Evaluation Model of Light Pollution Based on Analytic Hierarchy Process

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Abstract. With the rapid development of the global economy, light pollution has become a new type of environmental pollution. In this paper, we propose a model to measure the degree of light pollution in a certain region (MDLP). In MDLP, we first consider the light demand of an area in relation to its population, GDP, and built-up area, so we apply the hierarchical analysis method (AHP) to calculate the weight of each factor and finally arrive at the light demand rate. Then we take into account environmental, social, amount of light factors when designing MDLP model. Finally, the results obtained for different indexes were graded and fitted with Bortle grades. This thesis provides theoretical support for study by sorting out the factors of the light pollution in city at night.

Keywords: Light pollution, Light demand, AHP, EWM.

1. Introduction

With the development of cities, light has become a new source of pollution. The light decorates the city and brings a lot of negative effects. It disrupts our circadian rhythms, and the strong glare can cause traffic accidents. Meanwhile, light pollution also affects the normal growth of plants. Many wild animals are misled by artificial light in the migration process.

In 1973, Walker [1] established a numerical model of night sky brightness increment $\triangle I = f(r)$ ($r$ means distance) by accumulating measurement data. It can be estimated the sky increment $\triangle I$ within a certain range from the city center, which is also an earlier night sky brightness distribution model. German scholar Andreas H [2] conducted a cumulative study on the growth of light pollution, energy consumption, and light pollution prevention and control in Germany from 1980 to 1998 In 1997. A. R. Upgren [3] used optical photometric method to measure sky brightness and introduced the method of measuring sky brightness by visual photometric method. In 2007. The brightness of the sky on a cloudy night has been calculated by Garstang, R.H [4] using a simple model of the city and a uniform cloud layer. In 2009, the literature of Ming Liu [5] using Back Propagation Neural Network shows that the night sky brightness distribution shows a nonlinear and time varying information structure. And his student, Xiaowei Guo [6] uses spatial test model to analyze the main influencing factors of the light pollution in 2019.

However, it is impractical to avoid light pollution, which will not only affect economic development, but also increase the crime rate in some areas. So this thesis learns abundant related articles and sorting relevant theories and innovatively proposed the light demand rate based on the development of society. The theme requires us to develop a metric for light pollution risk levels based on indicators and test its validity. We also propose intervention strategies for reducing light pollution risk levels. Our main work is presented in the figure 1.
2. Preliminary

2.1. Assumption

Assumption1: We ignore the impact of weather conditions on the measurable results.

Justification1: Since the weather conditions are very complex and changeable, it is difficult to maintain stability. The purpose of establishing the model is to propose controllable and effective policies. The weather conditions are difficult to be changed under the existing means and are uncontrollable factors, so the inference of weather conditions is not considered.

Assumption2: Light pollution is closely related to the degree of economic development, so it is feasible to take GDP into consideration.

Justification2: The urban economy has a high degree of development, and the daylight generated by its lights at night can affect places several kilometers away, resulting in strong light pollution.

Assumption3: We assume the research data is accurate.

Justification3: We assume that the EI and GDP do not show obvious measurable deviation, so we can establish a more reasonable module on it.

2.2. Notations

Table 1. Notations

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQM</td>
<td>Sky quality meter</td>
</tr>
<tr>
<td>Area</td>
<td>The urban area</td>
</tr>
<tr>
<td>B</td>
<td>The Built-up area</td>
</tr>
<tr>
<td>EI</td>
<td>Ecological Environment Index</td>
</tr>
<tr>
<td>( L_R )</td>
<td>Radiance</td>
</tr>
<tr>
<td>DN</td>
<td>Gray level of the images</td>
</tr>
<tr>
<td>R</td>
<td>Reflectivity</td>
</tr>
<tr>
<td>Q</td>
<td>Surface incident light intensity</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Light demand rate</td>
</tr>
<tr>
<td>L</td>
<td>Light source</td>
</tr>
<tr>
<td>E</td>
<td>Environment</td>
</tr>
<tr>
<td>S</td>
<td>Society</td>
</tr>
<tr>
<td>F</td>
<td>Metric value of light pollution level</td>
</tr>
</tbody>
</table>

3. The Degree of Light Pollution

It can be concluded that the influence of light pollution on sky brightness is significant and positively correlated, so the degree of light pollution can be judged from sky brightness. In February
2001, Porter published the Porter dark space classification method, which can preliminarily determine the level of sky brightness by combining the knowledge of astronomy with the color base, the limiting magnitude and the telescope limit magnitude.

