Optimization of transportation problem algorithm based on MATLAB

Yun Lu *, Yuejiao Zhang

Department of Foundational Courses, Tianjin Collage, University of Science & Technology Beijing, Tianjin 301830, China

* Corresponding Author Email: 13745@sdjzu.edu.cn

Abstract. Whether it is the transportation problem with balanced production and marketing or the transportation problem with unbalanced production and marketing, the transportation problem with balanced production and marketing can be obtained by transformation. The transportation problem is usually solved by directly applying the transportation simplex method on the table of the freight rate table to find the optimal solution. The selected initial transportation scheme needs to adjust the input variables for many times to get the optimal transportation scheme, resulting in a relatively large amount of calculation of the mathematical model. Using the powerful computing power of MATLAB software, write the corresponding program, and realize the solution of the optimal scheme of transportation problem through running the program.

Keywords: Transpotation problem, Balanced problem, Unbalanced problem, Matlab.

1. Introduction

Transportation Model is a special linear planning model, and many decision-making models belong to the type of transportation model, so the content is very rich. When solving, it is necessary to obtain a basic feasible solution through the method of minimum element, the method of lower left corner, the method of lower right corner, etc., and then adjust the feasible solution through the closed loop method or the potential method to get the optimal solution [1]. Every adjustment needs to re-apply the potential method and recalculate. In order to solve the optimal solution more quickly, we join in the MATLAB software to implement transportation problem solving way. This paper summarizes the comparison of the Table-Manipulation method and the MATLAB solution method for transportation problems in two cases.

2. Production and marketing balance problem

By establishing the mathematical model, the transportation simplex method can be used to solve the problem, or the MATLAB software can be directly used to solve the problem of production and marketing balance [2].

Example 1. The network diagram is shown below. There are three breadbaskets: $A_i(i = 1,2,3)$, the food available is respectively 10, 8 and 5 (tons). Food is now being delivered to four areas: $B_j(j = 1, 2, 3, 4)$. The quantity demanded is 5, 7, 8 and 3 (tons). The figure below on the right is the freight rate table (Yuan/tons). Question: How to arrange a shipping schedule so that the total freight is minimized?

The network diagram is:

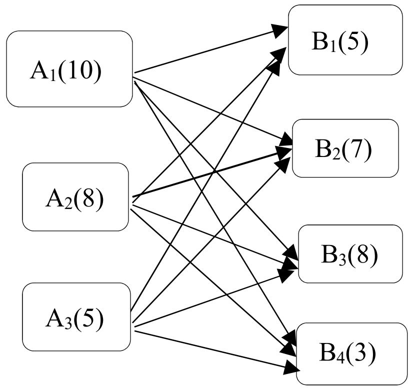

Table 1. The network diagram

Table 1. Freight list

Demand Place / Place of origin	B_1	B_2	B_3	B_4	Supply quantity (tons)
A_1	3	2	6	3	10
A_2	5	3	8	2	8
A_3	4	1	2	9	5
Quantity demanded (tons)	5	7	8	3	Total: 23

Assume x_{ij} is the volume from the i producing place to the j demand place (tons), b_i is the production from place i, d_j is the quantity demanded at j place of demand. So we get the following:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{14} \\ \vdots & \ddots & \vdots \\ x_{31} & \cdots & x_{34} \end{bmatrix}, A = \begin{bmatrix} 3 & 2 & 6 & 3 \\ 5 & 3 & 8 & 2 \\ 4 & 1 & 2 & 9 \end{bmatrix} b = (10,8,5),\ d = (5,7,8,3) \tag{1}$$

The total freight matrix is expressed as:

$$f = AX^T = \begin{bmatrix} 3 & 2 & 6 & 3 \\ 5 & 3 & 8 & 2 \\ 4 & 1 & 2 & 9 \end{bmatrix} . \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \\ x_{13} & x_{23} & x_{33} \\ x_{14} & x_{24} & x_{34} \end{bmatrix} \tag{2}$$

