Modeling and design of maximum output power of wave energy device

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Abstract. This paper mainly focuses on a wave energy device modeling and analysis, research in different wave action, the device how to weigh the size of various damping coefficients, based on Newton's second law and bending moment equilibrium theorem to establish a state space model, so as to maximize the average output power. According to Newton's second law, the equation of motion under the state of motion equilibrium is established. By establishing the state space model, the equation of motion is reduced to obtain the standardized state equations of space. Then using the fourth order Runge Kutta solution, the vertical displacement and velocity of the oscillator and float under different damping coefficients are obtained.

Keywords: wave energy device, Motion state model, Force equilibrium equation.

1. Introduction

With the depletion of fossil fuels in the world, developing renewable energy industry has become the consensus of all countries in the world. Therefore, in order to achieve the goal of carbon peaking and carbon neutrality, China has issued a series of support measures to vigorously develop renewable energy, so as to build a "1+N" policy system for carbon peaking and carbon neutrality. Wave energy, as a kind of renewable energy, uses the energy of waves as the power to produce electric energy. The power of sine wave per meter crest width is $P \approx HT \text{ kW/m}$. Where, $H$ is the wave height and $T$ is the wave period. Through the wave energy device, the wave energy can be converted into mechanical, pneumatic or hydraulic energy, and then through the transmission mechanism, gas turbine, water turbine or hydraulic motor to drive the generator to generate electricity. Globally, the economically valuable amount of wave energy mined is estimated at 100 to 1 billion kW. The theoretical reserves of wave energy in China are around 70 million kW. Therefore, how to efficiently utilize wave energy devices to maximize their output power is a problem worth solving[1].

In a wave energy device, the damping force of the linear damper is proportional to the relative velocity of the float and the oscillator, and the proportionality coefficient is the damping coefficient of the linear damper. Based on the motion model of the device and considering that the float only makes heave motion in the wave, the motion model of the float and the oscillator is established. At the initial moment, the float and vibrator are balanced in still water. According to the known parameter values in the research background, the paper calculates the floating displacement and velocity of the float and vibrator under the action of wave excitation force within the first 40 wave cycles with a time interval of 0.2s under the following two conditions: (1) the damping coefficient of the linear damper is 10000 N·s/m; (2) the damping coefficient of the linear damper is directly proportional to the power of the absolute value of the relative velocity of the float and the oscillator, where the proportionality coefficient is 10000 and the power exponent is 0.5[2].

2. Simplify the physical model

Between the float and the oscillator is a spring and a linear damper, a mechanical system that is common in mechanical control. But unlike the usual two-degree-of-freedom mass-spring-damping series system, where the oscillator is located inside the float, only the main force on the float is considered when the water is acting on it, which in problem 1 is called the vertical excitation force[3].
As shown in Figure 1, this paper only considers the floating floating motion in waves, and does not consider the displacement of the device caused by wind force and other factors. Therefore, the horizontal movement of the device can be ignored, and the model can be simplified to the analysis of the two-degree-of-freedom mass-spring-damping series system in two-dimensional space. A cartesian coordinate system is established in two-dimensional space, and the float center of gravity is taken as the origin of coordinates[4].

![Fig. 1 Schematic diagram of the simplified physical model in cartesian coordinate system](image)

**3. System force analysis**

3.1. Force analysis of shaker and float

For the whole system, the external contact force can only act on the float, but in the process of solving the problem, it is necessary to solve the vertical displacement and velocity of the float and the oscillator at the same time, so in the force analysis, it is necessary to find the force connection between the two, and carry out the force analysis of the oscillator and the float[5].

1. Vibrator force analysis

The vibrator is subjected to gravity \( G_{m1} \) as well as the tension of the spring \( F_{k1} \) on it \((k_1 \) represents the stiffness of the spring), and \( F_{b1} \) resistance of the linear damper on it \((b_1 \) the damping coefficient of the linear damper[6].

2. Float force analysis

Considering that gravity and buoyancy are equal when the float remains stationary and the whole system maintains equilibrium[7], since gravity remains constant and the resultant force of the two can be expressed as a force varying with the depth of the float's draft is expressed as \( F_{k2} \) \((k_2 = \pi \times r^2 \times \rho \times g \)). Like the oscillator, the float is also subject to the tension of the spring on it \( F_{b1} \) \((b_1 \) denoting the stiffness of the spring), the resistance of the linear damper on it \( F_{k3} \) \((k_3 \) denoting the damping coefficient of the linear damper), the additional inertial force \( F_g \), the wave exciting force varying with time \( u(t) = f \cos(at) \).

Schematic diagram of the wave energy device's force is shown as figure2.
3.2. Analysis of floating floating motion state

In the process of motion state, it is necessary to consider the float may appear in two states: the first case is that the float in the process of hanging motion to keep the cone does not reveal the horizontal plane; The second case is that the float will make the conical surface of the water during the dangling motion[8].

