Modeling for Trajectory of Ballistic Missile Target Group

Zhanke Li, Junfeng Jin, Zheng Li

China Electronic Technology Group Corporation KLAASA of CETC38, Hefei, China

Abstract. Ballistic target tracking and fusion is important to missile defense systems. A vital problem of in tracking and fusion is how to model the trajectory of ballistic missile target group accurately. This paper proposes a method to model the trajectory of ballistic missile, decoys and debris. The method uses pseudo-spectral method to determine the trajectory of a ballistic missile, is based on release model to establish the trajectory of decoys, and considers vanishing condition to model the trajectory of debris. The simulation results show the proposed method can obtain the trajectory of the ballistic missile target group and approximate the real situation with appropriate parameters.

Keywords: ballistic missile target group; trajectory model; warhead; pseudo-spectral method; decoys; debris.

1. Introduction

Ballistic missile launch schemes are highly variable, and the forces are complex. For penetration purposes, decoys are usually released to form a space target group containing warheads, decoys and debris. It brings great challenges to the situation awareness of missile defense systems. Therefore, it is necessary to study the trajectory model of ballistic missile target group to support the tracking and fusion algorithm of ballistic target group [1].

The entire trajectory of a ballistic missile is commonly divided into three basic phases: boost, ballistic and reentry. The boost phase is the powered flight, which lasts from launch to thrust cutoff or burnout. It is followed by the ballistic phase, which is an exo-atmospheric, free-flight motion, continuing until the atmosphere is reached again. The atmospheric reentry begins when the atmospheric drag becomes considerable and endures until impact. Giving the initial value, Runge-Kutta methods is usually used to are integrate trajectory of a ballistic missile. However, it cannot be used to determine the trajectory of a ballistic missile, given only the launch position and targeted impact point [2].

Beside of warheads, a space target group contain decoys and debris. Due to the differences of the motion characteristics at different phases, the trajectory of coast decoy is influenced to release model [3]. And boost debris trajectories are under the consideration of several factors: the number, mass, initial state, and vanishing condition [4]. It brings difficult to model trajectory of ballistic missile target group.

In order to model the trajectory of ballistic missile target group, a phased modeling method for ballistic missile target group is proposed. The research uses pseudo-spectral method to determine the trajectory of a ballistic missile, given only the launch position and targeted impact point, establishes the release model of coast decoy in symmetric release structure, and designs the generation process of boost debris trajectories considering vanishing condition. The simulation results show that the proposed method can approximate the real situation with appropriate parameters and greatly improve the tracking ability of anti-missile radar to complex group targets.

2. Mathematical model

2.1 Dynamical model

According to the different force conditions, the trajectory of a ballistic missile usually divided into boost, ballistic and reentry.

The boost phase is mainly affected by thrust, gravity and drag. After the booster is turned off, the missile is in the ballistic phase and moves under the action of the earth's gravity. In the reentry, the missile is affected by drag in addition to gravity. In the non-inertial frame, it is necessary to consider the Coriolis inertial force and the associated inertial force brought by the rotation of the Earth to establish the target motion model. In the ECEF non-inertial coordinate system, the dynamic model of ballistic missile is [5]:

$$\dot{\mathbf{v}} = \mathbf{a}_T + \mathbf{a}_D + \mathbf{g} - 2\boldsymbol{\omega}_e \times \mathbf{v} - \boldsymbol{\omega}_e \times (\boldsymbol{\omega}_e \times \mathbf{r}) \tag{1}$$

