

Research on weighted carrier phase three difference precision velocity measurement method based on BDS / GPS integrated system

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Abstract. Aiming at the problem of velocity measurement accuracy in single system carrier phase differential velocity measurement and the possible cycle slip in the process, a carrier phase three difference method is proposed to further improve the velocity measurement accuracy. The use of carrier phase triple difference positioning has been very mature, but there is not much research in velocity measurement. Therefore, this paper uses carrier phase triple difference for velocity measurement. At the same time, in order to improve the cycle slip problem in the process, an integral Doppler double difference combined with carrier phase triple difference is proposed, and then a method to improve the cycle slip based on weight is obtained. Both static and dynamic tests show that the carrier phase triple difference method can improve the speed measurement accuracy, and the weighted carrier phase triple difference method can improve the cycle slip.

Keywords: BDS / GPS, integrated system, weighted carrier phase.

1. Introduction

Precise velocity measurement is of great significance for dynamic applications such as automatic guidance and control of unmanned aerial vehicle (UAV) and calibration of inertial navigation system (INS) [1]. There are three main methods of GNSS velocity measurement in common use [2]: high-precision position differential velocity measurement can reach the accuracy of cm/s level [3]. The velocity of the original Doppler observation value can reach about 1cm/s [4], and the velocity accuracy is low [5]. The carrier phase differential speed measurement can reach below 1cm/s [4] [6]. In order to further improve the speed measurement accuracy, this paper uses carrier phase triple difference to measure the speed. The carrier phase triple difference has been widely used in positioning, but the research on velocity measurement is less. Document [7] uses BDS data as carrier phase epoch difference to measure the dynamic receiver, indicating that the dynamic accuracy of the receiver is at cm/s. The Doppler measurement value indicates the speed of the user at a certain time, while the carrier phase indicates the average speed of the user over a period of time [3] [8], and the Doppler measurement result is significantly affected by the motion of the moving carrier [9]. Therefore, when the carrier movement is stable, the accuracy of the method based on the carrier phase difference between epochs is better [10]. The dynamic and static data were analyzed by the carrier phase difference method between epochs in literature [11], but the clock difference between satellite and receiver was not completely eliminated in the process, so the accuracy of speed measurement can still be further improved. The combined system has a certain improvement in stability compared with the single system. Literature [10] analyzes the speed measurement accuracy of different combined systems using BDS data. Literature [12] shows that the residual ionospheric and tropospheric errors after the inter-era difference are very small. Reference [13] uses GPS to measure the receiver speed, and further improves the accuracy, and does not need to use satellite speed to solve.

This paper will study the carrier phase triple difference method based on the carrier phase, use the BDS/GPS integrated system to accurately fix the speed of the receiver, and eliminate the impact of the satellite clock error caused by the carrier phase difference between epochs, satellites and stations, and further improve the accuracy of the speed measurement. At the same time, in order to improve the effect of cycle slip that may exist in the process, this paper uses the method of integrating Doppler

double difference and carrier phase triple difference to get a method based on weight to improve cycle slip.

2. Analysis of speed measurement method

2.1 Carrier phase three difference method

The carrier phase three-difference velocity measurement model is to use the carrier phase to carry out the carrier phase inter-era difference, carrier phase inter-station difference and carrier phase inter-satellite difference successively. The carrier phase received by the receiver can be expressed as [6]:

$$\lambda\varphi = p + c(\delta t_r - \delta t^s) + \lambda N - \delta_{iono} + \delta_{trop} + \delta_{rev} + \varepsilon \quad (1)$$

Where: λ is the carrier wavelength; φ represents the carrier phase measurement value; p represents the distance between the receiver and the corresponding satellite; c is the speed of light; δt_r is the receiver clock error; δt^s indicates satellite clock error; N represents the ambiguity of the whole cycle; δ_{ip} represents the ionospheric error; δ_{tp} represents tropospheric error; δ_r represents relativistic correction; ε includes various errors including multipath effect, solid tide, ocean tide and receiver noise [14] [15].

