The Establishment and Practical Application of UAV Fixed-Point Delivery Model Based on Difference Method

Haoyu Sun [1, *], Wenkai Han [1], Siwen Wang [2]

[1] College of Science, China University of Petroleum, Qingdao, China, 266580

[2] College of New Energy, China University of Petroleum, Qingdao, China, 266580

* Corresponding Author Email: m19854217856@163.com

Abstract. The problem of UAV fixed-point delivery is of great significance in the field of military and relief supplies delivery. In order to better calculate the distance of UAV delivery, this paper establish a UAV fixed-point delivery model. At the same time, this paper use the difference method to calculate, and the error analysis of the model, which verifies that the model has good applicability.

Keywords: UAV, Calculus of Differences, Air Friction, Stability Analysis.

1. Introduction

With the development of science and technology, UAVs are widely used in many fields. The delivery accuracy of a fixed-point airdrop of a UVA depends not only on the operating technique, but also on the state and environment in which UAV performs the task. Researchers from different countries have carried out systematic research on the UAV problem, but the research focus is still different, including focusing on different research modules such as target recognition, tracking strategy, simulation theory and so on. In this paper, by using the relevant laws of aerodynamics, the relevant models of UAV fixed-point delivery are established, the delivery strategies of spheres under different circumstances are solved and the stability analysis of UAV is carried out. It can enrich the theoretical research in this field in China, provide reference for similar research, and provide guidance value for the better application of UAV in express delivery, military, emergency rescue and other fields.[1-2]

2. The basic funamental of UAV fixed-point delivery model

2.1. Establishment of UAV fixed-point delivery model

We only consider the case of UAV flying in horizontal mode.At the same time, we consider the shape of the material to be spherical.The schematic of the aircraft dropping material is shown in Figure 1.

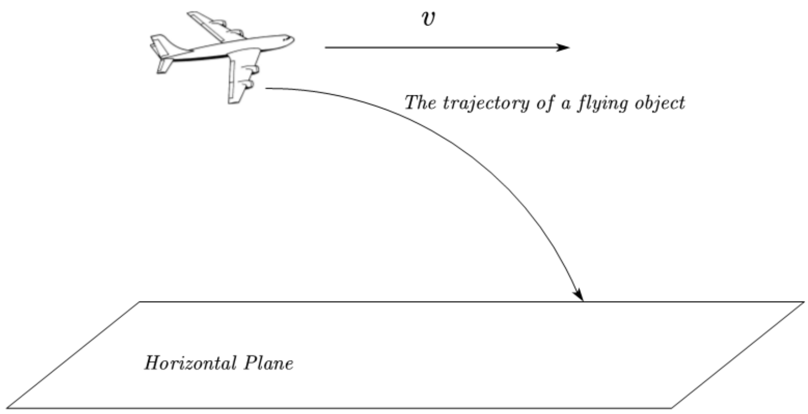

Figure 1. Illustration of UAV delivery of supplies

According to reference [3-6], the air resistance of ball flying is

$$f_{wind} = -\frac{1}{2} C_d \rho S v^2 \tag{1}$$

In this formula C_d Is the drag coefficient(This paper argues that C_d is 0.53),ρ is the density of air，S is the projected area of the ball object.Now the straight-line distance between the launching point and the target point is solved. It is easy to know that the component force of air resistance in the horizontal plane of UAV movement is

$$f_{\mathrm{wind}_x} = f\cos\theta \tag{2}$$

$$\Rightarrow f_{\mathrm{wind}_x} = -\frac{1}{2} C_d \rho S v^2 \frac{v_x}{\sqrt{{v_x}^2 + {v_y}^2}} = -\frac{1}{2} C_d \rho S v_x \sqrt{{v_x}^2 + {v_y}^2} \tag{3}$$

Similarly, the component force of air resistance in the vertical direction can be obtained as

$$f_{\mathrm{wind}_y} = -\frac{1}{2} C_d \rho S v^2 \frac{v_y}{\sqrt{{v_x}^2 + {v_y}^2}} = -\frac{1}{2} C_d \rho S v_y \sqrt{{v_x}^2 + {v_y}^2} \tag{4}$$

The air resistance decomposition is shown in the Figure 2.