Where $\mathbf{v}$ is the velocity, $\mathbf{a}_T$ is the thrust acceleration, $\mathbf{a}_D$ is the drag acceleration, $\mathbf{g}$ is the acceleration of gravity, $\boldsymbol{\omega}_e = [0, 0, \omega_z]$ is the angular velocity of the earth's rotation, and $\mathbf{r}$ is the distance from the target to the center of the earth. The target thrust acceleration is:

$$\mathbf{a}_T = \frac{\mathbf{T}}{m} \tag{2}$$

where $\mathbf{T}$ is the engine thrust, m is the target mass, and its change rate is:

$$\dot{m} = \frac{\|\mathbf{T}\|}{g_0 I_{sp}} \tag{3}$$

where g_0 is the acceleration of gravity in the sea level, I_{sp} is the specific impulse, and the target drag acceleration is:

$$\mathbf{a}_D = -\frac{1}{2} C_D S \rho \|\mathbf{v}\| \mathbf{v} \Big/ m \tag{4}$$

where C_D is the drag coefficient, S is the equivalent cross-sectional area of the target, and the atmospheric density ρ is the height function:

$$\rho = 1.226 e^{-h/7254.24} \tag{5}$$

where, h is the altitude, and the target gravity acceleration is:

$$\mathbf{g} = -\frac{\mu}{\|\mathbf{r}\|^3} \mathbf{r} \tag{6}$$

where, μ is the earth's gravitational constant.

2.2 Decoy release model

The decoy is released by internal release devices or spin passively to achieve the purpose of penetration. To analyze the motion of the decoy, the coordinate system is defined as follows:

For horizontal coordinate system $o - x_g y_g z_g$, coordinate origin o is located at the missile center of mass, ox_g is perpendicular to the target meridional plane pointing to the east, oz_b is perpendicular to the local horizontal plane pointing to the zenith, and oy_b points to the north [6].

For ballistic coordinate system $o - x_b y_b z_b$, coordinate origin o is located at the center of mass of missile, ox_b is along the direction of missile velocity, oy_b is located in the local horizontal plane vertically pointing to the right, and oz_b forms the right hand coordinate system. Where the spin velocity of the capsule is ω, the decoys are placed in a symmetric structure, and the initial placement angle is ε_0. After the interval Δt, the azimuth angle of the decoy is $\varepsilon = (\varepsilon_0 + \omega \Delta t) \bmod 2\pi$, the angle between the release device and the decoy is α, and the release angle is β. The release device increases the decoy speed by v_d. When the decoy is released with tangential velocity v_r caused by spin, the initial relative velocity in the capsule ballistic coordinate system is , $\mathbf{v}_{rb} = [v_d \cos\alpha, -v_r \sin\varepsilon + v_d \cos\beta, v_r \cos\varepsilon - v_d \sin\beta]$. When the velocity of the capsule at the time of release is v, the initial velocity of the decoy is:

$$\begin{cases} v_{xd} = v + v_d \cos\alpha \\ v_{yd} = -v_r \sin\varepsilon + v_d \cos\beta \\ v_{zd} = v_r \cos\varepsilon - v_d \sin\beta \end{cases} \tag{7}$$

According to the initial velocity in the ballistic coordinate system, the decoy initial velocity in the horizontal coordinate system can be obtained as follows:

$$\mathbf{v}_g = \begin{bmatrix} \cos\gamma\sin\psi & -\cos\psi & \sin\gamma\sin\psi \\ \cos\gamma\cos\psi & \sin\psi & \sin\gamma\cos\psi \\ -\sin\gamma & 0 & \cos\gamma \end{bmatrix} \begin{bmatrix} v_{xb} \\ v_{yb} \\ v_{zb} \end{bmatrix} \tag{8}$$

where γ is the trajectory inclination angle, ψ is the heading angle, and the initial velocity $\mathbf{v}_g$ is transformed into the ECEF coordinate system:

$$\mathbf{v}_{decoy} = \begin{bmatrix} -\sin\theta & -\sin\phi\cos\theta & \cos\phi\cos\theta \\ \cos\theta & -\sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} v_{xg} \\ v_{yg} \\ v_{zg} \end{bmatrix} \tag{9}$$

Assuming that the bait release is completed in an instant, and the effect of the decoy release on the speed and position of the capsule is ignored, the position of the capsule at the moment of release is taken as the initial position of the decoy.