Suppose $r^s(k)$ represents the position of the satellite at the k-epoch, and $r^s(k-1)$ represents the position at the k-1 epoch. $r_r(k)$ indicates the position of the receiver of the k epoch, $r_r(k-1)$ indicates the position of the receiver of the k-1 epoch. $r_b(k)$ indicates the position of the k epoch reference station, $r_b(k-1)$ indicates the location of the k-1 epoch reference station. Δr_r represents the change of receiver position coordinates between epochs. l represents the unit observation vector between the reference station and the receiver. Therefore, the distance between the receiver and the satellite can be expressed as

$$p_r(k) = \tilde{l}_r(k)[\tilde{r}^s(k) - \tilde{r}_r(k)] \quad (2)$$

$$p_r(k-1) = \tilde{l}_r(k-1)[\tilde{r}^s(k-1) - \tilde{r}_r(k-1)] \quad (3)$$

Assuming that the reference satellite is represented by the symbol j and other satellites are represented by the symbol s, three differences can be obtained:

$$\begin{aligned} \Delta p_{rb}^{sj} = & [\tilde{l}_r^s(k) - \tilde{l}_b^s(k)]\tilde{r}^s(k) - [\tilde{l}_r^s(k-1) - \tilde{l}_b^s(k-1)]\tilde{r}^s(k-1) - [\tilde{l}_r^j(k) - \tilde{l}_b^j(k)]\tilde{r}^j(k) \\ & + [\tilde{l}_r^j(k-1) - \tilde{l}_b^j(k-1)]\tilde{r}^j(k-1) - [\tilde{l}_r^s(k) - \tilde{l}_r^s(k-1) - \tilde{l}_r^j(k) + \tilde{l}_r^j(k-1)]\tilde{r}_r(k-1) \\ & + [\tilde{l}_b^s(k) - \tilde{l}_b^s(k-1) - \tilde{l}_b^j(k) + \tilde{l}_b^j(k-1)]\tilde{r}_b(k-1) - [\tilde{l}_r^s(k) - \tilde{l}_r^j(k)]\Delta\tilde{r}_r \end{aligned} \quad (4)$$

At the same time, the difference of lambda $\lambda\Delta\varphi$ can be obtained

$$\begin{aligned} & [\tilde{l}_r^s(k) - \tilde{l}_r^j(k)]\Delta\tilde{r}_r - \Delta\varepsilon_{rb}^{sj} \\ = & [\tilde{l}_r^s(k) - \tilde{l}_b^s(k)]\tilde{r}^s(k) - [\tilde{l}_r^s(k-1) - \tilde{l}_b^s(k-1)]\tilde{r}^s(k-1) - [\tilde{l}_r^j(k) - \tilde{l}_b^j(k)]\tilde{r}^j(k) \\ & + [\tilde{l}_r^j(k-1) - \tilde{l}_b^j(k-1)]\tilde{r}^j(k-1) - [\tilde{l}_r^s(k) - \tilde{l}_r^s(k-1) - \tilde{l}_r^j(k) + \tilde{l}_r^j(k-1)]\tilde{r}_r(k-1) \\ & + [\tilde{l}_b^s(k) - \tilde{l}_b^s(k-1) - \tilde{l}_b^j(k) + \tilde{l}_b^j(k-1)]\tilde{r}_b(k-1) \\ & - \lambda^{sj}\Delta\varphi_{rb}^{sj} \end{aligned} \quad (5)$$

From this $\Delta\tilde{r}_r$. According to the time interval between epochs, the average speed between adjacent epochs can be obtained.

2.2 Weighted carrier phase triple difference method for joint integrated Doppler

For equation (5), if the cycle jump occurs during the process, the cycle jump should be subtracted from the right side of the equation, namely

