The State-Of-Art Applications of Conventional Probability Issues

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Abstract. Contemporarily, the applications of conventional statistics to state-of-art problems become increasingly ingenious and complicated. Specifically, poker games, secretary puzzles, and the newsvendor theory will be fully discussed and elaborated in this paper. First, we discuss how to select the initial hand poker cards with the same card power research. The experiment proves that the method of equal card power generation is effective and feasible. In addition, we discuss the secretary puzzle, based on the optimal stopping theory, with the basic condition discussed by using different methods, probability, and calculus. Finally, we elaborate on the application of the newsvendor theory to the hotel overbooking problem. The theory is suitable for helping make decisions in manufacturing and service industries. When resolving the hotel overbooking problem, we successfully decide the number of rooms that the hotel reservations need to overbook in order to maximize the profit through the newsvendor model and the probabilistic approach, whose results turn out to be the same. Overall, these results shed light on guiding further exploration of statistics and probability.

Keywords: Poker cards, Secretary puzzle, The newsvendor theory, Probability.

1. Introduction

More than four hundred years ago, gambling, especially dice, was popular among aristocrats in many European countries. Based on the idea of winning at gambling, people began to explore probability theory. With the development of science in the 18th and 19th centuries, it was noticed that some biological, physical, and social phenomena were similar to games of chance, so the probability theory was applied to these fields and promoted itself with the birth of normal distribution, Poisson's Theorem etc. Nowadays, probability theory, together with the statistics principle based on it, plays an indispensable role in many fields, including science, engineering, industrial and agricultural production. Our research aims are closely related to Probability and Statistics, with three aspects: poker cards, secretary puzzle as well as the newsvendor theory.

For the poker game, we may assume that the score probability distribution of the initial hand is calculated by the statistical real card spectrum. Besides, the initial hand is divided into several categories based on the score probability distribution. Poker cards belong to competitive games; each game can be regarded as an examination, and the initial hand dealt with the player can be regarded as an examination question. The calculation of test item differentiation can refer to the famous Classical Test Theory (CTT) in psychology [1, 2]. Under the framework of CTT, test item differentiation can be reflected by the correlation between each test item and the total test score. Based on the statistics of the real card spectrum, Yipeng calculates the mean score and variance of the initial hand, and the initial hand is divided into 10 categories by means of the mean score and variance [3]. Besides, it is considered that the hand in the same category has the same power. However, the classification method that divides the interval by means of score mean and variance has a significant error. For this reason, Li improved it by calculating the score probability distribution of the initial hand through statistics of the spectrum of real cards [4] and clustering with the score probability distribution as the feature to divide the initial hand into several categories but two problems exist. In order to ensure the reliability of the rating difficulty evaluation index of a deck of cards, this paper proposes a method of using multiple different levels of "two play one AI program" instead of human playing cards to generate card charts.

With regard to the secretary puzzle, the situation is that interviews are taken to select only one new secretary, and in the end, the position must be filled. There are n candidates applying, who are interviewed individually in random order. After each interview, the decision must be made immediately: HIRE or PASS. If the candidate is selected to be HIRED, the interview just stops, which means there is no chance to interview the rest of the candidates; if the choice is PASS, the candidate is dismissed, and there is not second chance to turn back to hire he/she. The only aim is to select the best candidate from the application group.

In fact, numerous extensions can be classified on three standards [5]: the number of candidates selected, the number of attributes for evaluating each candidate, and information available for a decision-maker [6]. In terms of real-life application, the online auctions problem is one of these examples [7], with consideration of time-discounted revenue and incentive compatibility [8]. Additionally, to give a more precise formula, machine learning is used to improve the algorithm [9, 10].

As the general ability of candidates is unknowable and an interview is the only chance to measure their level, to find a standard criterion, the first k people can be regarded as a sample among the population of n, which means no matter how good they are, they have to be rejected. Afterwards, the candidate who is better than anyone before he/she will be hired. The result is to find the proportion of sample size and population (value of k/n) with the success rate.

In terms of the newsvendor theory, the hotel reservation always suffers from the last-minute cancellations, i.e., a few guests tend to cancel their reservations abruptly due to some emergencies. Therefore, most hotel reservation systems would use an overbooking policy in order to reduce the potential loss [11]. Nevertheless, when the hotel is overbooked, it needs to find a room in another hotel and pay the room fee for the customer or directly pay the penalty fee, which generates an expensive cost. Therefore, exactly how many rooms to overbook becomes an extremely crucial problem for the hotel managers to consider carefully since it directly relates to the eventual profit they can make [12].