Establish mathematical model as:

$$\min f = \sum_{i=1}^{3}\sum_{j=1}^{4} a_{ij}x_{ij}$$

$$s.t.\begin{cases} \sum_{i=1}^{3} x_{ij} = d_j \\ \sum_{j=1}^{4} x_{ij} = b_i \\ x_{ij} \geq 0,\ \ i = 1,2,3; j = 1,2,3,4 \end{cases} \tag{3}$$

Using the transport simplex method to find the optimal transport plan and the minimum total freight rate:

1. First use the minimum element method to get the initial dispatching plan:

Table 2. The initial dispatching plan

Demand Place / Place of origin	B_1	B_2	B_3	B_4	Supply quantity (tons)	u_i
A_1	5^* 3	2^* 2	3^* 6	3	10	0
A_2	5	3	5^* 8	3^* 2	8	2
A_3	4	5^* 1	2	9	5	-1
Quantity demanded (tons)	5	7	8	3	23	
v_j	3	2	6	0		

The test number obtained by potential algorithm method is potential as: $\lambda_{14}=0 \quad \lambda_{21}=0, \lambda_{22} < 0$, and adjust the input base variable by the closed loop method:

Table 3. The adjusted table

Demand Place / Place of origin	B_1	B_2	B_3	B_4	Supply quantity (tons)	u_i
A_1	5^* 3	2	5^* 6	3	10	0
A_2	5	2^* 3	3^* 8	3^* 2	8	2
A_3	4	5^* 1	2	9	5	0
Quantity demanded (tons)	5	7	8	3	23	
v_j	3	1	6	0		

3. The test number obtained by potential algorithm method is potential as: $\lambda_{12} > 0, \lambda_{14} > 0, \lambda_{21} = 0, \lambda_{31} > 0, \lambda_{33} < 0$. And continue to adjust the input base variable by the closed loop method:

Table 4. The adjusted table

Demand Place / Place of origin	B_1	B_2	B_3	B_4	Supply quantity (tons)	u_i
A_1	5^* 3	2	5^* 6	3	10	0
A_2	5	5^* 3	3^* 8	3^* 2	8	-2
A_3	4	2^* 1	3^* 2	9	5	-4
Quantity demanded (tons)	5	7	8	3	23	
v_j	3	5	6	4		

The test number obtained by potential algorithm method is potential as: $\lambda_{12} < 0$. And continue to adjust the input base variable by the closed loop method:

Table 5. The optimal table

Demand Place / Place of origin	B_1	B_2	B_3	B_4	Supply quantity (tons)	u_i
A_1	5^* 3	2^* 2	3^* 6	3	10	0
A_2	5	5^* 3	8	3^* 2	8	1
A_3	4	1	5^* 2	9	5	-4
Quantity demanded (tons)	5	7	8	3	23	
v_j	3	2	6	1		

At this time, all the test numbers are greater than or equal to zero, we get the optimal solution: $X=\begin{bmatrix}5&2&3&0\\0&5&0&3\\0&0&5&0\end{bmatrix}$, and the optimal value is 680,000 Yuan. However, we also find that the solving steps of transportation simplex method are very cumbersome.

Then, we can use the MATLAB software to solve this problem.

The MATLAB program is as follows:

```
>>c= [3;2;6;3;5;3;8;2;4;1;2;9];
>>Aeq = [1 1 1 1 0 0 0 0 0 0 0 0;
          0 0 0 0 1 1 1 1 0 0 0 0;
          0 0 0 0 0 0 0 0 1 1 1 1;
          1 0 0 0 1 0 0 0 1 0 0 0;
          0 1 0 0 0 1 0 0 0 1 0 0;
          0 0 1 0 0 0 1 0 0 0 1 0;
          0 0 0 1 0 0 0 1 0 0 0 1];
>>Beq = [10;8;5;5;7;8;3];
>>lb = [0,0,0,0,0,0,0,0,0,0,0,0];
>>ub=[]; x0=[]; A=[]; b=[];
>>options = optimset ('LargeScale','off','Simplex','on');
>>[x, fval, exitflag, output, lambda] = linprog(c, A, b, Aeq, Beq, lb, ub, x0, options)
```