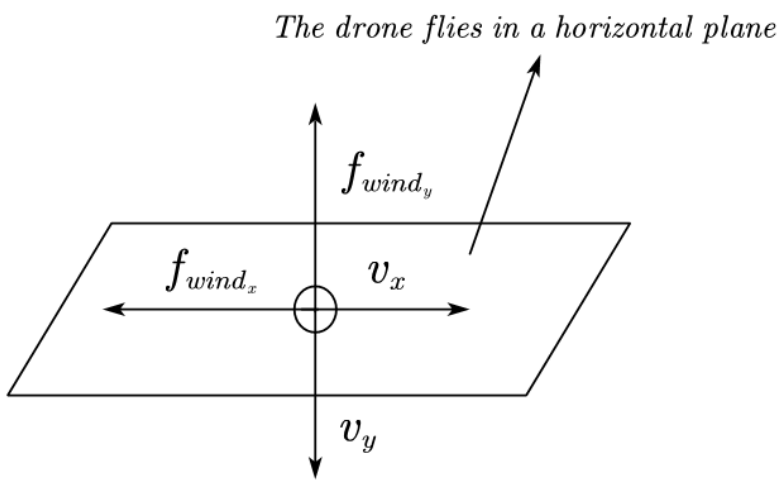

Figure 2. Schematic diagram of air resistance

The spherical material is subjected to force analysis. It is affected by air resistance in the horizontal direction, gravity and air resistance in the vertical direction, and the relationship between air resistance and displacement can be obtained according to the relationship between acceleration in each direction and the external force in each direction

$$\begin{cases} \dfrac{d^2x}{dt^2} = \dfrac{1}{m} f_{wind_x} \\ \dfrac{d^2y}{dy^2} = \dfrac{1}{m} f_{wind_y} + g \end{cases} \tag{5}$$

The integral transformation is applied to the differential equation.

$$\frac{d^2x}{dt^2} = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{1}{m} f_x \tag{6}$$

$$\Rightarrow v_x \frac{dv_x}{dx} = -\frac{1}{2m} C_d \rho S v_x \sqrt{v_x^2 + v_y^2} \tag{7}$$

Based on the initial conditions

$$\begin{cases} \dfrac{dx}{dt}\Big|_{t=0} = v_0 \\ \dfrac{dy}{dt}\Big|_{t=0} = 0 \end{cases} \tag{8}$$

The above equation can be obtained by integrating it.

$$\int_0^x dx = \int_{v_0}^{v_x} -\frac{2m dv_x}{C_d \rho S \sqrt{v_x^2 + v_y^2}} \tag{9}$$

$$\Rightarrow x = -\frac{2m}{C_d \rho S}\left[\ln\left(v_x + \sqrt{v_x^2 + v_y^2}\right) - \ln\left(v_0 + \sqrt{v_x^2 + v_y^2}\right)\right] \tag{10}$$

At the end of the flight,$y = h$.The straight-line distance between the transmitting point and the target point has the following expression

$$r^2 = x^2 + y^2 \tag{11}$$

$$\Rightarrow r^2 = \left\{-\frac{2m}{C_d \rho S}\left[\ln\left(v_x + \sqrt{v_x^2 + v_y^2}\right) - \ln\left(v_0 + \sqrt{v_x^2 + v_y^2}\right)\right]\right\}^2 + h^2 \tag{12}$$

$$\Rightarrow r = \sqrt{\left\{-\frac{2m}{C_d \rho S}\left[\ln\left(v_x + \sqrt{v_x^2 + v_y^2}\right) - \ln\left(v_0 + \sqrt{v_x^2 + v_y^2}\right)\right]\right\}^2 + h^2} \tag{13}$$

Therefore, the model of UAV delivery distance is obtained.On this basis, the influence of wind speed is introduced. The wind speed can be regarded as the relative speed of the object and the air, so the problem can be regarded as the air is still in the case of no wind, and then the wind speed can be regarded as the speed of the spherical material itself. By vector adding the wind speed to the original speed of the material:

$$v = \sqrt{v_0^2 + v_{wind}^2 + 2 v_0 v_{wind} \cos\theta} \tag{14}$$

Among them$\theta$is the angle between the wind speed and the material speed. It is necessary to consider the impact of material speed and wind speed simultaneously when using this model.

2.2. The difference method is used for the calculation

Although the flight distance can be calculated directly, the solution of the horizontal flight speed and vertical flight speed is related to y, and the integration of y is extremely complex, so the idea of difference is adopted. [7-10]

The horizontal acceleration and vertical acceleration of spherical materials continue to change with the change of velocity：

$$\begin{cases} v_x = v_x + \dfrac{d^2 x}{dt^2} t \\ v_y = v_y + \dfrac{d^2 y}{dt^2} t \end{cases} \tag{15}$$