For these two cases, the height of the cone part is 0.5m, and the shell part of the cylinder is 3m. Obviously, if the cone comes out of the water, it will tip unreasonable. Therefore, the first case is assumed here, assuming that the cone will not come out of the water during the dangling motion. Then verify whether the conical part is exposed to the surface of the water. If it is true, the hypothesis is correct and the opposite is wrong, and the results are used to test the hypothesis and put in the result analysis[9].

4. Establish motion state model based on Newton's second law

4.1. Establishment of equilibrium equation

(1) Symbol description: Since the problem needs to be solved is the displacement and velocity of the float and oscillator, the coordinate system is set in the simplified model, so the displacement of the float and oscillator along the direction x is set as an independent variable about the change of time, denoting as $x_1(t)$, $x_2(t)$. From a physical point of view, the derivative of the displacement of time $x_1(t), x_2(t)$ means the velocity of the float and the oscillator. At the same time, $x_1(t), x_2(t)$ it can also lead to the acceleration of the float and the oscillator[10].

(2) The equilibrium equation of the force changing with time

1) vibrator force balance equation

Within the elastic limit, the force of the spring is related to the deformation and displacement of the spring, so the force provided by the spring is proportional to the displacement difference between the float and the oscillator, and the proportion coefficient is the stiffness of the spring, so the force
provided by the spring, and the oscillator is also subjected to the linear damper, and the force on the oscillator by the linear damper is proportional to the relative speed of the oscillator and the float. According to Newton’s second law, the equilibrium equation of the oscillator is obtained:

\[ k_1(x_2(t) - x_1(t)) + b_1(x_2(t) - x_1(t)) = m_2\ddot{x}_1(t) \]  

(1)

2) Balance equation of float
The equilibrium equation of the float force is as follows:

\[ u(t) - k_1(x_2(t) - x_1(t)) - b_1(x_1(t) - \dot{x}_2(t)) - k_2x_2(t) - k_3\dot{x}_2(t) = m_2\ddot{x}_2 \]  

(2)

4.2. Solution based on motion state space model

The displacement of each part of the device can be regarded as the change of state caused by the wave excitation force, and can be converted into a linear time-invariant system (n states, r inputs and outputs). When the wave excitation force equation is input, the corresponding output of float and oscillator displacement and velocity should be obtained through the system, so the state-space model can be introduced, and its system and control process are shown in Figure 3.

![Schematic diagram of the system control process](Image)

Fig. 3 Schematic diagram of the system control process

The solution process is as follows:

**Step P1:** Establish the time-varying differential equations of the system through the equilibrium equations of the oscillator and the float

\[
\begin{align*}
k_1(x_2(t) - x_1(t)) + b_1(x_2(t) - x_1(t)) &= m_2\ddot{x}_1(t) \\
u(t) - k_1(x_2(t) - x_1(t)) - b_1(x_1(t) - \dot{x}_2(t)) - k_2x_2(t) - k_3\dot{x}_2(t) &= m_2\ddot{x}_2(t)
\end{align*}
\]

(3)

**Step 2:** However, since this is a second-order differential system, it is difficult to solve, so the order is reduced. Here, by analogy with the motion state space model, the second-order system is reduced, the time term is omitted, and only the state variable can be written down.

\[ z = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \]

(4)

**Step 3:** According to the replacement of parameters in the second step, the derivative (differential) of each state vector \( \dot{Z} \) with respect to time can be obtained by sorting the system into the first-order differential equations of each state parameter to obtain four first-order differential equations, which are the same as the standard equations of the state space: \( x = Ax + Bu \).

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \frac{1}{m_1} (-k_1z_1 - b_1z_2 + k_4z_3 + b_1k_4) \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= \frac{1}{m_2} [k_1z_1 - (k_1 + k_2)z_3 + b_1z_2 - (b_1 + k_3)z_4]
\end{align*}
\]

(5)
Step4: Observing the formula, it can be found that this is a linear system, so the system can be expressed as a matrix form:

\[ \dot{z} = Az + Bu \]

Where, A is the system state matrix, B is the output matrix, and u is the input vector:

\[
A = \begin{bmatrix}
  z_1 & z_2 & z_3 & z_4 \\
 0 & k_1/m_1 & k_1/m_1 & 0 \\
 0 & k_1/m_1 & k_1/m_1 & 0 \\
 k_1/m_2 & b_1/m_2 & -(k_1+k_2)/m_2 & -(b_1+k_3)/m_2
\end{bmatrix}
\]

(6)

\[
B = \begin{bmatrix}
  0 & 1 & 0 \\
  1 & 0 & 0 \\
  0 & 0 & 1
\end{bmatrix} \quad \quad u = [u_1 \ u_2]^T
\]

(7)

Step5: However, it is found that it is difficult to obtain the analytical solution for such a system of ordinary differential equations. In the solving process, fourth-order Runge-Kutta is used for numerical solution. By obtaining the system state space equation, input vector and output matrix, the displacement and velocity of the vertical motion can be obtained.

