An iterative mass-to-drag ratio estimation method for ballistic targets

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Abstract. Ballistic target tracking and identification is a key task for missile defense systems. A vital problem in target track and recognition is how to estimate the target mass-to-drag ratio fast and accurately. Based on the constraints of energy conservation, this paper proposes an iterative mass-to-drag ratio estimation method. This algorithm first estimates the position and velocity of the target using unscented Kalman filter, and then estimates the mass-to-drag ratio of the target by using energy conservation. By iterating, the final value is outputted. The simulation results show that this method can estimate the mass-to-drag ratio of the target at a high altitude, which provides a precise way for mass-to-drag ratio estimation.

Keywords: ballistic target; mass-to-drag ratio; unscented Kalman filter; energy conservation.

1. Introduction

Among modern weapons, ballistic missile has become the focus of military powers in the world because of its advantages such as wide range of attack, huge power, fast speed and strong targeted attack capability. The flight trajectory of ballistic missile is divided into active and passive stages. The passive stage is divided into free flight stage and reentry stage according to the influence of atmospheric drag. The free flight stage is not affected by atmospheric drag, while the reentry stage is affected by atmospheric drag, so the dynamic characteristics of the two stages are significantly different. Mass-to-drag ratio is the ratio of target mass to windward cross-sectional area, and is an important parameter of target reentry characteristics. It is a vital problem for target track and recognition to estimate the mass-to-drag ratio of target [1].

There is an analytical formula method, which is derived from the reentry equation of motion and the radar measurement equation [2]. This method is simpler to calculate, but because only the measurement in the current time interval is taken for calculation each time, the estimated residual is large. Current principal method for estimating target mass-to-drag ratio is filter method [3]. Based on the reentry equation of motion, different filters such as extended Kalman filter, unscented Kalman Filter (UKF), etc. are used, and the mass-to-drag ratio is estimated as a state vector [4, 5]. However, the initial value of the mass-to-drag ratio and the value of zero-mean Gaussian white noise will affect the precision of result.

The mass-to-drag ratio is an important parameter that determines the reentry motion characteristics. In the mass-to-drag ratio estimation, the estimation is a convergence process from the initial value to the actual value. In order to quickly and accurately estimate the mass-to-drag ratio of the target, a mass-to-drag ratio estimation method based on constraint of energy conservation is proposed in this paper. The initial position and velocity of the target are estimated through the measurements of the trajectory, and the mass-to-drag ratio of the ballistic target is estimated by using the conservation of energy. The estimated mass-to-drag ratio is used to update the initial position and velocity until to minimize the track residual. The state estimation problem is transformed into an iterative problem of mass-to-drag ratio estimation, which simplifies the algorithm of the mass-to-drag ratio estimation and speeds up the convergence process.

2. Reentry target model

2.1 Target dynamic model

After reentering the atmosphere, the missile is mainly affected by gravity and air resistance. In the non-inertial coordinate system of earth-centered-earth-fixed (ECEF), the dynamic model of ballistic missile is as follows [6]:

$$\dot{\mathbf{v}} = \mathbf{a}_D + \mathbf{g} - 2\boldsymbol{\omega}_e \times \mathbf{v} - \boldsymbol{\omega}_e \times (\boldsymbol{\omega}_e \times \mathbf{r}) \tag{1}$$

Where $\mathbf{v}$ is the velocity, $\mathbf{a}_T$ is the thrust acceleration, $\mathbf{a}_D$ is the drag acceleration, $\mathbf{g}$ is the acceleration of gravity, $\boldsymbol{\omega}_e = [0, 0, \omega_z]$ is the angular velocity of the earth's rotation, and $\mathbf{r}$ is the distance from the target to the center of the earth. The target drag acceleration is:

$$\mathbf{a}_D = -\frac{1}{2} C_D S \rho \|\mathbf{v}\| \mathbf{v} \Big/ m \tag{2}$$

where C_D is the drag coefficient, S is the equivalent cross-sectional area of the target, and the atmospheric density ρ is the height function:

$$\rho = 1.226 e^{-h/7254.24} \tag{3}$$

where, h is the altitude, and the target gravity acceleration is:

$$\mathbf{g} = -\frac{\mu}{\|\mathbf{r}\|^3} \mathbf{r} \tag{4}$$

where, μ is the earth's gravitational constant. The mass-to-drag ratio is defined as:

$$\beta = \frac{m}{C_D S} \tag{5}$$

The state vector of the reentry target in the ECEF coordinate system is $\mathbf{X}(t) = [x(t), y(t), z(t), v_x(t), v_y(t), v_z(t)]^T$.

where $x(t), y(t), z(t)$ is position, $v_x(t), v_y(t), v_z(t)$ is velocity. The reentry ballistic missile can be modeled as:

$$\begin{cases} \dot{x}(t) = v_x(t) \\ \dot{y}(t) = v_y(t) \\ \dot{z}(t) = v_z(t) \\ \dot{v}_x(t) = -\frac{1}{2} \frac{\rho v_x(t)}{\beta \|\mathbf{v}\|} - \frac{\mu x(t)}{\|\mathbf{r}\|^3} \\ \dot{v}_y(t) = -\frac{1}{2} \frac{\rho v_y(t)}{\beta \|\mathbf{v}\|} - \frac{\mu y(t)}{\|\mathbf{r}\|^3} \\ \dot{v}_z(t) = -\frac{1}{2} \frac{\rho v_z(t)}{\beta \|\mathbf{v}\|} - \frac{\mu z(t)}{\|\mathbf{r}\|^3} \end{cases} \tag{6}$$

2.2 Measurements model

The measurements of the ground-based radar are usually acquired in the RAE coordinate system (r, φ, θ), and the position of the target can be confirmed by the range r, the azimuth angle φ and the elevation angle θ.The measurements (r, φ, θ) can be transformed from the RAE coordinate system to the East-North-Up(ENU) coordinate system. The transformation is as follows:

$$\begin{cases} r = \sqrt{x_e^2 + y_e^2 + z_e^2} \\ \varphi = \tan^{-1}(x_e / y_e) \\ \theta = \tan^{-1}\left(z_e \Big/ \sqrt{x_e^2 + y_e^2}\right) \end{cases} \tag{7}$$

where x_e, y_e, z_e are the positions on the ENU coordinate system. Considering the measurement noise, the measurements model can be written as:

$$\mathbf{Z} = h\left(T\left(\mathbf{X}\right)\right) + \mathbf{w} \tag{8}$$

where $\mathbf{Z} = \left[r \quad \varphi \quad \theta\right]^T$ and $\mathbf{w} = \left[w_r \quad w_\varphi \quad w_\theta\right]^T$ denoting the measurements and the measurement noise, respectively and the covariance can be chosen according to the measurement accuracy of the radar, $h(\cdot)$ denotes the transformation function from the ENU coordinates to the RAE coordinates, and $T(\cdot)$ denotes the transformation function from the ECEF coordinates to the ENU coordinates [7].

3. Filter model

3.1 Unscented Kalman model

Because of the nonlinear characteristic of state and measurement model, we use the nonlinear filter UKF as the estimator. The state vector X is n-dimensional, $\bar{\mathbf{X}}$ is the mean is X, $\mathbf{P}$ is the covariance matrix, and 2n+1 Sigma sampling points are firstly selected. The specific algorithm process is as follows:

Step 1: Set the initial filtering value, set the initial state $\mathbf{X}(1|1)$ and the value of the initial covariance matrix $\mathbf{P}(1|1)$;

Step 2: Calculate sampling points and obtain a set of Sigma points and corresponding weights by UT transformation;

Step 3: Make state prediction:

Calculate the one-step prediction value of 2n+1 Sigma points:

$$\mathbf{X}^{(i)}\left(k+1|k\right) = f\left[k, \hat{\mathbf{X}}^{(i)}\left(k|k\right)\right], i = 1, \cdots, 2n+1 \tag{9}$$

Calculate the one-step prediction $\hat{\mathbf{X}}(k+1|k)$ and covariance prediction $\mathbf{P}(k+1|k)$ of the state vector.