Among them $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ can be viewed as a function of time t

$$\frac{d^2x}{dt^2} = f(t) \tag{16}$$

$$\frac{d^2y}{dt^2} = g(t) \tag{17}$$

The continuously varying acceleration is not conducive to the calculation, so the idea of difference is used. First, a time unit Δt is established. When Δt is small enough, the motion of the sphere in this time unit can be regarded as a uniform acceleration motion. At the end of one-time unit, the acceleration $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ are updated, and the next time unit is started until the vertical displacement is equal to the flight height. In the loop, there are.

$$\begin{cases} v_x = v_x + \dfrac{d^2x}{dt^2}\Delta t \\ x = x + \dfrac{dx}{dt}\Delta t \\ v_y = v_y + \dfrac{d^2y}{dt^2}\Delta t \\ y = y + \dfrac{dy}{dt}\Delta t \end{cases} \tag{18}$$

The initial state is defined as

$$\begin{cases} \dfrac{d^2x}{dt^2}\Big|_{t=0} = -\dfrac{1}{2}C_d \rho S v_0^2 \\ \dfrac{d^2y}{dt^2}\Big|_{t=0} = g \end{cases} \tag{19}$$

3. Results

3.1. Practical application of the model

In order to further test the applicability of the model, we give the calculation results of the model in a specific case. We assume that the material is spherical, with a radius of 20cm and a weight of 50kg, and that the UAV has a flying height of 300m, a flying speed of 300km/h, and a wind speed of 5m/s, and the wind direction is parallel to the horizontal plane. We calculate the delivery distance of the UAV under three conditions: the UAV flying direction is the same as the wind direction (the Angle is 0 degrees), the opposite (the Angle is 180 degrees), and the vertical (the Angle is 90 degrees).

Calculate by computer programming. The specific process is as Figure 3

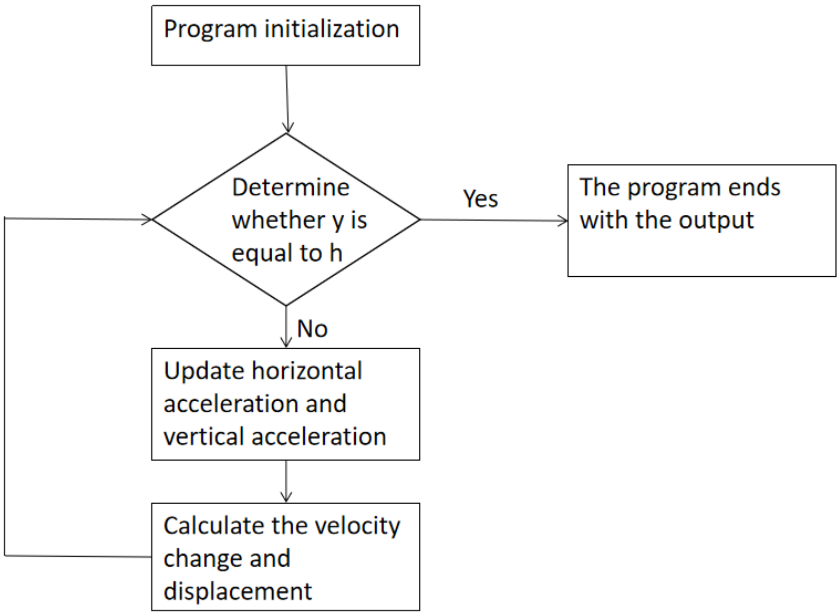

Figure 3. Program flow chart

Finally, the trajectory diagram of the spherical material in the same wind direction is shown as Figure 4

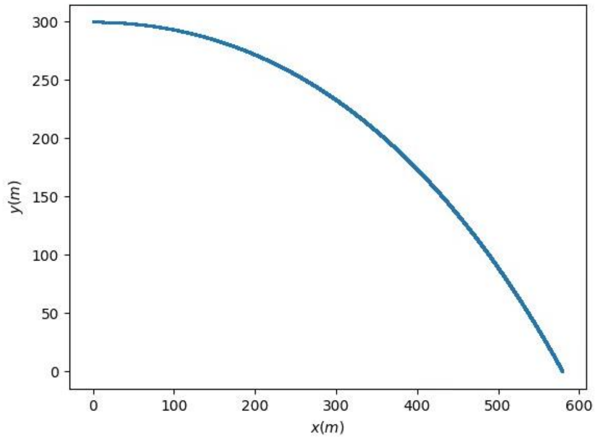

Figure 4. Trajectory diagram for the same wind direction

The trajectory diagram of the spherical material in the opposite wind direction is shown as Figure 5.