The result of running the program:

```
>>Optimal solution found.
>>x =5 2 3 0 0 5 0 3 0 0 5 0
>>fval =68
>>exitflag =1
```

With the help of the powerful operation function of MATLAB, we can get the optimal solution of this problem very quickly. Obviously the optimal solution to this problem is when $x_{11}=5$, $x_{12}=2$, $x_{13}=3$, $x_{14}=0$, $x_{21}=0$, $x_{22}=5$, $x_{23}=0$, $x_{24}=3$, $x_{31}=0$, $x_{32}=0$, $x_{33}=5$, $x_{34}=0$, the freight is the least and the total freight is at least 680,000 Yuan.

3. Production and marketing imbalances problem

We first to solve the problem of unbalanced production is greater than the sales of production and marketing. The core method to solve the problem of unbalance of production over sales is to transform the problem of production over sales into the problem of balance of production and sales, that is, to make a virtual sales place (inventory), let the excess output be transported to the virtual sales place, and the sales volume is "total output - total sales volume", and at the same time, let the unit freight rate of the virtual sales place be zero.

Example 2. Three factories of a food company, A1, A2 and A3, produce convenience foods. It has to be shipped to point of sale B1, B2 and B3. The freight rate of convenience food per ton from the three factories to each point of sale (freight unit: Yuan/tons) is shown in the table below. Question: How does the food company arrange the transportation plan to minimize the total freight when it meets the sales volume at each point of sale?

Table 6. Freight list

Point of sale / The factory	B_1	B_2	B_3	Quantity of supply (tons)
A_1	50	90	20	15
A_2	30	10	70	18
A_3	60	20	80	17
Quantity of sale(tons)	18	12	16	

The tariff table (tariff unit: Yuan/tons) can be expressed as follows:

Table 7. Freight list

Point of sale / The factory	B_1	B_2	B_3	B_4	Quantity of supply (tons)
A_1	50	90	20	0	15
A_2	30	10	70	0	18
A_3	60	20	80	0	17
Quantity of sale(tons)	18	12	16	4	Total: 50

Assume x_{ij} is the volume (tons) from the i factory to the j point of sale, b_i is the output from the i factory, and d_j is the demand from the j point of sale. So we get the following:

$$X=\begin{bmatrix} x_{11} & \cdots & x_{14} \\ \vdots & \ddots & \vdots \\ x_{31} & \cdots & x_{34} \end{bmatrix}, \quad A=\begin{bmatrix} 50 & 90 & 20 & 0 \\ 30 & 10 & 70 & 0 \\ 60 & 20 & 80 & 0 \end{bmatrix}$$

$$b=(15,18,17), \quad d=(18,12,16,4) \tag{4}$$

The total freight matrix is expressed as:

$$f=AX^T=\begin{bmatrix} 50 & 90 & 20 & 0 \\ 30 & 10 & 70 & 0 \\ 60 & 20 & 80 & 0 \end{bmatrix}\cdot\begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \\ x_{13} & x_{23} & x_{33} \\ x_{14} & x_{24} & x_{34} \end{bmatrix} \tag{5}$$

Establish mathematical model as:

$$\min f=\sum_{i=1}^{3}\sum_{j=1}^{4}a_{ij}x_{ij}$$

$$s.t.\begin{cases} \sum_{i=1}^{3}x_{ij}=d_j \\ \sum_{j=1}^{4}x_{ij}=b_i \\ x_{ij}\geq 0, \quad i=1,2,3; j=1,2,3,4 \end{cases} \tag{6}$$

Using the transport simplex method to find the optimal transport plan and the minimum total freight rate:

1. First use the minimum element method to get the initial dispatching plan:

Table 8. The initial dispatching plan

Place of origin \ Demand Place	B_1	B_2	B_3	B_4	Supply quantity (tons)	u_i
A_1	50	90	15^* 20	0	15	0
A_2	6^* 30	12^* 10	70	0	18	30
A_3	12^* 60	20	1^* 80	4^* 0	17	60
Quantity demanded (tons)	18	12	16	4	50	
v_j	0	-20	20	-60		

2. The test number obtained by potential algorithm method is potential as: $\lambda_{11} > 0, \lambda_{12} > 0, \lambda_{14} > 0, \lambda_{23} > 0, \lambda_{24} > 0, \lambda_{32} < 0,$ and adjust the input base variable by the closed loop method:

Table 9. The optimal table

Place of origin \ Demand Place	B_1	B_2	B_3	B_4	Supply quantity (tons)	u_i
A_1	50	90	15^* 20	0	15	0
A_2	18^* 30	0^* 10	70	0	18	50
A_3	60	12^* 20	1^* 80	4^* 0	17	60
Quantity demanded (tons)	18	12	16	4	50	
v_j	-20	-40	20	-60		

At this time, all the test numbers are greater than or equal to zero, we get the optimal solution: $X = \begin{bmatrix} 0 & 0 & 15 & 0 \\ 18 & 0 & 0 & 0 \\ 0 & 12 & 1 & 4 \end{bmatrix}$, and the optimal value is 1160 Yuan. The amount of calculation is relatively large.

Then, we can use MATLAB to solve this problem.

The MATLAB program [3] is as follows:

```
>>c=[50;90;20;0;30;10;70;0;60;20;80;0];
>>Aeq=[1 1 1 1 0 0 0 0 0 0 0 0 ;
        0 0 0 0 1 1 1 1 0 0 0 0 ;
        0 0 0 0 0 0 0 0 1 1 1 1 ;
        1 0 0 0 1 0 0 0 1 0 0 0 ;
        0 1 0 0 0 1 0 0 0 1 0 0 ;
        0 0 1 0 0 0 1 0 0 0 1 0 ;
        0 0 0 1 0 0 0 1 0 0 0 1];
>>Beq =[15;18;17;18;12;16;4];
>>lb = [0 0 0 0 0 0 0 0 0 0 0 0];
>>ub=[];x0=[];A=[];b=[];
>>options = optimset('LargeScale','off','Simplex','on');
>>[x,fval,exitflag,output,lambda] = linprog(c,A,b,Aeq,Beq,lb,ub,x0,options)
```

The result of running the program:
>>Optimization terminated.
>>x =0 0 15 0 18 0 0 0 0 12 1 4
>>fval =1160
>>exitflag =1

The optimal solution to this problem is when x_{11}=0, x_{12}=0, x_{13}=15, x_{14}=0, x_{21}=18, x_{22}=0,x_{23}=0, x_{24}=0, x_{31}=0, x_{32}=12, x_{33}=1, x_{34}=4. The third factory has four tons of food stored locally, the freight is the least and the total freight is at least 1160 Yuan.

By assuming a virtual place of sale, the unbalanced transportation problem of production exceeding sales is transformed into a balanced transportation problem, which can be solved by MATLAB. It make we can very quickly get the optimal solution of problem.

Then, similar to the way to solve the problem of unbalance of production over sales, the core method to solve the problem of unbalance of sales over production is to convert the problem of sales over production into the problem of balance of production and sales, that is, to make a virtual production area, its output is "total sales - total production", and at the same time, the unit freight rate from the virtual production area to the sales place is zero. It also can be quickly solved by MATLAB after it is transformed into a production-sales balance problem.

4. Conclusion

It is feasible for us to use the transportation simplex method to solve the transportation problem, but the heavy calculation is not conducive to our solution. Using MATLAB to solve the transportation optimization problem, the complex operation process is simplified and the efficiency of transportation planning is improved and it can find the optimal solution of transportation scheme, so as to reduce logistics costs and win more benefits for the company. The introduction of MATLAB to solve traffic problems improves people's ability to model and solve models. At the same time, combined with the actual way, it also makes the application of transportation problems closer to solving problems in real life.

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