Calculate the measured predicted value of Sigma point $\mathbf{Z}^{(i)}(k+1|k)$:

$$\mathbf{Z}^{(i)}\left(k+1|k\right) = h\left[\mathbf{X}^{(i)}\left(k+1|k\right)\right], i = 1, 2 \cdots, 2n+1 \tag{10}$$

The predicted mean and covariance matrix of the system can be obtained by the weighted sum of the predicted observations.

Calculate the Kalman gain:

$$\mathbf{K}\left(k+1\right) = \mathbf{P}_{x_k z_k} \cdot \mathbf{P}_{z_k z_k}{}^{-1} \tag{11}$$

Update the state parameters:

$$\hat{\mathbf{X}}\left(k+1|k+1\right) = \hat{\mathbf{X}}\left(k+1|k\right) + \mathbf{K}\left(k+1\right)\left[\mathbf{Z}\left(k+1\right) - \hat{\mathbf{Z}}\left(k+1|k\right)\right] \quad \mathbf{P}\left(k+1|k+1\right) = \mathbf{P}\left(k+1|k\right) + \mathbf{K}\left(k+1\right)\mathbf{P}_{z_k z_k}\mathbf{K}\left(k+1\right)' \tag{12}$$

To estimate the unknown parameter β, the state is extended as $\mathbf{X}(t) = \left[x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), \beta(t)\right]^T$, and use the extended state X for the filter. To guarantee that the filter value.

To guarantee that the filter value of the mass-to-drag is non-negative, we build the mass-to-drag ratio model as [8]:

$$\beta = \beta_0 e^{q(t)} \tag{13}$$

where β_0 is the mass-to-drag ratio of a certain typical target, and $q(t)$ is the zero-mean Gaussian white noise.

When the nonlinear degree of the state equation is high, the prediction error may be large, which will lead to the divergence of the filter. Runge-Kutta integral method can be used in one-step prediction of states [9].

3.2 An iterative method of mass-to-drag ratio estimation

Although the estimation method mentioned above can obtain mass-to-drag ratio, its accuracy and speed are closely related to β_0 and $q(t)$ in (13). In order to quickly and accurately estimate the mass-to-drag ratio of the target, a mass-to-drag ratio estimation method based on constraint of energy conservation is proposed.

Once the initial position and velocity of trajectory are determined, the state of reentry trajectory can be updated by used UKF. The reentry target obeys law of conservation of energy, and is subjected to gravity and atmospheric drag in the atmosphere. So, mass-to-drag ratio is calculated according to the following formula:

$$\frac{1}{2}m\left\|\mathbf{v}_0\right\|^2-\frac{m\mu}{\left\|\mathbf{r}_0\right\|}=\frac{1}{2}m\left\|\mathbf{v}_f\right\|^2-\frac{m\mu}{\left\|\mathbf{r}_f\right\|}-\int_{\mathbf{r}_0}^{\mathbf{r}_f}\frac{1}{2}\frac{m\rho\mathbf{v}\left\|\mathbf{v}\right\|}{\beta}d\mathbf{r} \tag{14}$$

where the subscript represents the time of parameter. The calculation of β is as follows:

$$\beta=\frac{\int_{\mathbf{r}_0}^{\mathbf{r}_f}\rho\frac{1}{2}\mathbf{v}\left\|\mathbf{v}\right\|d\mathbf{r}}{\frac{1}{2}\left\|\mathbf{v}_f\right\|^2-\frac{\mu}{\left\|\mathbf{r}_f\right\|}-\frac{1}{2}\left\|\mathbf{v}_0\right\|^2+\frac{\mu}{\left\|\mathbf{r}_0\right\|}} \tag{15}$$