$$\begin{aligned} & [\tilde{l}_r^s(k) - \tilde{l}_r^j(k)]\Delta\tilde{r}_r - \Delta\varepsilon_{rb}^{sj} - \lambda^{sj}\Delta N_{rb}^{sj} \\ = & [\tilde{l}_r^s(k) - \tilde{l}_b^s(k)]\tilde{r}^s(k) - [\tilde{l}_r^s(k-1) - \tilde{l}_b^s(k-1)]\tilde{r}^s(k-1) - [\tilde{l}_r^j(k) - \tilde{l}_b^j(k)]\tilde{r}^j(k) \\ & + [\tilde{l}_r^j(k-1) - \tilde{l}_b^j(k-1)]\tilde{r}^j(k-1) - [\tilde{l}_r^s(k) - \tilde{l}_r^s(k-1) - \tilde{l}_r^j(k) + \tilde{l}_r^j(k-1)]\tilde{r}_r(k-1) \\ & + [\tilde{l}_b^s(k) - \tilde{l}_b^s(k-1) - \tilde{l}_b^j(k) + \tilde{l}_b^j(k-1)]\tilde{r}_b(k-1) \\ & - \lambda^{sj}\Delta\varphi_{rb}^{sj} \end{aligned} \quad (6)$$

Because the integral value of Doppler frequency shift between two epochs is equal to the carrier phase difference without cycle hopping, f represents Doppler frequency shift. Through inter-station and inter-satellite difference, we can get:

$$-\int_{k-1}^k \lambda^{sj} f_{rb}^{sj} dt = \lambda^{sj} \Delta \varphi_{rb}^{sj} \quad (7)$$

Take Formula (7) into Formula (6), we can get:

$$\begin{aligned} & [\tilde{l}_r^s(k) - \tilde{l}_r^j(k)] \Delta \tilde{r}_r - \Delta \tilde{\varepsilon}_{rb}^{sj} \\ &= [\tilde{l}_r^s(k) - \tilde{l}_b^s(k)] \tilde{r}^s(k) - [\tilde{l}_r^s(k-1) - \tilde{l}_b^s(k-1)] \tilde{r}^s(k-1) - [\tilde{l}_r^j(k) - \tilde{l}_b^j(k)] \tilde{r}^j(k) \\ &+ [\tilde{l}_r^j(k-1) - \tilde{l}_b^j(k-1)] \tilde{r}^j(k-1) - [\tilde{l}_r^s(k) - \tilde{l}_r^s(k-1) - \tilde{l}_r^j(k) + \tilde{l}_r^j(k-1)] \tilde{r}_r(k-1) \\ &+ [\tilde{l}_b^s(k) - \tilde{l}_b^s(k-1) - \tilde{l}_b^j(k) + \tilde{l}_b^j(k-1)] \tilde{r}_b(k-1) \\ &+ \int_{k-1}^k \lambda^{sj} f_{rb}^{sj} dt \end{aligned} \quad (8)$$

The difference between formula (8) and formula (6) can be obtained,

$$\lambda^{sj} \Delta N_{rb}^{sj} + \Delta \varepsilon_{rb}^{sj} - \Delta \tilde{\varepsilon}_{rb}^{sj} = \lambda^{sj} \Delta \varphi_{rb}^{sj} + \int_{k-1}^k \lambda^{sj} f_{rb}^{sj} dt \quad (9)$$

At this time, if the corresponding satellite in equation (9) does not have cycle jump, the value on the right of the equation will be close to zero. It can also be seen that the closer the value on the right side of the equation is to zero, the smaller the probability of the satellite's cycle jump is, and the more trustworthy it is. Therefore, each satellite can be assigned a weight, and the smaller the weight, the higher the weight. Assuming that there are N satellites in the process, $N-1$ equations can be obtained. Remember

$$\beta = \begin{bmatrix} \beta^{1j} \\ \beta^{2j} \\ \dots \\ \beta^{N-1j} \end{bmatrix} \quad (10)$$

In order to prevent the occurrence of zero value in the process, a small value $\Delta\alpha$ should be added, so

$$\alpha = \begin{bmatrix} \alpha^{1j} \\ \alpha^{2j} \\ \dots \\ \alpha^{N-1j} \end{bmatrix} = \begin{bmatrix} \beta^{1j} - \beta_{min} \\ \beta^{2j} - \beta_{min} \\ \dots \\ \beta^{N-1j} - \beta_{min} \end{bmatrix} \quad (11)$$

Then take the reciprocal of this value as the weight matrix C , that is

$$C = \begin{bmatrix} \frac{1}{\alpha^{1j} + \Delta\alpha} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\alpha^{N-1j} + \Delta\alpha} \end{bmatrix} \quad (12)$$

