Euler's Formula-Based Research on the Dynamics of Cycling Competition

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Abstract. Speed and endurance are tested in the sport of road cycling, and competitors must appropriately distribute their physical strength based on the road's conditions. This study establishes a bicycle dynamics model based on the rider's motion state and power limitations, as well as the rider's constant power, using the Euler formula in order to obtain the rider's ideal riding strategy. The road conditions are divided into four categories by creating a three-dimensional space coordinate model of the track: uphill, downhill, sharp turns, and flat ground. The cyclist dynamics model is then meticulously optimized to forecast the best performance of various riders on various tracks. The predicted result is improved by the outcome and the reference power allocation scheme.

Keywords: Cyclist Dynamic Model, Euler Formula; Power Distribution.

1. Introduction

Speed and endurance are tested in the sport of road cycling. The surroundings and scenery along the way provide coziness and warmth to a highly difficult sport. Standard, team, and individual time trials are used in the sport. The race that results in the fastest individual time trial is the one that takes the shortest path along a particular route. The type of rider, the amount and length of power they can produce, the weather, the style of track, and many other variables all have an impact on performance. The first step is to create a model that can be applied to various sorts of riders. This model establishes the connection between the rider's force application and location on the course. Additionally, as a result of the various genders and types of riders and their various skills, determining the power curves for the two different types of riders is our second task. We must examine the fundamental causes of the game's effects as well as instances where the model doesn't apply. We will expand the model and provide a set of more all-inclusive competitive strategies based on the aforementioned study. To get the best riding strategy for the rider, (Gordon, 2005)[1] study optimising distribution of power during a cycling time trial. (Zignoli et al., 2017)[2] made a contribution by solving the problem of predictive dynamics of sub-maximal cycling using an optimum control computational approach that employs an indirect method. A 2D two-legged seven bodies three degrees of freedom model of a cyclist's lower limbs was built and verified against the average behavior of eight well-trained riders pedaling at varied sub-maximal intensities (100, 220, 300 W) at constant cadence (90 rpm). (Maier et al., 2017)[3] made a contribution by comparing the accuracy of a large number of current mobile cycling power meters used by professional and recreational riders to a first principle-based mathematical model of treadmill cycling. (Blocken et. al., 2020)[4] study aerodynamic benefits for a cyclist by drafting behind a motorcycle. A cyclist power model is used to convert these drag reductions into potential time gains.(Mayerhofer et al., 2020)[5] aim to modify the prescribed cadence of a cyclist to modulate mechanical power production in outdoor riding. Wind tunnel measurements of rider drag for 32 distinct parallel and staggered cyclist-motorcycle configurations are presented by (Blocken et al., 2021)[6].

To overcome the issue, we primarily created three models and established a number of cycling road time trial racing methods. We created the model, then improved it and included more cases. The impact of various elements on the competition is then examined in terms of how sensitively Finally, the model is evaluated, and the sensitivity analysis is run.

2. The application of the target power

In order to achieve the best result in cycling time trial, the rider needs to maintain his target power during the race. For this, we establish the dynamic equation model of bicycle rider. The model is used to analyze different types of cyclists on different tracks and calculate their optimal target power respectively.[7]

2.1. Bicycle rider dynamic equation model

In this section, we conducted stress analysis, body form analysis and physical state analysis for different types of riders, and established the bicycle rider dynamic equation model of the stress state and power of riders changing with speed and geographical position. We divided the track into several sections according to different road sections. Given the data of the rider's body shape, initial speed, initial acceleration, and initial position information, the applied force of the rider at any position on the track can be determined and a scheme can be devised to minimize the race time.

2.2. The rider's continuous state equation of force

Considering the four main frictions suffered by bicycle riders in cycling: inertia force related to bicycle acceleration, gravity during climbing, tire rolling friction and air resistance, we analyzed the forces on bicycle and rider and described the continuity equation with physical expression as follows:

$$F = F_{sys} + f_{gra} + f_{roll} + f_{air} \tag{1}$$

Where
F_{sys} is the total driving force of people and bicycles
F_{gra} is human and bicycle gravity versus the horizontal component of the road (when climbing)
F_{roll} is mechanism for tire friction
F_{air} is air resistance to people and bicycles
On a stretch of track, where$F_{sys}, f_{gra}, f_{roll}, f_{air}$ can be written as:

$$F_{sys} = mA \tag{2}$$

$$f_{gra} = Gsin\theta \tag{3}$$

$$f_{roll} = GC_s cos\theta + 2C_d V \tag{4}$$

$$f_{air} = \frac{1}{2}KSDV^2 \tag{5}$$

Then, the continuous equation of state for the rider becomes:

$$F = mA + Gsin\theta + GC_s cos\theta + 2C_d V + \frac{1}{2}KSDV^2 \tag{6}$$

And according to the instantaneous power formula $P = FV$, after simplification, the equation is given by:

$$A = \frac{P}{m}\frac{1}{V} - g(sin\theta + C_S cos\theta) - 2C_d\frac{V}{m} - \frac{1}{2}\frac{K}{m}V^2 \tag{7}$$

Where g is the acceleration of gravity, this is the rider's continuous state equation of force. The figure1 is the stress analysis diagram of rider and bike.

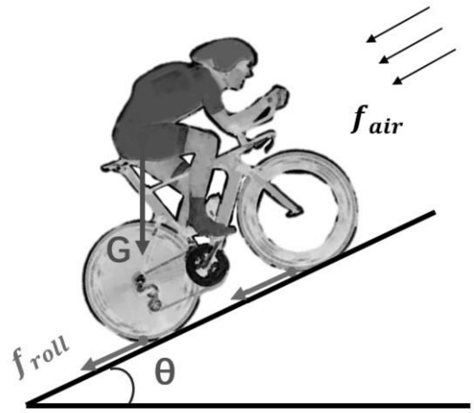


Figure 1. Stress analysis diagram of rider and bike

2.2.1 Equation of state for bicycle riders

As we all know, it is very important to calculate the riding status data of the rider. Therefore, on the basis of the rider's stress state equation above, the numerical function of time in relation to velocity and acceleration, can be deduced respectively by using Euler equation in this link, so as to further understand the rider's driving state.

The derivation process of a driving state equation is as follows:

$$\begin{cases} f(V) = A = \frac{P}{m}\frac{1}{V} - g(sin\theta + C_S cos\theta) - 2C_d\frac{V}{m} - \frac{1}{2}\frac{K}{m}V^2, \\ \frac{dV}{dt} = A \end{cases} \tag{8}$$

$V\ (0), A(0)$ are the initial race speed and the initial acceleration respectively, Euler formula is used to recursively obtain:

The first recursion

$$\begin{cases} V(\Delta t_i) = V(0 + \Delta t_i) \approx V(0) + A(0)\Delta t_i, if\ \ j = 1 \\ A(\Delta t_i) = f\big(V(\Delta t_i)\big) \end{cases} \tag{9}$$

And then,

$$\begin{cases} V(j\Delta t_i) = V\left(\sum_{i=1}^{j} \Delta t\right) \approx V[(j-1)\Delta t_i] + A[(j-1)\Delta t_i]\Delta t_i, if\ j \geq 2 \\ A(j\Delta t_i) = f\big(V(j\Delta t_i)\big) \end{cases} \tag{10}$$

Where t_i is the race time from section i track, j is the recursion number.

In this way, when the known parameters are determined and the constant power, initial speed and initial time are substituted, the numerical function of the relationship between speed and time and the numerical image of the relationship between acceleration and time can be obtained at the i-section. The figure 2 shows the simulated V-ln(t), A-ln(t) image:

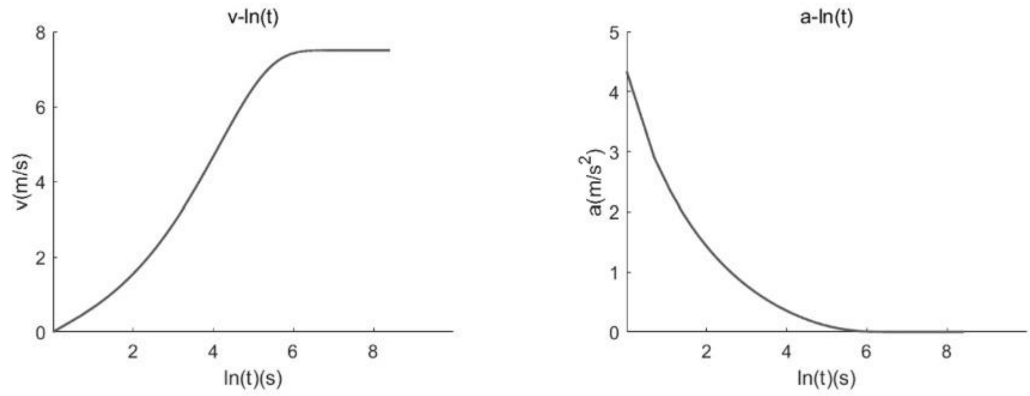


Figure 2. left: V-ln(t), right: A-ln(t)