2.3 Debris model

During booster separation and decoys release, a large number of debris are generated with different mass and initial velocity. During the reentry phase, these debris are subjected to aerodynamic heating, which may lead to the ablation phenomenon of full and partial melting. In order to describe the debris trajectory more realistically, the debris generation process and reentry ablation model are designed as follows.

1) n debris are randomly generated, and the parameter n follows the discrete uniform distribution.

2) The generation position of debris is the missile position, and the velocity increment meets the zero-mean normal distribution.

3) The debris is equivalent to a solid sphere, and the debris reentry model is established as follow. The calculation of the drag coefficient is closely related to Knudsen number (Kn) [7].

4) When $Kn \le 0.01$, it is the continuous flow of the atmosphere; when $0.01 < Kn < 0.1$,it is the atmospheric transition zone; when $Kn \ge 0.1$,it is the free molecular flow zone of the atmosphere.

In the continuous flow region, the average heat flow is:

$$q_{cont} = 0.275 \frac{1.103117 \times 10^8}{\sqrt{R}} \left(\frac{\rho}{\rho_0}\right)^{0.5} \left(\frac{v}{v_0}\right)^{3.15} \tag{10}$$

where, ρ_0 is the standard air density, v_0 is the first cosmic velocity, and the average heat flow in the molecular flow region is

$$q_{fm} = 0.25 \frac{0.9 \rho v^2}{2} \tag{11}$$

In the transition flow region, the average heat flow can be calculated by interpolation:

$$q_{trans} = q_{cont} \left[1 - \exp\left(\frac{q_{fm}}{q_{cont}}\right)\right] \tag{12}$$

5) When the overall temperature of the debris reaches the melting point, the temperature will not rise until the heat absorbed by the debris exceeds the latent heat of melting. The condition for complete melting of debris is [8]:

$$\int Sq \mathrm{d}t > mC(T_m - T_0) + mQ \tag{13}$$

where, C is the specific heat, T_m is the melting temperature, T_m is the initial temperature, and Q is the latent heat of melting.

3. Pseudo-spectral method

Legendre-Gauss-Radau (LGR) pseudo-spectral method is one of the direct methods for solving optimal problems, which can effectively deal with trajectory optimization problems with many constraints [9]. The basic solution idea is as follows. Firstly, Legendre polynomial is used to determine the optimal range of N Gaussian node. Then, using the Lagrange interpolation polynomial approximation N nodes structure state variables and control variables, and using numerical integral approximate the objective function of the integral item, so that the original optimization problem into a nonlinear quadratic programming problem. In the research, pseudo-spectral method is used to solve the shortest time optimization problem of trajectory under path constraints and control constraints.

The equation of state of the system is:

$$\dot{\mathbf{X}}(t) = f\left[\mathbf{X}(t), \mathbf{U}(t), t\right] \tag{14}$$

The initial state is $\mathbf{X}(t) = \mathbf{X}(0)$, the terminal constraint $\mathbf{X}(t_f)$ satisfies the terminal constraint equation:

$$N\left[\mathbf{X}(t_f), t_f\right] = 0 \tag{15}$$

The control $\mathbf{U}(t)$ satisfies the inequality constraint:

$$g\left[\mathbf{X}(t), \mathbf{U}(t), t\right] \geq 0 \tag{16}$$

The performance indicator function is:

$$J = \phi\left[\mathbf{X}(t_f), t_f\right] + \int_{t_0}^{t_f} L\left[\mathbf{X}(t), \mathbf{U}(t), t\right] \tag{17}$$

Under the conditions of equality and inequality constraints, the optimal control is found $\mathbf{U}(t)$ to make the index J minimal.

The steps of solving the above control problem using LGR are as follows:

1) The time domain optimized by the pseudo-spectral method is $[-1,1]$, and the original time domain is normalized:

$$\tau = -1 + \frac{2(t - t_0)}{t_f - t_0} \tag{18}$$

2) Construct Lagrangian interpolation polynomials to approximate the state variables:

$$\mathbf{X}(\tau) \approx \tilde{\mathbf{X}}(\tau) = \sum_{i=0}^{N} \tilde{\mathbf{X}}(\tau_i) L_i(\tau) \tag{19}$$

where $L_i(\tau)$ is Lagrange interpolation basis function.