The weight in the least squares method is thus constructed, namely

$$WG\Delta x = Wb \quad (13)$$

Where $G = \begin{bmatrix} [\tilde{l}_r^1(k) - \tilde{l}_r^j(k)] \\ [\tilde{l}_r^2(k) - \tilde{l}_r^j(k)] \\ \dots \\ [\tilde{l}_r^{N-1}(k) - \tilde{l}_r^j(k)] \end{bmatrix}, \Delta x = \Delta \tilde{r}_r, C = W^T W$

$$\begin{aligned} \gamma^{ij} &= [\tilde{l}_r^i(k) - \tilde{l}_b^i(k)] \tilde{r}^i(k) - [\tilde{l}_r^i(k-1) - \tilde{l}_b^i(k-1)] \tilde{r}^i(k-1) - [\tilde{l}_r^j(k) - \tilde{l}_b^j(k)] \tilde{r}^j(k) \\ &+ [\tilde{l}_r^j(k-1) - \tilde{l}_b^j(k-1)] \tilde{r}^j(k-1) - [\tilde{l}_r^i(k) - \tilde{l}_r^i(k-1) - \tilde{l}_r^j(k) + \tilde{l}_r^j(k-1)] \tilde{r}_r(k-1) \\ &+ [\tilde{l}_b^i(k) - \tilde{l}_b^i(k-1) - \tilde{l}_b^j(k) + \tilde{l}_b^j(k-1)] \tilde{r}_b(k-1) \\ &- \lambda^{ij} \Delta \varphi_{rb}^{ij} \end{aligned} \quad (14)$$

Then

$$b = \begin{bmatrix} \gamma^{1j} \\ \gamma^{2j} \\ \dots \\ \gamma^{N-1j} \end{bmatrix} \quad (15)$$

According to the weighted least squares formula,

$$\Delta x = (G^T C G)^{-1} G^T C b \quad (16)$$

For static conditions, it can be weighted without introducing Doppler measurements, because the position itself will not change.

3. Test analysis

3.1 Cyclic jump dynamic test

The dynamic test also uses Panda Space-Time as the reference station, Beidou Xingtong as the mobile station, and uses the trolley to collect data. The dynamic sampling frequency is 1Hz. In order to compare the data, the RTK positioning technology is used to compare the speed obtained by the carrier phase triple difference method with the speed obtained by the RTK positioning position difference. In order to compare the repair degree of different cycle slips of the system, this paper compares the two cycle slips introduced by GPS satellite artificially, and introduces cycle slips every one minute in the process. The results are shown in the figure