Use variable substitution to transform into integral of time:

$$\beta=\frac{\int_{t_0}^{t_f}\frac{1.226\mathrm{e}^{-h/7254.24}}{2}\left\|\mathbf{v}\right\|\left(v_x^2+v_y^2+v_z^2\right)dt}{\frac{1}{2}\left\|\mathbf{v}_f\right\|^2-\frac{\mu}{\left\|\mathbf{r}_f\right\|}-\frac{1}{2}\left\|\mathbf{v}_0\right\|^2+\frac{\mu}{\left\|\mathbf{r}_0\right\|}} \tag{16}$$

By integrating the time numerically, the results can be obtained. We can calculate the sum of the distance difference of the last N points between the measure position and the position estimated by β. Compared with the sum of difference based on the β0 of last iterative step, the β is updated or outputted until meeting the setting conditions. The flow chart is as follow:

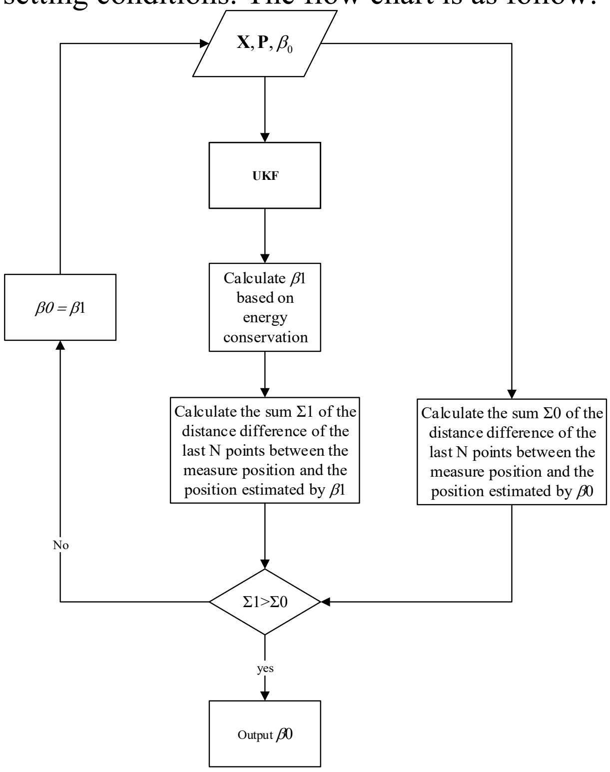

Figure 1. Flow chart of the iterative method

In fact, the calculation of (16) is an iterative process. After solving (16), the sum of difference can be obtained, and the estimated mass-to-drag ratio can be used to correct the initial mass-to-drag ratio. After several iterations, a more accurate mass-to-drag ratio of the target can be obtained.

4. Simulation example

In this section, the performance of the proposed method is verified. The radar is located at the impact point of the ballistic missile, and the sample data rate is 1Hz, the measurement accuracy on the range, azimuth and elevation are 10m, 0.01degree and 0.01degree, respectively. In order to show that the method is suitable for different mass-to-drag ratio cases, two cases with values of 8000 and 800 are designed.