LGR integral points are used to determine N nodes in the optimization time domain, where boundary nodes τ_N is equal to 1, and other integral points $-1 = \tau_1, \tau_2, \cdots, \tau_{N-1} < 1$ are the roots of the following formula:

$$l_{N-2}(\tau) + l_{N-1}(\tau) = 0 \tag{20}$$

where $l_N(\tau)$ is Legendre polynomial of degree N.

3) Similarly, Lagrangian interpolation polynomials are constructed to approximate the control variables without boundary constraint nodes:

$$\mathbf{U}(\tau) \approx \tilde{\mathbf{U}}(\tau) = \sum_{i=0}^{N-1} \tilde{\mathbf{U}}(\tau_i) L_i(\tau) \tag{21}$$

Taking the first derivative of the discrete state with respect to time, $\dot{L}_i(\tau)$ is expressed as follows:

$$D_{ki} = \dot{L}_i(\tau) = \sum_{l=0}^{N} \frac{\prod_{j=0, j\neq i}^{N} (\tau_k - \tau_j)}{\prod_{j=0, j\neq i}^{N} (\tau_i - \tau_j)} \quad k = 1 \cdots N \quad i = 0 \cdots N \tag{22}$$

The original dynamic constraint is rewritten as:

$$\sum_{i=0}^{N} D_{ki}\tilde{\mathbf{X}}_i - \frac{t_f - t_0}{2} f\left[\mathbf{X}_k, \mathbf{U}_k, \tau_k; t_0, t_f\right] = 0 \tag{23}$$

The performance index function of the original optimization problem is rewritten as:

$$J = \phi\left[\mathbf{X}(t_f), t_f\right] + \frac{t_f - t_0}{2} \sum_{k=1}^{N} \lambda_k L\left[\mathbf{X}_k, \mathbf{U}_k, \tau_k; t_0, t_f\right] \tag{24}$$

λ_k is the weight function of LGR integral node:

$$\lambda_1 = \frac{2}{(N-1)^2} \tag{25}$$

$$\lambda_k = \frac{1}{(N-1)^2} \frac{1-\tau_k}{\left[L_{N-2}(\tau_k)\right]^2}, k = 2, 3, \cdots, N-1 \tag{26}$$

The terminal constraint is rewritten as:

$$N\left[\mathbf{X}_f, t_f\right] = 0 \tag{27}$$

The control constraint is rewritten as:

$$g\left[\mathbf{X}_k, \mathbf{U}_k, \tau_k; t_0, t_f\right] \geq 0 \tag{28}$$

The above equation transforms the original optimal control problem into a nonlinear quadratic programming problem, which can be solved quickly by using optimization algorithms.

4. Simulation example

A three-stage boosted ballistic missile is taken as the object for simulation, and the parameters are shown in Table I.

Table 1. Basic simulation parameter

Parameter	Value
Total mass of each boost(kg)	32680 11148 3583
Fuel quality of each boost(kg)	27680 8648 2583
Working time of each boost(s)	75 100 46
Total thrust of each boos(kN)	1117 280 117
Number of decoys	6
Release angle α (°)	10
Release angle β (°)	0
The increment of velocity v_d /(m/s)	10
Maximum debris size (m)	1
Standard deviation of velocity increment/(m/s)	60
Debris density/(kg/m3)	7871
Constant-pressure specific heat/(J/(kg·K))	4480
Melting temperature(K)	1801
Latent heat of fusion/(J/kg)	2.07×10^5

The launch position of the missile is 70° E and 10° N, and the landing position is 135° E and 45° N. The pseudo spectral method is used to solve the missile trajectory, and the time of flight is 1501s.

The flight trajectory of the warhead in the ECEF coordinate system is shown in the following Fig. 1. The red line is the trajectory of the boost phase and blue line is the trajectory of the passive phase.