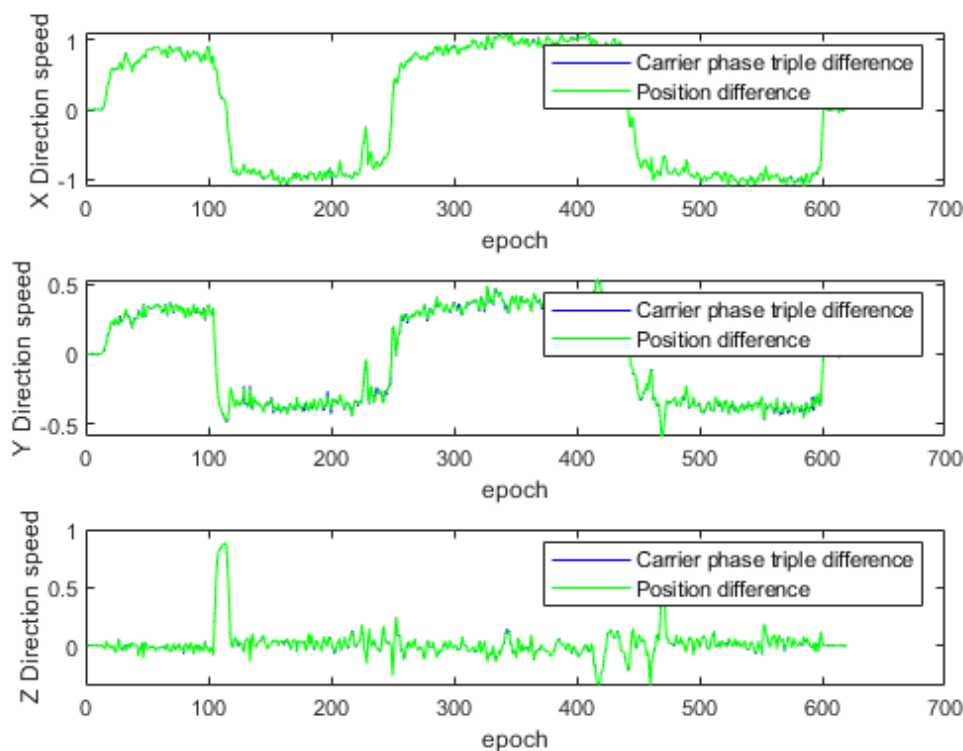


Figure 1. Comparison between RTK position differential velocity measurement and carrier phase triple difference velocity measurement

Table 1. Two-cycle dynamic RMS error of single system and combined system

scheme	X/(mm/s)	Y/(mm/s)	Z/(mm/s)
Weighting BDS	3.3	6.0	2.7
Weighting GPS	8.1	10.3	4.8
Weighting BDS/GPS	3.5	5.9	2.8
BDS	2.2	4.5	2.1
GPS	8.3	19.7	6.0
BDS/GPS	2.4	8.5	2.5

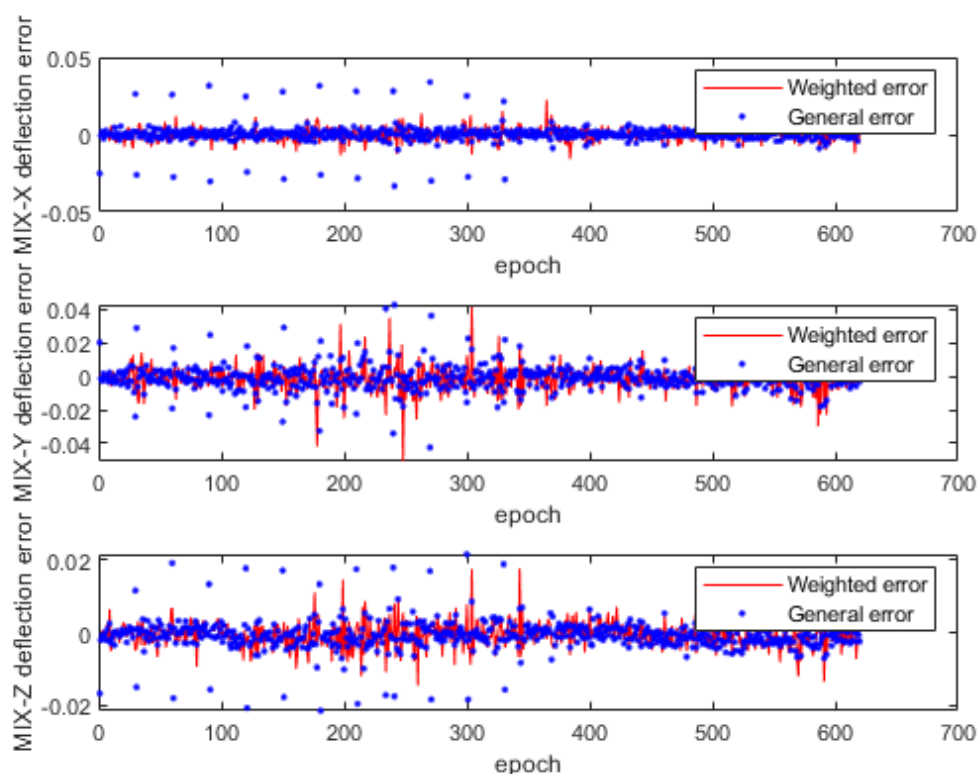


Figure 2. Error diagram of weighted and unweighted BDS/GPS combined system for two-cycle cycle slip in dynamic test

4. Conclusion

This paper introduces the basic principle of carrier phase triple difference method and weighted method in detail, and introduces the influence of single system and combined system on the accuracy of carrier phase triple difference method and weighted carrier phase triple difference method. The advantages and disadvantages of weighted carrier phase compared with carrier phase triple difference method in dynamic and static aspects are compared. The following conclusions are obtained:

- (1) The accuracy of carrier phase difference method is improved compared with that of carrier phase difference method between epochs.
- (2) When there is cycle slip dynamically, the weighted carrier phase triple difference method can improve the error caused by cycle slip.
- (3) The double system has stronger anti-interference ability and better accuracy than the single system.

