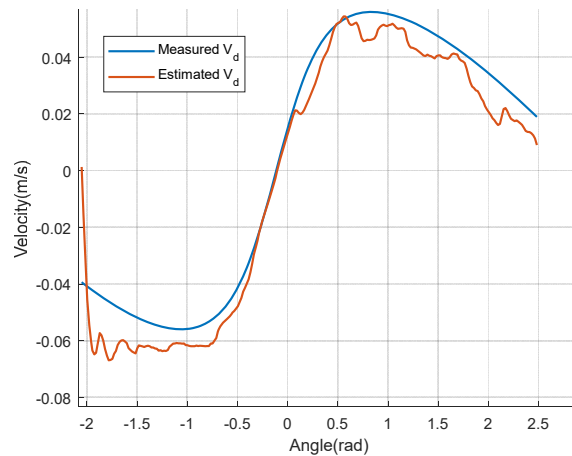


Figure 4. Comparison of measured  $V_d$  and estimated  $V_d$

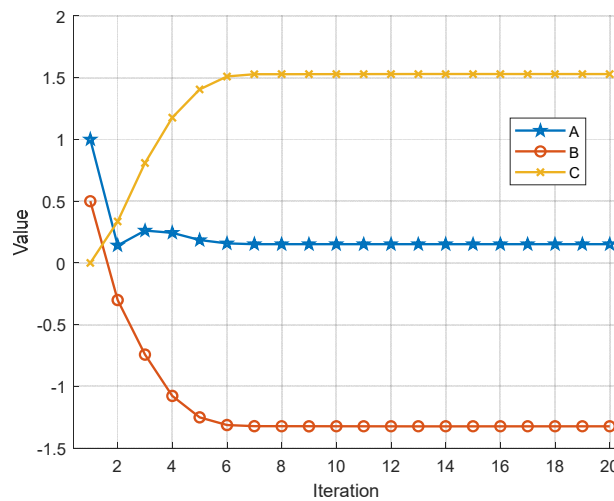


Figure 5. Estimation iteration

Table 2. Rotation-disturbance measured and estimated parameters.

Parameter	Initial Value	Measured Value	Estimated Value
$\hat{A}$	1.00	0.07	0.15
$\hat{B}$	0.50	-1.40	-1.32
$\hat{C}$	0.00	1.82	1.53

Fig. 6 Shows the comparison between the calculated unwinder radius  $\frac{V_T}{\omega_u}$  and the estimated radius when unwinding runs at 250m/min. The result shows that the radius estimator can estimate an accurate and stable unwinder radius. Therefore, in section 5.3 the estimated radius is used to generate the radius compensation law.

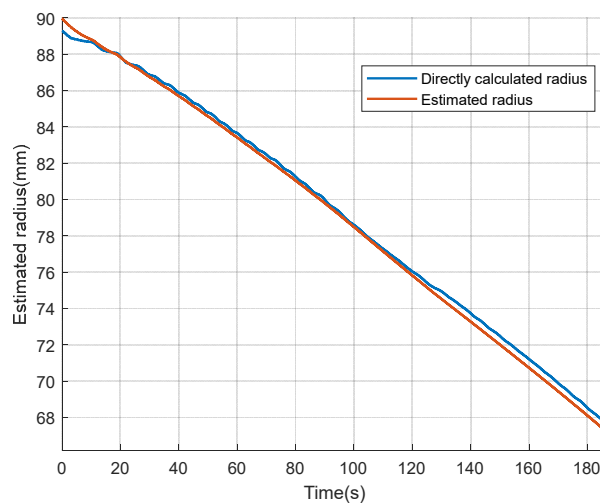


Figure 6. Estimated radius

### 5.3 Tension error with nonlinear compensation

This section shows a set of controlled experiments. The experiment variable is the controller, one is PID, and the other is the controller with nonlinear compensation proposed in this paper. The experiment process is as follows. Firstly, keep the unwinding speed at 250m/min and maintain 188s. This is to verify the control effect of the time-varying radius. Then, while keeping the given speed constant, the rollover stand starts to rotate. This is to verify the control effect on rotation-disturbance.

It can be seen from Fig. 7 that when the controller is only PID without nonlinear compensation, the tension error continues to increase. When the controller is with nonlinear compensation, the tension error stays near 0. After 188s, the rollover stand starts to rotate, simple PID controller has a large tension fluctuation, and dancer voltage increases from 0.5 to 1.5, while dancer voltage only slightly increases from 0 to about 0.2 under the proposed controller.

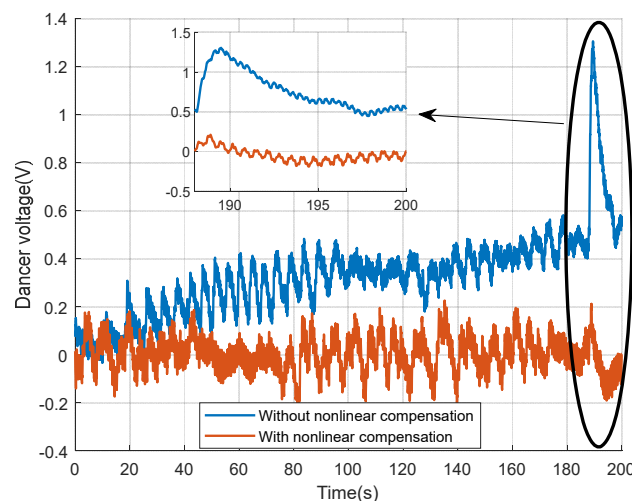


Figure 7. Comparison of dancer voltage with or without nonlinear compensation

## 6. Conclusion

In this paper, Gauss-Newton algorithm and Kalman filter are used to estimate the rotation-disturbance parameters and time-varying radius, respectively. Based on the estimated parameters and the nonlinear mechanism model, we designed a nonlinear compensation controller for the unwinding system. We first verify that the parameter estimation algorithm can accurately estimate the parameters of the nonlinear model. Then we verify that the nonlinear compensation controller based on estimated

parameters can effectively suppress the effects on the tension caused by rotation-disturbance and time-varying radius.

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