The problem, also known as the single-period problem (SPP) or the newsboy problem, is to find the order quantity which maximizes the expected profit in a single period probabilistic demand framework. Interest in the SPP remains unabated, and many extensions to it have been proposed in the last decade. These extensions include dealing with different objectives and utility functions, different supplier pricing policies, different newsvendor pricing policies and discounting structures, different states of information about demand, constrained multi-products, multiple products with substitution, random yields, and multi-location models [13]. Exactly how many rooms to overbook becomes an extremely crucial problem for the hotel managers to consider carefully since it directly relates to the eventual profit they can make. Except for the application on the hotel overbooking problem, the newsvendor theory can also be applied to other resembling single-period inventory problems such as the overbooking of airline flights and ordering of fashion items.

The rest part of the paper is organized as follows. Sec. II to IV will introduce these examples in detail, respectively. Eventually, a summary will be given in Sec. V.

2. Equal card force competition system of competitive two against one game

Experiment 1 generated multiple DDZAI with differentiation. In this paper, 10 tables of DDZAI were randomly selected for the doubles match, as shown in Figure 1. Apparently, the probability of the 10 DDZAI gaining each place and the ranking of each DDZAI are shown. The experimental results show that the probability values of 10 DDZAI in different rankings are different, which is basically consistent with the rankings obtained by the double competition, indicating that the generated DDZAI has a certain degree of differentiation.

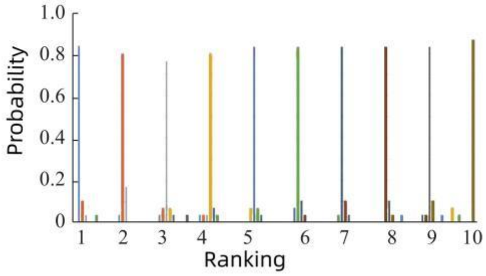

Fig. 1 Probability vs. The order.

As for Experiment 2, the Initial hand test with equal power. In this paper, the initial hand Center difficulty GDI= (0.96,0.91,0.79) was taken as a reference, the parameter ε=0.15 was set, and 5 GDI with different difficulty were selected, as shown in Table 1. Eq. (1) is used to calculate the distance from the center respectively:

$$d_{12} = \sqrt{\sum_{j=1}^{n} \left(g_{1j} - g_{2j}\right)^2} \tag{1}$$

Tab. 1 Five GIDS with different difficulties

central point	g1	g2	g3
center1	0.950	0.950	0.809
center2	0.971	0.971	0.824
center3	1.000	0.85	0.720
center4	0.736	0.677	0.618
center5	1.000	1.000	1.000

Tab. 2 Probability of each DDZAI winning the first five places in a duplicate tournament.

game session	DDZAI	first	second	third	fourth	fifth	DDZAI ranking
5	0	0.70	0.20	0	0	0.05	1
5	1`	0.05	0.75	0.15	0	0.05	2
5	2	0	0.05	0.70	0.20	0.05	3
5	3	0.10	0	0.10	0.70	0.10	4
5	4	0.05	0.05	0.05	0.10	0.75	5
10	0	0.71	0.20	0.04	0	0.05	1
10	1	0.05	0.75	0.15	0	0.05	2
10	2	0	0.04	0.71	0.20	0.05	3
10	3	0.10	0	0.10	0.70	0.10	4
10	4	0.05	0.10	0	0.10	0.75	5
20	0	0.70	0.25	0	0	0.05	1
20	1	0.05	0.75	0.15	0	0.05	2
20	2	0	0.05	0.70	0.20	0.05	3
20	3	0.10	0	0.10	0.70	0.10	4
20	4	0.05	0.05	0.05	0.10	0.75	5

Tab. 3 Probability of each DDZAI winning the first five places in equal card force.