References

- [1] Xi Chen and Wenyun Gao and Yunheng Wan. Revisiting the doppler filter of LEO satellite GNSS receivers for precise velocity estimation[J]. Journal of Electronics (China), 2013, 30(2): 138-144.
- [2] He Haibo, Yang Yuanxi, Sun Zhongmiao Comparative analysis of several GPS velocity measurement methods [J] Journal of Surveying and Mapping, 2002, 31(3): 217-221. DOI: 10.3321/j.issn:1001-1595.2002.03.007.
- [3] Zhang L, Chang G, Chen C, et al. Adaptive GNSS Velocimetry Combining Doppler and Carrier Phase Measurements Based on Online Variance Component Estimation[J]. IEEE Sensors Journal, 2020, 21(3): 3278-3288.
- [4] Min Li et al. Real-time high-precision earthquake monitoring using single-frequency GPS receivers[J]. GPS Solutions, 2015, 19(1): 27-35.

- [5] Ye S, Yan Y, Chen D. Performance analysis of velocity estimation with BDS[J]. The Journal of Navigation, 2017, 70(3): 580-594.
- [6] Fuhong Wang and Xiaohong Zhang and Jingsong Huang. Error analysis and accuracy assessment of GPS absolute velocity determination without SA[J]. Geo-spatial Information Science, 2012, 11(2) : 133-138.
- [7] FREDA, PIERLUIGI, ANGRISANO, ANTONIO, GAGLIONE, SALVATORE, et al. Time-differenced carrier phases technique for precise GNSS velocity estimation[J]. GPS Solutions, 2015, 19(2): 335-341. DOI:10.1007/s10291-014-0425-1.
- [8] Min Li et al. Real-time high-precision earthquake monitoring using single-frequency GPS receivers[J]. GPS Solutions, 2015, 19(1): 27-35.
- [9] Yan Yongwei, Ye Shirong, Xia Jingchao. Research on BDS carrier phase inter-era differential velocity measurement method [J]. Surveying and mapping science, 2016, 41(07): 193-196+210. DOI: 10.16251/j.cnki.1009-2307.2016.07.0034.
- [10] Liu Chaoying, Chen Guo, Zhao Qile, etc BDS single point velocity measurement principle and accuracy analysis [J] Geodesy and Geodynamics, 2014, 34(6): 114-118.
- [11] Yin Xiao, Chai Hongzhou, Xiang Mingzhi, etc GPS/BDS precision single-point velocity measurement algorithm based on carrier phase difference between epochs [J] Chinese Journal of Inertial Technology, 2020, 28(2): 226-230. DOI: 10.13695/j.cnki.12-1222/o3.2020.02.014.
- [12] YE, SHIRONG, YAN, YONGWEI, CHEN, DEZHONG. Performance Analysis of Velocity Estimation with BDS[J]. Journal of navigation, 2017, 70(3): 580-594. DOI: 10.1017/S0373463316000813.
- [13] Weidong Ding and Jinling Wang. Precise Velocity Estimation with a Stand-Alone GPS Receiver[J]. Journal of Navigation, 2011, 64(2): 311-325.
- [14] Van Graas F, Soloviev A. Precise velocity estimation using a stand-alone GPS receiver[C]//Proceedings of the 2003 National Technical Meeting of The Institute of Navigation. 2003: 262-271.
- [15] Junping Chen et al. Modeling and Assessment of GPS/BDS Combined Precise Point Positioning[J]. Sensors, 2016, 16(7): 1151.
- [16] Yao Yao, Zhang Fan, Du Hongsong, etc Precise carrier phase velocity measurement method based on joint Doppler [J] Chinese Journal of Inertial Technology, 2018, 26(3): 310-315. DOI: 10.13695/j.cnki.12-1222/o3.2018.03.006.