2.2.2 Integral equation

We imagine that the length of each section of track is compared with the distance the rider has traveled since the start of timing, in order to determine the $i$track the rider is on and the time the rider has traveled each section. For this, we use the integral equation to determine the distance the rider has traveled. So let's first integrate the velocity of t_i:

$$L_i = \int_0^{t_i} V dt \tag{11}$$

$$X(t_i) = L_i \tag{12}$$

Where, L_i is distance covered by a rider in section I from time.

Let $X(t_i)$ be the primitive function of $V(t_i)$,and $X(0)$= 0, therefore there is a Inverse function:

$$t_i = [\, X(Li)]^{-1} \tag{13}$$

Once we have determined the time and speed, we can calculate the time for the cyclist to pass each section of the track by using equation (11) (12). By adding time,

$$\sum_{i=1}^{n} t_i = T \tag{14}$$

n is the number of sections of the track to be divided

T is the total time it takes a rider to complete a race

When we into all of the known parameters and use of the above formula and the rider in the transient state in the process of game data (speed, acceleration, race time and distance), through consulting a large number of data, we can change every reasonable track the value of the constant power to determine the best constant power size, each track Further planning to complete the race in the least time.

Since there is a limit to the total energy that a rider can consume during the whole race, after a clear study of the whole race process of a bicycle rider, we can deduce the total energy that a rider can consume with a limit through the law of energy conservation.

2.3. Boundary conditions of bicycle rider dynamic equation model

Considering that there is a limit to the total amount of energy a rider can expend over the course of the race, as well as limitations that accumulate due to past aggression and exceeding power curve limits, we can summarize these limitations as follows:

$$\sum_{i=1}^{n} t_i = T \tag{15}$$

$$lb \leq [P_1, P_2, \cdots, P_n] \leq ub \tag{16}$$

$$\sum_{i=1}^{n} t_i \leq T(P_0) \tag{17}$$

Expression (15):P_i is constant power of section $i$track, E_{total} is the total amount of energy a rider can expend during the course.

Expression (16):lb is the power lower limit of each track, ub is the power upper limit of each track.

Expression (17):$T(P_0)$ is the sum of the time spent driving each section of the track at constant power, P_0 is Reference power.

The above is the bicycle rider dynamic equation model designed by us.

2.4. Power curve definition for the rider

As we all know, power curve is a graphic form of power that can be generated in a period, which can clearly show the mechanical power and the ability of applying force provided by the rider in the process of a competition, and has a crucial impact on the further analysis of the rider's competition state and the study of competition strategy. Since the amount and duration of energy generated by different types of riders vary, in this section we analyze the characteristics of timing experts and sprinters by consulting a large amount of data and real race data, and define their power curves.[8]

2.4.1 Analysis of rider characteristics

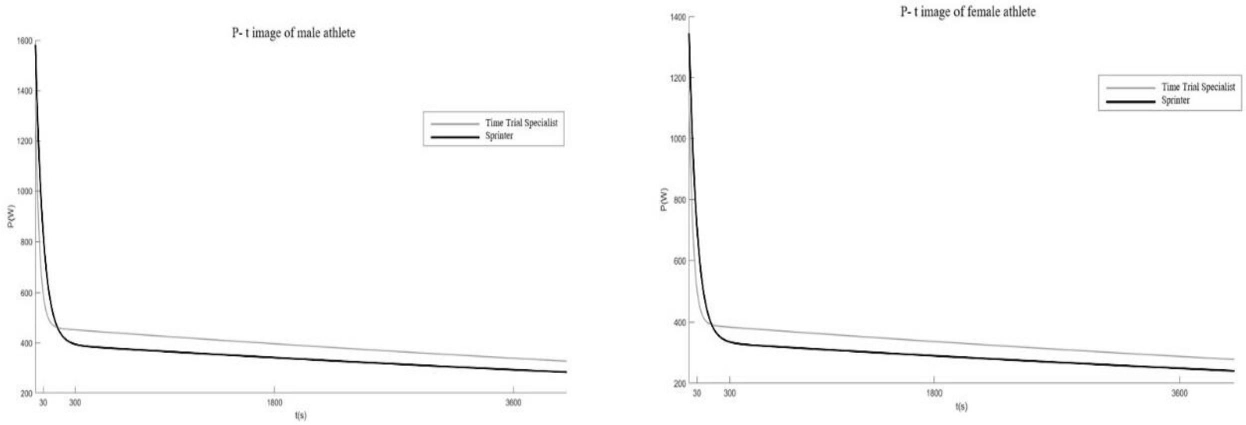
Through consulting a large number of data and real race data, we found that the power output of sprinters varies greatly in different sections during the race, and the slope of the power curve changes more obviously. Therefore, we selected timing experts and sprinters as the research objects.