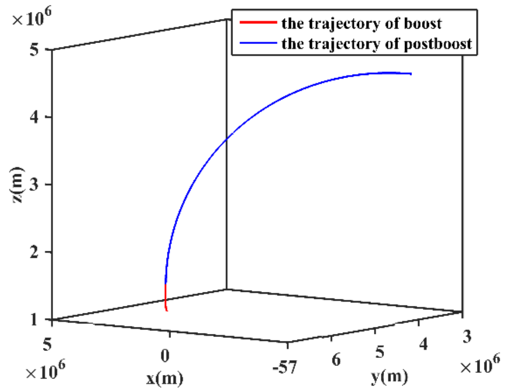

Figure 1. Main target trajectory in ECEF

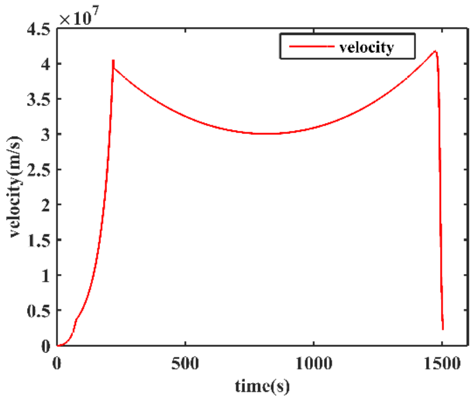

Figure 2. Velocity curve of missile

The velocity of the missile varying with time as shown in the following Fig. 2. It can be seen that under the action of thrust, the speed of the missile is constantly increasing. With engine shutdown, the speed firstly decreases and then increases. In the latter part of the flight, the speed decreases dramatically because of drag.

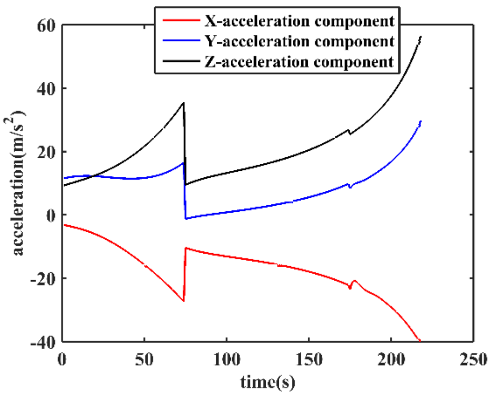

Figure 3. Acceleration curve of the booster

The acceleration curve of the booster varying with time as shown in the Fig. 3. It can be seen that the target acceleration in the booster phase varies greatly. Because of the shutdown and separation of the sub-engine, the acceleration curve has obvious mutation points.

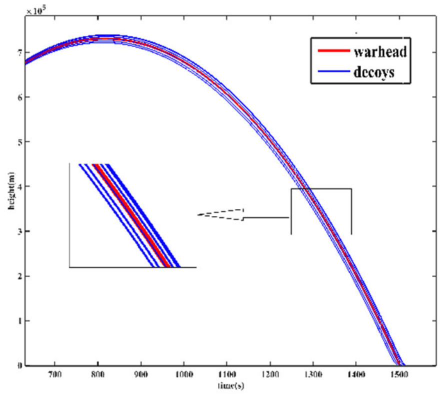

Figure 4. Velocity curve of missile

The decoy is placed in the capsule around the warhead at 60° intervals, which is released simultaneously during the missile's 400s flight. As can be seen in Fig. 4, a threatening target group has formed in space to surround the warhead.