However, the purpose of this model is to find feasible intervention strategies, and the selection of some indicators should involve factors that can be considered as interventions. Therefore, we get the light pollution index function from the three advanced levels of light source, environment, and society:

\[ F = \omega_1 L + \omega_2 E + \omega_3 S \]  \hspace{1cm} (1)

3.1. Select detailed indicators

In the hypothesis, we believe that the degree of light pollution in a certain area is closely related to the level of local economic development. The common indicators to judge the level of economic development in a certain area are the population, GDP, and the area level of the construction area. The large population, high GDP, large construction area, to a certain extent, can explain the large number of nightlife, rich nightlife, and ultimately lead to light pollution.

Authors examined the Gross Domestic Product (GDP), total population of 21 prefecture-level cities in Guangdong and built-up area of cities in Guangdong. Figure 2-5 visualize the data.
Figure 4. Heat map of light pollution distribution in Guangdong

Figure 5. Built-up area of cities in Guangdong (Unit: square kilometer)

Although the amount of artificial light generated varies from city to city, we cannot judge the level of light pollution risk in an area from this aspect alone. Because certain cities have a large population and a developed economy leading to a further increase in light demand, we propose that the light demand rate:

$$e = \sigma_1 x_1 + \sigma_2 x_2 + \sigma_3 x_3$$  \hspace{1cm} (2)

$x_1, x_2, x_3$ are the values of population, GDP and built-up area after de-quantization respectively.

We can divide the source of light into natural light and artificial light. Natural light mainly comes from the moon, while artificial light mainly comes from street lights and car lights, which are the source of light pollution. There is no doubt that the source of light is one of the important indicators. We consider the luminous efficiency of the lamp, the illuminance of the moon, the radiation flux, and the intensity of the incident light on the surface. Although they can reflect the degree of light pollution because the difficulty and completeness of obtaining different kinds of data are different, we finally choose the incident light intensity on the surface.

For the surface incident light intensity, we query the data of Luojia No.1, and convert the gray value into the radiance value by using the radiation calibration equation given by the official website [7]. The formula is:

$$L = DN^3 \times 10^{-10}$$  \hspace{1cm} (3)

Because the lighting is mainly concentrated in the built-up area, the buildings outside the built-up area can be ignored. First, the building coverage is obtained by the following formula:
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\[ \eta = \frac{B}{Area} \]  

Then we can get:\n\[ Q = \frac{L_R}{(1-\eta) \times R} \]  

Optical demand rate:\n\[ Q_c = Q \times (1-\varepsilon) \]

3.2. Process the indicators

3.2.1 Normalize the index data

We forward all indicators and propose different methods for different indicators. For those bigger is better, the equation should be:\n\[ y_i = \frac{x_i - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \] (7)

For those smaller is better, the equation should be:\n\[ y_i = \frac{x_{\text{max}} - x_i}{x_{\text{max}} - x_{\text{min}}} \] (8)

For indicators with no clear upper limit, we consider Logistic function normalization:\n\[ y_i = \frac{1}{1 + e^{-\xi(x_i - x_0)}} \] (9)

Among them, the normalized index is \( y_i \). \( x_0 \) is the minimum standard for indicators. \( \xi \) is a parameter of the Logistic function. We assume: \( \xi \) is 1.

3.2.2 Calculate weight by AHP

Considering that each factor has a different degree of influence on the level of light pollution risk, we need to calculate the weight of the influence of each indicator. We use AHP (hierarchical analysis) to analyze the process systematically and hierarchically. A hierarchical model of target level - indicator level - program level is established. By pairwise comparison, the importance of different indicators relative to the calculated light pollution level is determined, and finally the weight coefficients of each indicator are obtained.

Due to the subjectivity of the score matrix, we adopt the multiple regression analysis model to analyze the significance of each index and light pollution, and then allocate the weight according to the significance. We obtain the urban data evaluated by the Porter’s dark space classification method, and obtain the significant size relationship of the indicators based on the stepwise backward regression:

\[ \text{SQM} > \text{GDP} > \text{Population} > Q > S > Ei \]

According to the above analysis, we can get:

\[ E = 0.8E_1 + 0.2E_2 \] (10)
\[ S_o = 0.5S_1 + 0.4S_2 + 0.1S_3 \] (11)
The results are in the Table 2:

**Table 2. Calculation Results of AHP**

<table>
<thead>
<tr>
<th>Index</th>
<th>Property vector</th>
<th>Weight (%)</th>
<th>Maximal characteristic root</th>
<th>CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment</td>
<td>0.963</td>
<td>31.343</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Social</td>
<td>1.326</td>
<td>43.172</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Light source</td>
<td>0.783</td>
<td>25.485</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After calculation, CI < 0.1, the consistency test passed.