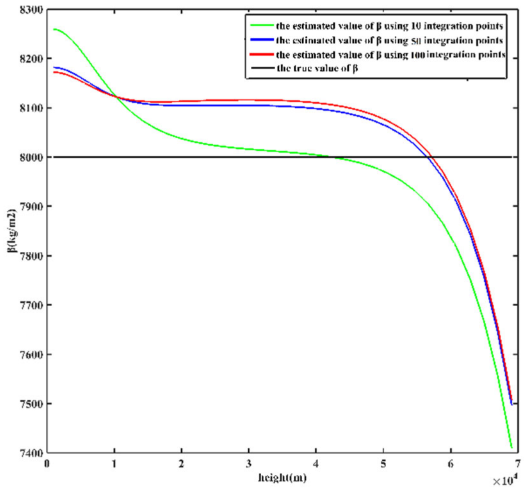

Figure 2. β curve of the value of 8000

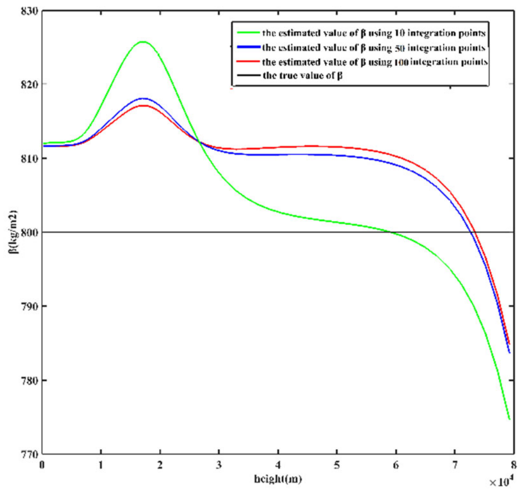

Figure 3. β curve of the value of 800

By integrating the time numerically used law of conservation of energy, mass-to-drag ratio is obtained at different integration points. Fig. 2 and Fig. 3 show the results integrated at 10, 50 and 100 points. As can be seen from figures, with the increase of integration points, the accuracy of integration is improved, and the value using 50 integration points is approximately to the value using 100 integration point. For the case with value of 8000, when the height is less than 52km, the estimated value converges to the true value. And for the case with value of 800, when the height is less than 68km, the estimated value converges to the true value.

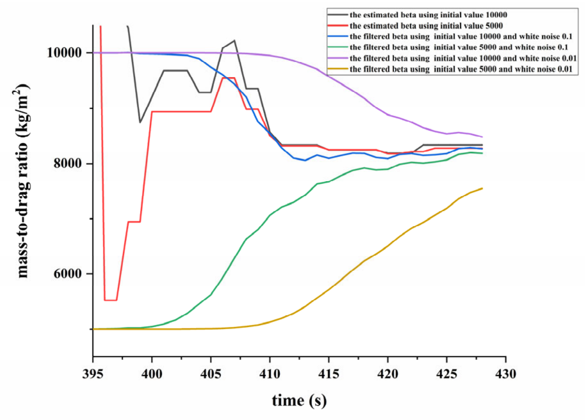

Figure 4. β curve of the value of 8000

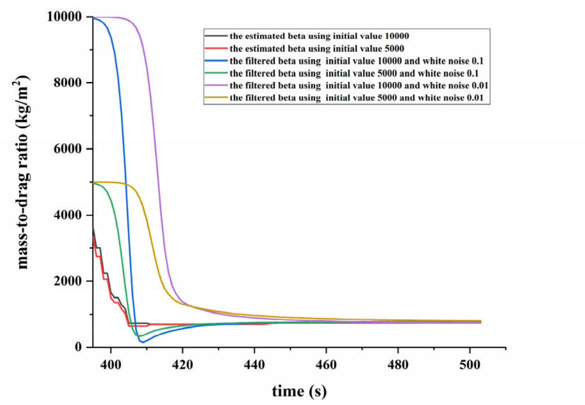

Figure 5. β curve of the value of 800

To verify the advantage of the proposed method, simulations with the proposed method and with the UKF are compared. The results are shown in Fig. 4 and Fig. 5. From Fig. 4, for the cases with value of 8000, the convergence time of iterative method is 412s and much faster than UKF. Meanwhile, the estimation precision of the proposed method is higher than UKF. For the cases with value of 800, the convergence time of iterative method is 415s and much faster than UKF.

5. Conclusion

We proposed an iterative mass-to-drag ratio estimation method based on energy conservation constraint. Combining the trajectory data of the reentry vehicles, the initial state of the reentry vehicles is obtained by using the unscented Kalman filter, and the mass-to-drag ratio is estimated by using energy conservation. Meanwhile, the iterative method proposed in this paper is verified and analyzed. The simulation results show that this method can estimate the mass-to-drag ratio of the target at a high altitude, which provides a precise way for mass-to-drag ratio estimation.

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