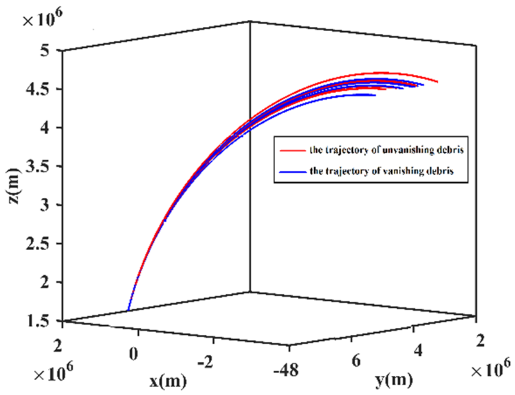

Figure 5. Debris trajectory at stage 3

Taking the debris generated at stage 3 as an example, the generated trajectory is shown in the following Fig. 5. The blue line is the trajectory of the unvanishing debris and the red line is the trajectory of the vanishing debris. It can be seen that as the debris descends, part of the debris vanish under the action of aerothermal.

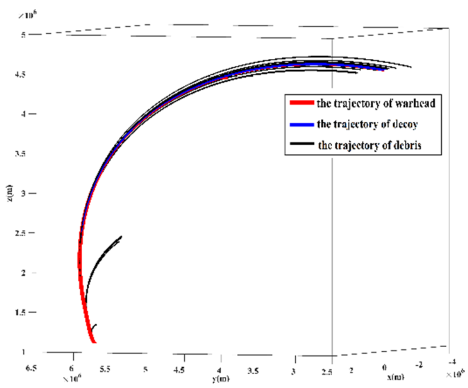

Figure 6. Trajectory curve of the target group in the ECEF

By integrating the simulation results, the motion trajectory of the target group in the ECEF coordinate system can be obtained. Fig. 6 shows the results. The proposed modeling and method can simulate the group target including warhead, decoy and debris under the given launch point and land point. Compared with the actual data, the proposed modeling simulation method can approximate the real situation under the condition of selecting the appropriate parameters, which verifies the effectiveness of the method.

5. Conclusion

We proposed a phased modeling method for ballistic missile target group. The method uses pseudo-spectral method to determine the trajectory of a ballistic missile, given only the launch position and targeted impact point, establishes the release model of coast decoy in symmetric release structure, and designs the generation process of boost debris trajectories considering vanishing condition. The proposed method is applied to simulate the trajectory of the ballistic missile, the decoy release process and the trajectory of multiple batches of debris. The trajectory of the ballistic missile target group is obtained with the method. The simulation results show that the proposed method can approximate the real situation with appropriate parameters.

References

[1] H.T. Li, Y. Zhao, and Y.X. Huang, "Missile Stage Separation Simulation Considering Complex Factors," IEEE Proceedings of the 34th Chinese Control Conference. Hangzhou, China, 8768-8772, 2015.

[2] B. Cockburn and C.W. Shu. "TVB Runge-Kutta local projection discontinuous Galerkin finite element method for conservation laws," Math. Comp., 1989.

[3] W. Zi, Y. Bi, and I.N. Sneddon, "Spatial Distribution Study of Decoys in Ballistic Missile Midcourse," Journal of Projectiles, Rockets, Missilesand Guidance, vol. 33, pp. 189-194, 2013.

[4] W. Wang, H. Shen, and Y. Li, "Study of Liquid Rocket Explosion Fragments Model," Aerospace Shanghai, vol. 30, pp. 35-39, 2013.

[5] X.R. Li and V.P. Jilkov, "A Survey of Maneuvering Target Tracking: Dynamic Models," In Proc. 2000 SPIE Conf. on Signal and Data Processing of Small Targets, vol. 4080, pp. 212-235, 2000.

[6] W.P. Library, "Horizontal coordinate system," Nadir, 2015.

[7] B. Ross, "Space debris reentry risk analysis," Aiaa Journal, 2000.

[8] R.F. Hu, Z.N. Wu, and X. Qu, "Debris reentry and ablation prediction and groundrisk assessment system," Acta Aeronautica et Astronautica Sinica, vol. 32, pp. 390-399, 2011.

[9] M. Bittner, F. Fisch and F. Holzapfel, "A Multi-Model Gauss Pseudo-spectral Optimization Method for Aircraft Trajectories," AIAA Atmospheric Flight Mechanics Conference, pp. 1-14, 2012.