### 3.2.3 Improve by EWM

We also applied the entropy weighting method to make the results out more objective and to make the weighting of each indicator to the light pollution risk level more effective, which is an objective weighting method and a relatively simple algorithm. We used the deviation standardization method to process the data of six evaluation indicators of 21 tests to obtain the standardized data and calculate the probability matrix. The indicator information entropy and entropy weighting coefficients were calculated according to the following formula.

We calculate the state transition probability:

$$ P_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}} $$

(12)

Calculate the entropy value of each index:

$$ E_{ij} = -\frac{1}{\ln n} \sum_{i=1}^{n} (p_{ij} \ln p_{ij}) $$

(13)

Calculate the entropy weight of the $j^{th}$ index

$$ W_j = \frac{1 - E_j}{\sum_{j=1}^{m} (1 - E_j)} $$

(14)

Since we hope to reduce the subjectivity of AHP, and EWM can supplement and optimize it, we obtain the final weight by weighted averaging the above calculation results.

We define the equation as follows:

$$ \omega = \alpha \omega_1 + \beta \omega_2 $$

(15)

We assume: $\alpha=0.7$, $\beta=0.3$.

Afterwards, we successfully assign weights to each indicator, we can use F as our metric to measure the risk level of light pollution in a certain area. We can divide the F value into 4 levels, indicating 4 levels of light pollution risk.

**Table 3. 4 kinds of light pollution risk level**

<table>
<thead>
<tr>
<th>Level</th>
<th>F</th>
<th>Medium-low</th>
<th>Medium-high</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0~0.2</td>
<td>0.2~0.4</td>
<td>0.4~0.7</td>
<td>0.7~1.0</td>
</tr>
</tbody>
</table>

We take each indicator for urban, suburban, rural, and protected areas into the model to obtain F-values. Then, we compare the values with our calculated values to derive the above graph to illustrate the level of light pollution based on the Porters dark sky classification method.
Table 4. Boteler degree and F P of 4 places

<table>
<thead>
<tr>
<th>Area</th>
<th>Protected</th>
<th>Rural</th>
<th>Suburban</th>
<th>Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boteler degree</td>
<td>1~2</td>
<td>3~4</td>
<td>5~7</td>
<td>8~9</td>
</tr>
<tr>
<td>F</td>
<td>0~0.2</td>
<td>0.2~0.4</td>
<td>0.4~0.7</td>
<td>0.7~1.0</td>
</tr>
</tbody>
</table>

Our metric fits basically with the Porters’ grade. It is evident that our metric has a wide range and validity.

4. Sensitivity Analysis and Model Evaluation

We consider the hierarchical analysis method and the entropy weighting method to get the final weights, we assume that $\alpha$ is 0.8, we will analyze the sensitivity of $\alpha$ and obtain the results to prevent this subjective operation from having a large impact on the model.

![Figure 7. Sensitivity Analysis](image)

The curves of each function in the figure 9 show the MDLP scores of each city when $\alpha$ is changed. The slope of each curve is extremely small, and the function is flat. Finally, we conclude the value of $\alpha$ does not affect our model.

4.1. Strengths

The data is reliable and accurate. The data we selected are from the authoritative official website, and all of them are data within 5 years so far, with timeliness.

The results of our model are consistent with real-world data and common sense. We use AHP and entropy weight method.

We cover all aspects and factors in the model to make it organized. Multi-purpose, that can be widely used in different regions.

We innovatively proposed the light demand rate based on the development of society.
4.2. Weaknesses

In our calculation, we ignore the factors that are difficult to measure and changeable. In fact, both weather conditions and climate and air quality have an impact on light pollution, and they have a weight that cannot be ignored.

In our model, due to the difficulty of data acquisition, the number of samples we selected is limited.

5. Conclusion

We used the MDLP model to establish a measurement of the risk level of light pollution in a region. Beside, we found that the level of light pollution in an area is influenced by society, environment, and light source. But the model still have some However, there are still shortcomings in the model. We can also start from the time of illumination and the type of light source to further improve the model. According to our model, we can quantify the light pollution, so we can further analyze the effect of light pollution.

References