ε	game session	DDZAI	1	2	3	4	5	DDZAI ranking
0.15	5	0	0.50	0.40	0.05	0.05	0	1
0.15	5	1	0.20	0.50	0.30	0	0	2
0.15	5	2	0.10	0	0.58	0.26	0.06	3
0.15	5	3	0.11	0.05	0.06	0.58	0.20	4
0.15	5	4	0.05	0.05	0.05	0.10	0.75	5
0.15	10	0	0.55	0.20	0.20	0	0.10	1
0.15	10	1	0.15	0.44	0.20	0.15	0.10	2
0.15	10	2	0.10	0.15	0.49	0.20	0.10	3
0.15	10	3	0.15	0.15	0.05	0.54	0.15	4
0.15	10	4	0.10	0.10	0.10	0.25	0.50	5
0.15	20	0	0.57	0.33	0.10	0.10	0	1
0.15	20	1	0.30	0.40	0.20	0.20	0	2
0.15	20	2	0.11	0.10	0.51	0.18	0.20	3
0.15	20	3	0.01	0.20	0.10	0.59	0.10	4
0.15	20	4	0.10	0.10	0.20	0.20	0.50	5
0.20	5	0	0.49	0.26	0.25	0	0.05	—
0.20	5	1	0.05	0.35	0.15	0.40	0.10	—
0.20	5	2	0.15	0.20	0.40	0.20	0.10	—
0.20	5	3	0.25	0.05	0	0.30	0.45	—
0.20	5	4	0.10	0.20	0.25	0.15	0.35	—

The calculation results show that the first three are all less than 0.15, which indicates that the three have the same card force. The distance between the last two calculations and the center is greater than 0.15, indicating that the two calculations do not have the same card force.

Experiment 3 verifies the feasibility of applying the same card force to competition. In this paper, 5 tables of DDZAI were selected to participate in the competition. The number of matches in duplicate, random card dealing, and equal card power matches were all 20, and the number of rounds in a match was set as 5, 10, and 20, respectively. Select the same power of the initial hand, set the Center difficulty as GDI=(0.96,0.91,0.79), ε set 0.15, 0.20, 0.25, respectively. Table II shows the probability of each DDZAI ranking 1~5. Table III shows the probability of each DDZAI ranking 1~5 after the same card force with different parameter ε values. Compared with Tables II and III, when ε = 0.15, the DDZAI ranking of the same card power game is consistent with the DDZAI ranking of the duplicate game and is not consistent with the DDZAI ranking of the random licensing game, indicating the reliability of the same card power generation method. Besides, when ε is determined, the probability distribution of DDZAI winning each ranking does not change with the increase in the number of rounds. Therefore, increasing the number of rounds will not reduce the difficulty of evaluating AI ranking in the same card power match.

In order to ensure the reliability of the experimental results, this paper innovatively proposes a method of generating card spectrum, generating the same card power, and verifying by using several different levels of “two play one AI program” instead of real people. The difficulty of the initial hand is expressed by GDI, and the method of judging the initial hand with the same power is given. By specifying DDZAI respectively for random licensing games, duplicate games, and equal card power games. The experimental results show that the proposed method is effective, and it is feasible to replace the current duplicate match with equal card match under the condition of choosing appropriate threshold value.

Tab. 4 Optimal sample size and success rate corresponding to various populations [14].

n	Optimal k	$\frac{k}{n}$	Success
3	1	33.33%	50.00%
4	1	25.00%	48.53%
5	2	40.00%	43.33%
6	2	33.33%	42.78%
7	2	28.57%	41.43%
8	3	37.50%	40.98%
9	3	33.33%	40.60%
10	3	30.00%	39.87%
11	4	36.36%	39.84%
12	4	33.33%	39.55%
13	5	38.46%	39.23%
14	5	35.71%	39.17%
15	5	33.33%	38.94%
16	6	37.50%	38.81%
17	6	35.29%	38.73%
18	6	33.33%	38.54%
19	7	36.84%	38.50%
20	7	35.00%	38.42%

3. Secretary Puzzle based on Optimal Stopping Theory

3.1. Models

When the population size is equal to 1, there is only one choice, and if it is 2, the probability is just half whether the sample is taken or not. Population from 3 to 20 is tried, and the following data is found. According to Table. IV, it is obvious that the optimal value of k/n is around 30% to 40%, the same as the rate of success is about 40%.

To make the proof more accurate and reliable, the probability mass function is used, which can be plotted as illustrated in Fig. 2. As the first k people can be just ignored, the best candidate must not be within the k, meaning the order of one will be the (k+1), (k+2), until the last one, n, and the total probability is the sum of whole these possibilities.