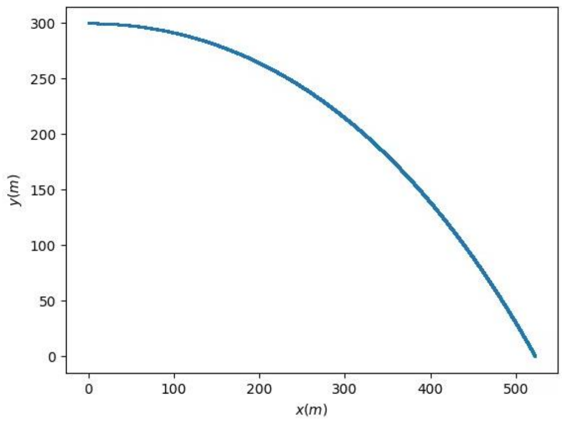

Figure 5. Trajectory diagram when the wind is opposite.

The trajectory diagram of the spherical material in the vertical case is shown as Figure 6

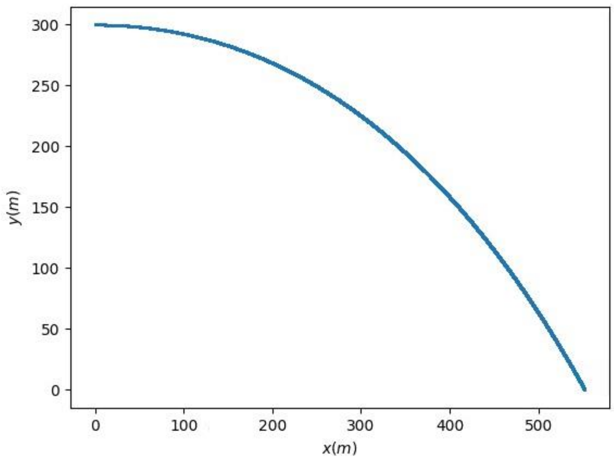

Figure 6. Trajectory diagram when the wind direction is vertical.

The following result can be obtained by calculation: When the wind direction is the same.
r=652.637m
When the wind is in the opposite direction
r=602.665m
When the wind is vertical
r=628.468m

3.2. Analysis of Calculation results

It is not difficult to find that using the UAV fixed-point delivery model based on the difference method can help us calculate the delivery distance better.But in the process of solving the delivery distance, the existence of error is inevitable. The launching distance will be affected by the change of flight height and flight speed (only the effects of flight height and flight speed are considered here because the air resistance is related to the ball itself and the flight speed). Through the computer, the flight height and flight speed are changed respectively to get some discrete data, and then these data are fitted to judge the impact of the change of flight height and flight speed on the launching distance. Here we choose to explore the degree of change in the delivery distance caused by the change rate of flight height and flight speed.

Changing the initial velocity v_0 and the initial height h_0, the changed velocity and height have the following relationship with the original velocity and height.

$$v = v_0(1+\alpha) \tag{20}$$

$$h = h_0(1+\beta) \tag{21}$$

Where α and β are the rate of change of flight speed and flight altitude, using the optimized model, the relationship between the changed delivery distance r and the original delivery distance r_0 can be obtained.

$$r = r_0(1+\gamma) \tag{22}$$

Where γ is the rate of change of the delivery distance. The diagram of the relationship between γ and α is as Figure 7