• The difference between a time trial specialist and a sprinter: the sprinter is more explosive, can pedal faster and for longer periods of time, but needs more time to recover at lower power.

• The differences between male and female riders: Male riders have stronger explosive, higher pedal frequency and longer sustained high- frequency output time, but the energy consumption level of male riders is higher than that of female riders, and the power output of female riders is more stable.

2.4.2 Define the power curve image

Through the analysis of the characteristics of the rider, and access to a lot of data about the rider competition, we defined the P-t images(Power curve image) of time-keepers and sprinters according to gender, which is shown in the Figure 3.



(a)The P-T image of the male time trial specialist and sprinter. (b)The P-T image of the female time trial specialist and sprinter.

Figure 3. The P-t images of time-keepers and sprinters according to gender

3. Work out the best game strategy

In the this section, we established the bicycle rider dynamic equation model to determine the relationship between the rider's time and speed and acceleration in the race, the time spent on each track, the total energy limit and other key factors. What we need to do is to carry out practical prediction analysis and application for various bicycle road race timing courses. Devise the best solution for the least amount of time for different types of riders.

3.1. Find the optimal solution based on optimization model

Our task is to optimize the bicycle rider dynamic equation model, which includes the optimization of the constraints of athletes, the influence of the weather environment, the influence of track position, etc.[9]

• We collected the geographic location and images of the track, and set up a three-dimensional coordinate system with X, Y and Z axes. The data information of the track was imported into MATLAB to obtain the coordinates and altitude of each point of the track. We also collected information about the riders' bodies and physiological indicators that were useful to the model.

• We learned a lot of literatures, critical power as an index, plays an important role in the power curve, the data according to the experimental results show that when the rider at less than or equal to the critical power, can maintain longer riding time, but with power more than the critical power, riding time was dropped substantially, therefore, The power curve in the process of the vast majority of players the game overall stability, fluctuates around in their critical power up and down slightly, while in the uphill, sprint, etc, will choose to temporarily exceed the critical power, and in the next hill, a little sleep, so we are more than the slope, slope and sprint for segmentation conditions, the circuit can be divided into several segments, easy to get more accurate results. The segmentation of the track is shown below:

According to the above discussion, the optimization model can be expressed as:

$$min\sum_{i=0}^{n} t_i \tag{18}$$

$$\begin{cases} A = \dfrac{P}{m}\dfrac{1}{V} - g(sin\theta + C_S cos\theta) - 2C_d\dfrac{V}{m} - \dfrac{1}{2}\dfrac{K}{m}V^2 \\ lb \le P_0 \le ub \\ P_i = \mu_i P_0 \\ t_i = [\,X(Li)]^{-1} \\ \sum_{i=1}^{n} P_i\, t_i \le E_{total} \\ \sum_{i=1}^{n} t_i \le T(P_0) \end{cases} \tag{19}$$

Where μ_i represents a coefficient which explains the proportional relationship between the power of the circuit in section I and the reference power P_0, and $-1 \le \mu_i \le 1$. We will perform a sensitivity analysis of this coefficient μ_i later. Where, we define the symbol of μ_i as μ_a in the uphill section and we define the symbol of μ_i as μ_b in the downhill section.

For the segmentation points of each track, we use MATLAB to simulate the reference power power and the minimum race time of riders in each section of the track. After optimization, we need to reasonably predict the following track and get the optimal solution:

• A track :2021 Olympic Time Trial course in Tokyo, Japan

• B track:2021 UCI World Championship time trial course in Flanders, Belgium

• C track : The custom track

A track is shown in the Figure 4, and B track is shown in the Figure 5.