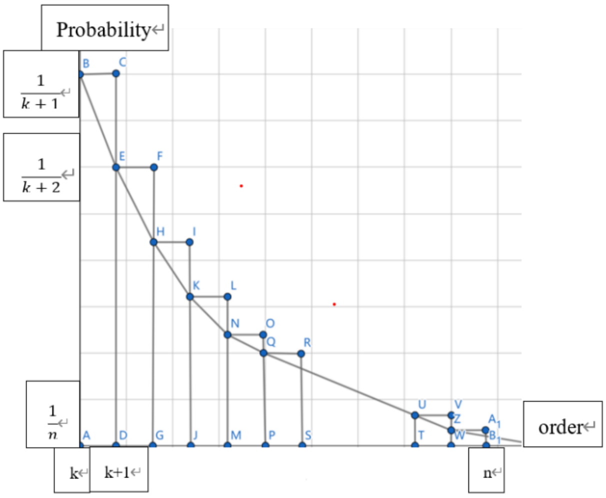

Fig. 2 Probability vs. The order of the best candidate.

According to the knowledge of calculus, the sum of probability is equal to the area under the curve, i.e., the integration of the graph:

$$P(k) = \int_{k}^{n} \frac{1}{x} dx = \frac{1}{n}[k\ln(n) - k\ln k] \quad (2)$$

$$P'(k) = \frac{1}{n}\left[\ln(n) - \left(\frac{k}{k} + \ln k\right)\right] = 0 \quad (3)$$

As the discrete data continues, the result is not so reliable. The other way is completely based on probability. Probabilities of the best candidate in the order from k+1 to n are added. In the following formula, 1/n represents the candidate selected as the best one among the whole group, and k/(k+integers) means that the candidates before the best one are not better than everyone in the sample.

$$P(k) = P(k+1) + P(k+2) + \cdots \ldots$$
$$+P(n) = \frac{k}{n}\sum_{i=k}^{n-1}\frac{1}{i} \quad (k \leq n, k \in N^{+}) \quad (4)$$

Let x=k/n (x means the ratio of sample and population):

$$P(k) = \frac{k}{n}\sum_{i=k}^{n-1}\frac{1}{i} \sim x\int_{x}^{1}\frac{1}{t}dt = -x\ln(x) \quad (5)$$

After taking the differential and making it equal to 0,

$$\frac{d}{dx}\left(-x\ln(x)\right) = 0 \quad (6)$$

One finds that x=1/e. Therefore, in both cases, the optimal value of k/n is calculated to be $1/e$.

3.2. Results

When the population size is great enough, the graph of the percentage of choosing the best candidate against the proportion of the sample overpopulation. As depicted in Fig. 3, the optimal sample size is $\frac{k}{n}$ (the proportion of the sample over the whole population), corresponding to approximately 36.79%, and the chance of success is also 36.79%. This article investigates the basic situation of the secretary puzzle. To find only one candidate who is the best among the population size, the proper method is to skip the first 36.79% candidate and then select the first one who is better than anyone before that, with the same value of 36.79% chance of ending up with the best candidate. This principle can also be applied to choosing a product or even a partner.

3.3. Evaluation

In real-life applications, there are still many limitations. The first restriction is that the result works when the population size approaches infinity. While in table V, the optimal sample size is not always equal to 36.79% of the population, and the rate of success varies. Moreover, it is possible for candidates to reject the offer, as they may take interviews with more than one company and receive a better one. According to an essay written by Smith, the data can be found as follows. Finally, many companies have more than one test to accurately measure the abilities of competitors, preferring to consider employees comprehensively rather than based on a single interview. In this case, there is no need to follow the 37% rule.

For future research, a great number of factors can also be considered to analyze the question, such as the cost to conduct each interview and the time that the employee plan to stay in the company, which relates to the efficacy. Meanwhile, there can be more applications in life from various respects, or even there may be cases where people use this rule without realizing it.

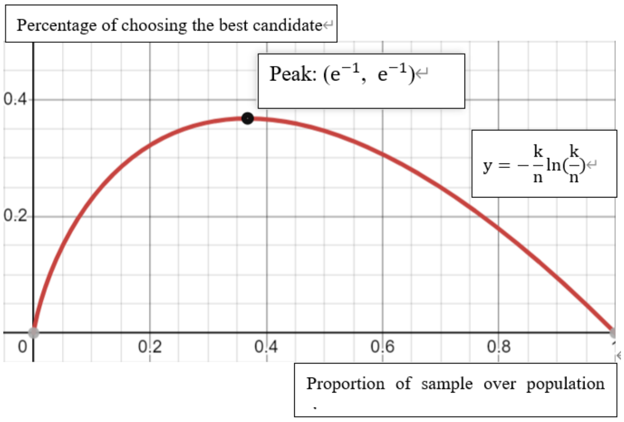

Fig. 3 Probability of choosing the best candidate vs. Proportion of sample over population.