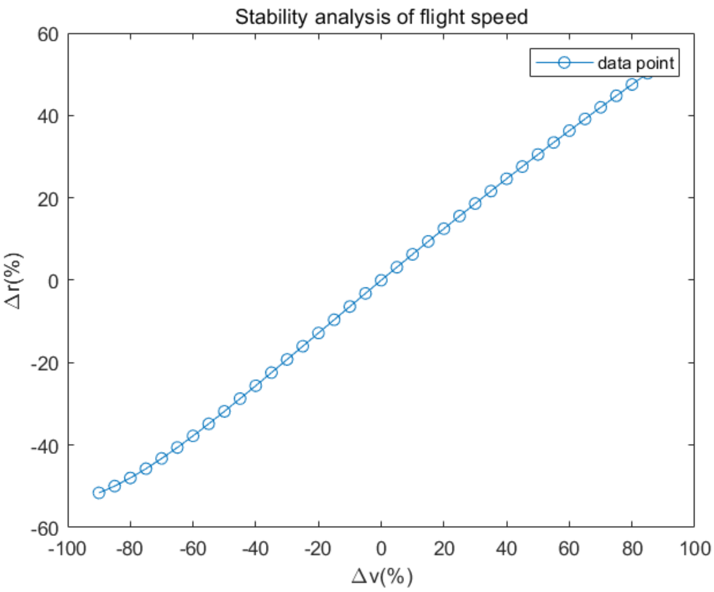

Figure 7. Stability analysis of flight speed

The diagram of the relationship between γ and β is as Figure 8

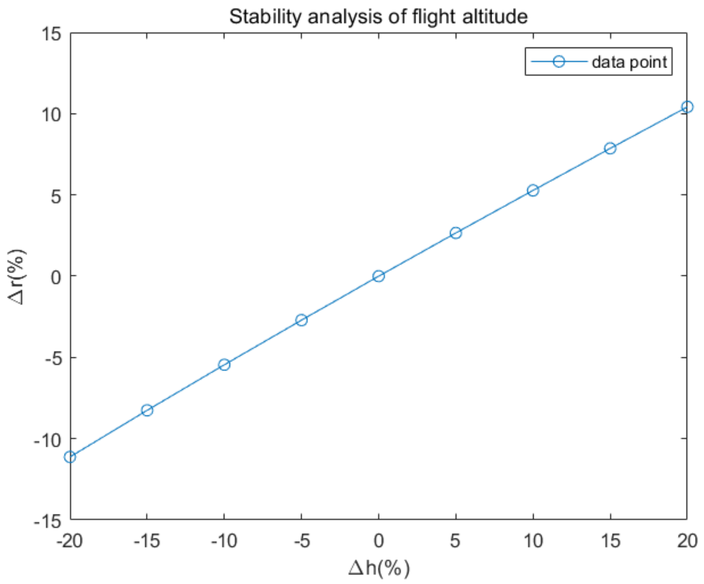

Figure 8. Stability analysis of flight altitude

It can be seen from the image that there is a good correlation between the launching distance and the flight height and speed. Therefore, the model can be used to analyze the relationship between the launching distance and the flight height, the linear distance between the launching point and the target point, and the air resistance.

4. Conclusions

The model takes into account the movement of the ball in the air and the friction and can still calculate the launching distance when the conditions change and there are errors. It has strong anti-interference and good robustness. However, this model can only deal with the situation when the ball is moving in the air. If the shape of the object is changed, this model is not applicable. At the same time, the model does not consider the rotation of the sphere itself, that is, it is idealized. In addition, the model can be further improved by adding the influence of the ball's own rotation on the air resistance, which makes the model more comprehensive.

References

[1] Steel City Drones Enters Partnership with SmartDrone[J]. Wireless News,2021.

[2] A. V S,V. V K,V. A T, et al. Experimental Estimates of Angular Coordinates and Location of Radio Emission Sources in Unmanned Aircraft Monitoring Systems[J]. Russian Aeronautics,2021,64(3).

[3] Zhou Yuqing, Ye Zhaoning, Wu Zonghan. Calculation and Analysis of Air Resistance in Ball Games [J]. Physics and Engineering,2002(01):55-59.

[4] Yu Xiaoming, Cui Yihe, Chen Fei et al. Research on accurate Measurement of Gravity Acceleration by dropping ball method considering air resistance and buoyancy [J]. Physics and Engineering,2010,20(05):26-28.

[5] Hao Chenghong, Huang Yaoqing, Liu Jukun. Considering air resistance football curveball trajectory [J]. Journal of college physics, 2018, 5 (6) : 19-21. DOI: 10.16854 / j.carol carroll nki. 1000-0712.170233.

[6] Golf-like surfaces can Reduce air drag or improve fuel efficiency [J]. Heilongjiang Science and Technology Information,2014(20):17-18.

[7] Liu Weiming, Sun Haibing, Cheng Xiaoliang. Probability calculation method of difference equations [J]. Journal of jilin normal university (natural science edition), 2023, 44 (3) : 50 to 55, DOI: 10.16862 / j.carol carroll nki issn1674-3873.2023.03.008.

[8] [1] PangRui. Two kinds of multi-objective optimization algorithm based on decomposition [D]. Shanxi normal university, 2019. The DOI: 10.27292 /, dc nki. Gsxfu. 2019.000432.

[9] Hui-ling jiang. Air - the fractional diffusion equation of quasi tight difference algorithm [D]. Xiangtan university, 2019. The DOI: 10.27426 /, dc nki. Gxtdu. 2019.000904.

[10] Zhang Mengru, PENG Haijun, Song Ningning. Semi-analytical Sensitivity Analysis Method for Strong Coupling Optimization of Flexible Multi-body Systems [C]// Chinese Society of Mechanics. The 12th multibody dynamics and control and the seventh aerospace dynamics and control and the analysis of the 15th national mechanical joint academic conference paper based. [publisher unknown], 2021-48. DOI: 10.26914 / Arthur c. nkihy. 2021.054631.