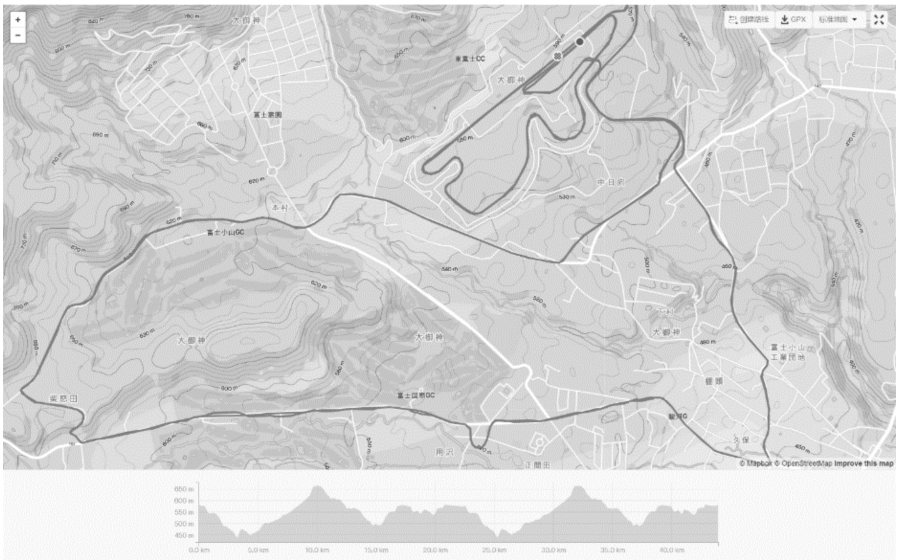


Figure 4. A Track Segmentation Map

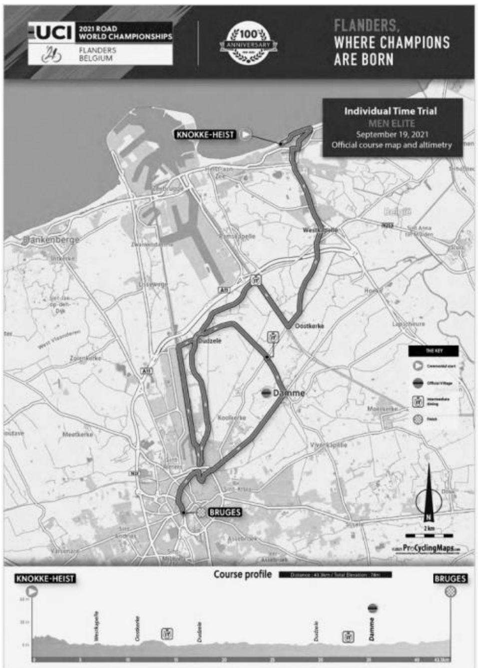


Figure 5. B Track Segmentation Map

A lap of the course is 25km, once for women (25km) and twice for men (50km), and the downhill slope is twice as steep as the uphill one.

The Figure 6 describes our algorithm:

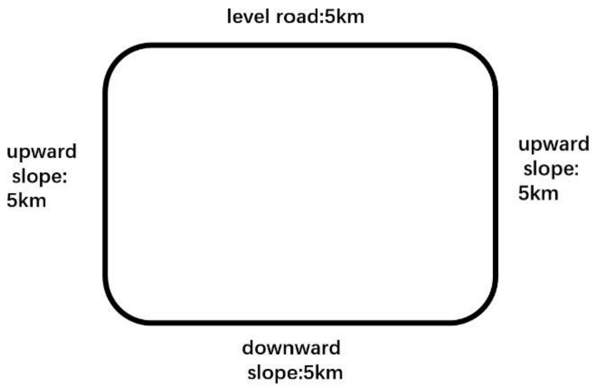


Figure 6. C Track Design

3.2. Analysis and results

We use the above optimization model to calculate the optimal race time and reference power power of different racetracks and riders with MATLAB. The Figure 7 is our result:

The above is the application of the bicycle rider dynamic equation model in different tracks, it is similar to real experimental data and has high reliability.[10]