Tab. 5 Optimal sample size corresponding to the probability of rejection [15]

Probability of rejection	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\frac{k}{n}$ value (%)	36.8	34.9	32.8	30.5	27.9	25.0	21.7	17.9	13.4	7.7	/

4. The Newsvendor Theory and Hotel Overbooking Problem

4.1. Models

In order to conduct thorough research on single-period inventory models such as the hotel overbooking problem, we first introduce a statistical method, i.e., the newsvendor problem, to help simplify the case. It is assumed that there's a newsboy who needs to decide how many papers to order on the weekend for the sales on Monday. He pays \$0.20 for each paper and sells each for \$0.50. After collecting sales data over a few months, he found that, on average, 90 papers were sold each Monday with a standard deviation of 10 papers [16]. Here, we assume that during this time, the papers were overstocked. In other words, we have $\mu_{demand} = 90$ and $\sigma_{demand} = 10$. For ease of exposition, we make the normality assumption that demand ~ $N(90,10^2)$. The problem we need to solve now is how many papers we should order. From the demand distribution with standard assumption, we can first find some specific examples. If we want to control the stocking out risk ≤ 50%, we should order more than 90. If one wants to control the stocking out risk ≤ 20%, we should order more than 99. This is because

$$P\{d \le \text{Order}\} = P\left\{\frac{d-\mu}{\sigma} \le \frac{\text{Order}-\mu}{\sigma}\right\} \ge 0.8 \tag{7}$$

i.e.,

$$\text{Order} \ge \mu + \Phi^{-1}(0.8)\sigma = 90 + 8.416 \approx 99 \tag{8}$$

Further, we generalize the way for calculation and find that if we want to control the stocking out risk ≤ (1-p), then we should order more than $\mu+\Phi^{-1}(p)\sigma$.

However, that is not enough since it does not discuss the profit or loss on the ordering, which is the ordering decision. Therefore, some basic economic concepts like marginal cost and marginal profit are involved, helping analyze the best stocking out probability (1-p). Recall what we assume about the newsboy is that he pays \$0.20 for each paper and sells each for \$0.50. Thus, the marginal cost for over-ordering is Cover = 0.20, which is the cost of each paper and the marginal cost for under-ordering is Cunder = 0.30, which is the lost sale. Thus, the expected marginal profit is $(1-p)C_u - pC_o$. In terms of marginal analysis, when the order increases, the probability that the 'next' unit can be sold and the expect marginal profit decrease. The optimal stocking level happens when the expected marginal profit reaches zero, i.e.,

$$(1-p)C_u - pC_o = 0 \tag{9}$$

where p is the probability that the 'next' unit cannot be sold; in other words, the probability of safe stocking [17].

Combining the statistical information and marginal analysis, we can finally resolve the newsvendor problem. As we mentioned earlier, the marginal cost for over-ordering is C_{over}=0.20, and the marginal cost for under-ordering is C_{under}=0.30. The best probability p* is

$$p^* = \frac{C_u}{C_o + C_u} = 0.6 \tag{10}$$

The best ordering level Order* is

$$\text{Order}^* = \mu + \Phi^{-1}\left(\frac{C_u}{C_o + C_u}\right)\sigma = 93 \tag{11}$$

After discerning this process, we can determine that the larger C_u is, the more orders should be made since the holding cost is relatively small. Industries that produce and sell cigarettes and alcohol are typical examples of such conditions since their gross profit margin is high. Therefore it is better for them to make a larger order.

4.2. Empirical analysis

Subsequently, we apply this method to solve the more complex real-life business problems, e.g., the hotel overbooking problem. Because the hotel reservations constantly confront the last-minute cancellations, most of them use an overbooking policy to curtail the potential loss. As introduced below, there are two main methods to help determine the amount of rooms hotels need to overbook in order to maximize their profit.