4.3. Analysis of model results

(1) The solution results when the damping coefficient is \(10000\, N \cdot s/m\)

The velocity and displacement of the oscillator and float are obtained respectively, as shown in Figure 4.

![Figure 4](image_url)

Fig. 4 Velocities and displacements of the oscillator and float under damping coefficient\(10000\, N \cdot s/m\)

A note on the assumptions in the analysis of the problem:

Analyzing the buoyancy of the float will reveal whether there is a conical surface above the water, as the buoyancy of the float is related to the drainage volume, which can be calculated by calculating the float draft volume. In the initial state, the buoyancy is in balance with the gravity of the two, so the draft depth at the equilibrium position can be calculated, and the results can be used to judge whether the lower cone will be above the water.
\[ F = (m_1 + m_2) \cdot g = \rho g v_0 \]  

Then the draft depth can be calculated according to the volume of the cone and cylinder

\[ \pi r^2 h_0 + \frac{1}{3} \pi r^2 \cdot 8 = v_0 \]

The solution \( h_0 = 2.4 \) is bigger than the heave displacement calculated by the float, so the hypothesis can be proved. Schematic diagram of float draft depth is shown as figure 5.

![Schematic diagram of float draft depth](image)

**Fig. 5** Schematic diagram of float draft depth

2) The damping coefficient is proportional to the power of the absolute relative velocity of the float and the oscillator, where the proportional coefficient is 10000 and the power exponent is 0.5 as the solution result. The velocity and displacement of the oscillator and the float are obtained, respectively. Waveform diagram in which the damping coefficient is proportional to the power of the absolute relative velocity of the float and the oscillator is shown as figure 6.

![Waveform diagram](image)

**Fig. 6** Waveform diagram in which the damping coefficient is proportional to the power of the absolute relative velocity of the float and the oscillator

Through the comparative analysis of the graph and its data, we find that under the two conditions, the displacement of the float and the oscillator shows a trend of first drastic fluctuation and then gradually stable, and the absolute value of the float motion parameter is always slightly lower than that of the oscillator at the peak of the wave. Considering that the float is more constrained by the additional inertia force and wave damping force than the oscillator, this is in line with the actual situation. For the convenience of calculation, it is assumed that the device in the initial state is located in still water. Therefore, it is of more practical significance to analyze the motion state of the device.
after it becomes stable. Table 1 and Table 2 show the vertical displacement and velocity of the float and oscillator at 10s, 20s, 40s, 60s and 100s under conditions (1) and (2).

**Table 1** The results of float and vibrator displacement and velocity under condition (1)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>float Displacement (m)</th>
<th>float Velocity (m/s)</th>
<th>vibrator Displacement (m)</th>
<th>vibrator Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.190592</td>
<td>0.640564</td>
<td>0.009838</td>
<td>0.693608</td>
</tr>
<tr>
<td>20</td>
<td>0.590531</td>
<td>0.240468</td>
<td>0.634068</td>
<td>0.272287</td>
</tr>
<tr>
<td>40</td>
<td>0.285456</td>
<td>0.313436</td>
<td>0.296610</td>
<td>0.333248</td>
</tr>
<tr>
<td>60</td>
<td>0.314436</td>
<td>0.479107</td>
<td>0.331355</td>
<td>0.515556</td>
</tr>
<tr>
<td>100</td>
<td>0.083590</td>
<td>0.604066</td>
<td>0.084037</td>
<td>0.643044</td>
</tr>
</tbody>
</table>

**Table 2** Displacement and velocity results of the float and oscillator under condition (2)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>float Displacement (m)</th>
<th>float Velocity (m/s)</th>
<th>vibrator Displacement (m)</th>
<th>vibrator Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.064898</td>
<td>1.076118</td>
<td>0.080357</td>
<td>1.274693</td>
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<tr>
<td>20</td>
<td>0.371953</td>
<td>0.559290</td>
<td>0.411087</td>
<td>0.679583</td>
</tr>
<tr>
<td>40</td>
<td>0.461128</td>
<td>0.465262</td>
<td>0.529273</td>
<td>0.546164</td>
</tr>
<tr>
<td>60</td>
<td>0.336653</td>
<td>0.448906</td>
<td>0.383493</td>
<td>0.506796</td>
</tr>
<tr>
<td>100</td>
<td>0.072144</td>
<td>0.645978</td>
<td>0.077099</td>
<td>0.736324</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper establishes the objective programming function which takes the maximum output power of the wave energy device as the target, comprehensively considers the damping coefficient and other influences, and considers the different motion states inside the device. In order to ensure the correct establishment of the model, we first roughly establish the range of the maximum power of the device by referring to the relevant literature, and check the calculation results to ensure the accuracy of the obtained results. Finally, using MATLAB programming, in the field of wave energy application has a high practical value.

References


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