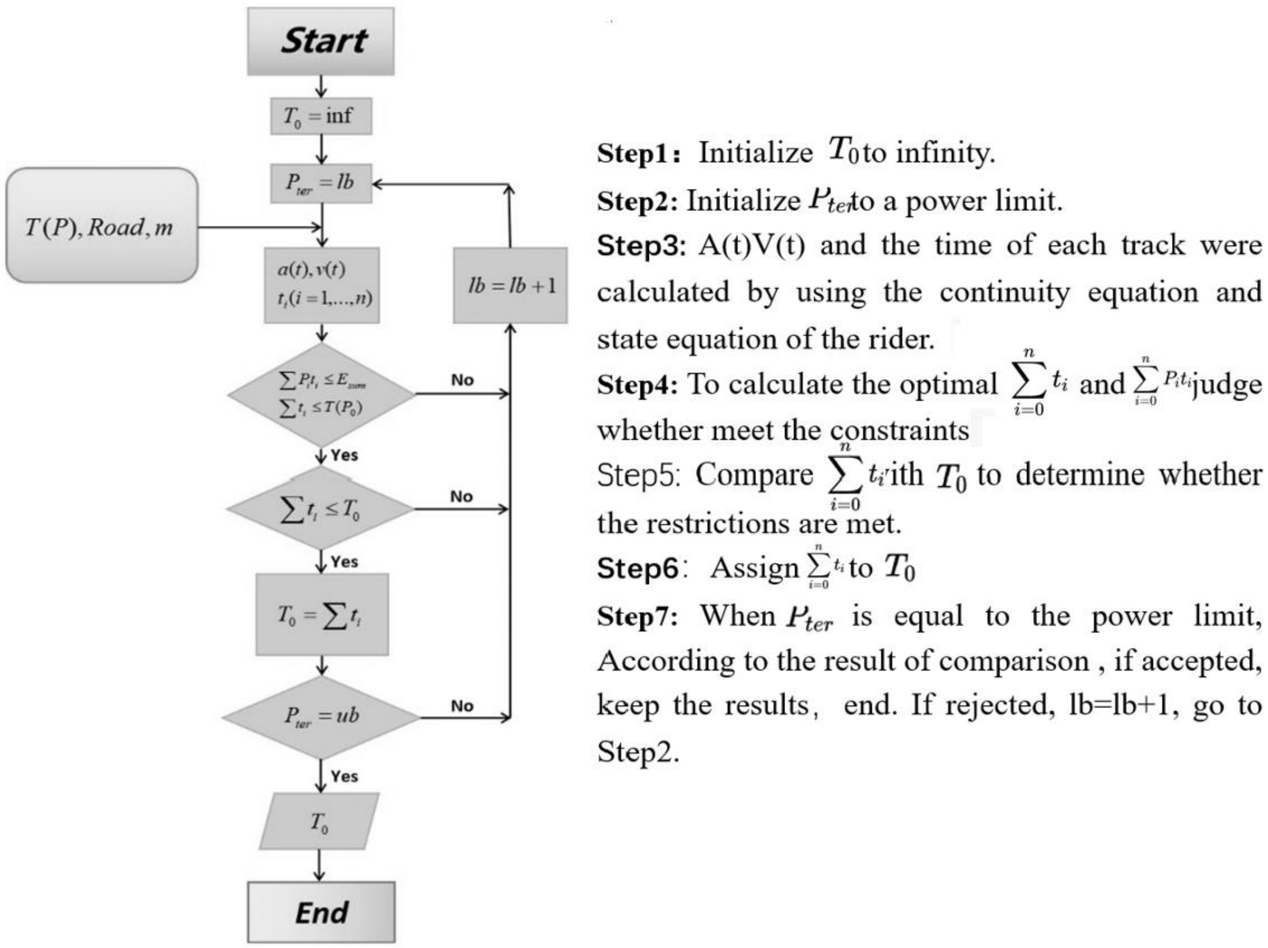


Figure 7. The step of calculation

4. Conclusions

The main topic of this article is how to use power to your advantage during bicycle road time trials. We construct a dynamic physical model, examine the physical behavior of various riders on various racecourses, take into account the impact of prospective race factors, and develop an efficient cycling strategy using Euler's formula, mathematical physics, and other techniques.

First, we looked at the rider's force during the race. We developed the cyclist dynamics model utilizing Euler's formula in accordance with the rider's motion state and power limits, as well as the observation that the majority of riders maintain a steady power throughout the race. For time trial specialists and sprinters of different genders, we have established power curves.

Second, we improved the cyclist dynamics model to forecast individual time trial performance and reference power for various riders on various tracks. The course is divided into four distinct sections: uphill, downhill, sharp curves, and flat land by creating a three-dimensional spatial coordinate model of the track, which yields the coordinates and height of each point of the rider's route. The predicted outcomes are more precise.

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