From the historical data of hotel reservation management, we find that the number of last-minute cancellations has a mean of 5 and a standard deviation of 3. It is assumed that the detailed information of the hotel is that the room rate is $80 and the hotel room fee for another hotel is $200 [18]. What we need to decide is how many rooms y* the hotel should overbook. Directly applying the newsvendor model, we first use the "Overbooking" to match the "Cancellation". The marginal cost for "Overbooking" < "Cancellation" (underestimating the cancellation number) is Cu= $80. The marginal cost for "Overbooking" > "Cancellation" (overestimating the cancellation number) is Co= $200. Thus, the best stock-safe probability is.

$$p^* = \frac{C_u}{C_o + C_u} = \frac{\$80}{\$200 + \$80} = 0.2857 \tag{12}$$

The best overbooking number y* is.

$$y^* = F^{-1}\left(\frac{C_u}{C_o + C_u}\right) = \mu + \Phi^{-1}(0.2857)\sigma = 3.29 \tag{13}$$

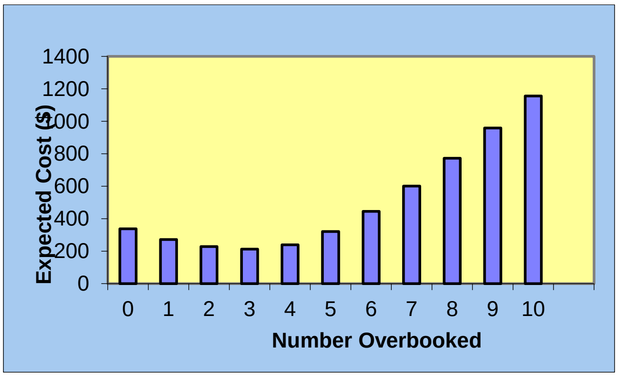

Fig. 4 Expected cost as a function number overbooked.

Another method we use to solve the problem is a probabilistic approach. Instead of using the (μ, σ), we directly use the Cancellation data. The discrete cancellation probability (c_i, p_i), $i \in [N]$ is adopted to maximize the expected profit [19]:

$$\text{Min} \sum_{i \in [N]} p_i(\text{Max}\{y - c_i, 0\}\$200 + \text{Max}\{c_i - y, 0\}\$80) \quad (14)$$

where c_i is the number of cancellations, p_i is the probability, and y is the number of overbookings.

The part before the plus sign means overestimating, while the part after the sign represents the situation of underestimating. When y < ci, Max{y- c_i, 0}is zero, Max{c_i-y, 0}is equal to c_i-y. When y > c_i, Max{y-c_i ,0}is equal to y-c_i, Max{c_i-y ,0}is zero. Next, we use the excel example to help visualize the solution [20].

Afterwards, one selects a random set of data to indicate the algorithm； for instance, when the number of units overbooked is 0, the total cost is $337.6. The results are given in Fig. 4. The same calculation mode is further applied to the other sets. By calculating all the remaining groups and comparing all the data of the total cost, we observe that when 3 units are overbooked, the total cost is minimized, which is just the best overbooking number. Juxtaposing the outcomes, one finds that they are identical.

4.3. Discussion

Although the principles of the newsvendor theory and a data-driven method are completely distinct, they both successfully solve the hotel overbooking problem and eventually lead to the same result. However, from the calculation process and results, the numerical values calculated by a data-driven method seems to be more accurate, and the images used are more intuitive and clearer. While the newsvendor theory focuses more on the mathematical and statistical routines to help solve the problem. Except for the application on the hotel overbooking problem, the newsvendor theory can also be applied to other resembling single-period inventory problems such as the overbooking of airline flight, ordering of fashion items, etc. [21].

5. Conclusion

In summary, this paper investigates poker games, the secretary puzzle, and the newsvendor theory mainly based on probability as well as statistics. Through the three examples, we clearly see how conventional statistical methods can be successfully applied to real-life issues. The statistical real card spectrum help calculate the score probability distribution of the initial hand, dividing it into several categories. The secretary puzzle helps find the correct method to determine only one best candidate, which can be further applied to choosing a product or a partner. The newsvendor theory provides a statistical breakthrough point for hotel managers to decide how many rooms to overbook in order to maximize the profit. Our eventual objective is to relate statistics with life to an extent as large as possible, and combining the conventional probability theories with the state-of-art applications makes the seemingly boring statistics more meaningful and practical. Although the current limitation of our paper is the. lack of consideration for the irrationality and subjectiveness, these results offer a guideline for real life applications as well as future research.

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