Analysis of Adaptive Equalization Algorithms

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Abstract. Adaptive equalization algorithms play a pivotal role in suppressing inter-symbol interference in wireless channels. Contemporarily, with the rapid development of science and technology, there is still a lack of unified cognition for adaptive equalization algorithms. Therefore, this study systematically discusses the research status and development process of adaptive equalization algorithms, focusing on the least mean square algorithm (LMS), constant modulus blind equalization algorithm (CMA) and neural network algorithm. Subsequently, based on Matlab simulation, their performance is analyzed visually. Finally, a table is listed to compare the three commonly used algorithms. From the aspects of practicability and application environment, it deeply analyzes the limitations of traditional adaptive equalization algorithms such as LMS and CMA in the current era, and demonstrates the superior performance of neural networks. On this basis, this paper emphasizes the powerful learning ability of neural networks and the opportunities for future research, which will lay the foundation for the development of next-generation communication networks.

Keywords: Adaptive equalization algorithm, LMS, CMA, neural network.

1. Introduction

In modern communication systems, factors such as multipath propagation, Doppler effect and noise of wireless channels will cause inter-symbol interference (ISI) between different channels [1]. The existence of ISI will reduce the bit error performance of the system, thereby reducing the performance of the communication system [2]. A very common technique is adaptive equalization. This is to pass the received signal through a filter whose characteristics are opposite to the channel's. This filter is called an equalizer. Since the characteristics of the channel may be unknown or change with time, the parameters of the equalizer need to be constantly adjusted to track the changes of the channel, and the adjustment of the parameters is an adaptive equalization algorithm. Before the 1960s, equalization parameters were manually input, and its performance was not ideal. In 1965, Lucky proposed a "zero-forcing adaptive equalizer", which was used to adjust the tap weight coefficients of the horizontal equalizer automatically, and initiated the research on adaptive equalization. In 1967, Austin proposed the decision-feedback equalizer, creating the most commonly used equalizer structure in modern times [3]. 1969 Gersho and Miller redefined the adaptive equalization algorithm using the minimum mean square error criterion. In 1974, Godard applied the Kalman filter theory to derive an efficient algorithm for adjusting the weighting coefficients of the taps of the transverse equalization filter - the fast Kalman algorithm [4]. In 1975, Sato proposed the concept of blind equalization, which greatly expanded the scope of application of the adaptive equalization algorithm. In 1978, Falconer and Ljung simplified the Fast Kalman algorithm to improve its practicality [5].

Nowadays, with the rapid development of digital communication technology, people have paid more and more attention to the research on adaptive equalization algorithms. This is mainly because equalization technology is required in the digital communication system, whether through the public switched telephone network, or through the short wave channel, microwave channel or satellite channel to transmit information. Zhang proposed a new multi-modulus and decision-oriented blind equalization algorithm based on the offset feedback fractionally spaced equalization (OF-FSE) framework, which can adaptively adjust the convergence state of various dynamic environments [6]. Wei designed a deep autoencoder network (DAN) and deep learning (DL) model that can be trained with a unified loss function and a unified optimization algorithm to implement various modems [7]. Ren designed an adaptive neural network equalizer structure based on decision feedback and error backpropagation genetic algorithm [8]. It can effectively eliminate the channel interference of the
nonlinear system and improve the performance of the equalizer under steady-state mean square error and bit error rate. Komeylian introduced a new blind adaptive algorithm with discrete periodic variables, i.e., discrete periodic variable algorithm [9]. The DPVA algorithm has the characteristics of low cost and low complexity, and it is used in the antenna array design. Yang proposed a momentum fractional MMA to improve the performance of MMA in blind equalization so that it can complete blind equalization and carrier phase recovery at the same time[10]. Masih applies LMS, RLS and LM-RS algorithms to the optical downlink system in the DWDM model, eliminating the need for bulky and expensive optical dispersion compensation units [11]. Khafaji proposed a new single-cycle genetic algorithm (GA) as a learning tool to optimize the coefficients of an adaptive linear equalizer [12]. This algorithm outperforms the least mean square (LMS) algorithm. In addition, single-cycle GA has good convergence performance and fast-tracking ability.

This study explores the significance of adaptive equalization algorithms in addressing interference within wireless channels. It delves into algorithms rooted in the least mean squares, blind equalization, and neural networks, highlighting their potential to enhance communication systems. A specific focus is placed on the promising future of neural network-based algorithms for adaptive equalization. The review’s structure encompasses a breakdown of the mathematical model, progressing from time domain principles to traditional adaptive equalization and introducing blind equalization concepts, culminating in a brief overview of equalization algorithms based on Deep Neural Network (DNN) structures. The subsequent section conducts in-depth analyses of the three algorithms through mathematical operations and visual simulations, followed by a comparative evaluation that outlines their strengths and weaknesses. Ultimately, the review concludes by summarizing its findings and predicting a positive developmental trajectory for neural network adaptive equalization algorithms. These algorithms hold broad applications, including 5G and 6G communications, wireless sensor networks, and the Internet of Things, while also extending their utility to domains like healthcare, transportation, and industry, ultimately benefiting society’s convenience and advancement [13-15].

2. Principle Analysis of Adaptive Equalization Algorithms

2.1. Traditional Adaptive Equalization

There are two basic approaches to equalization: one is frequency domain equalization, which makes the total transmission characteristics of the entire system including the equalizer meet the conditions for distortion-free transmission:

\[ H(\omega) = Y(\omega)/X(\omega) = K e^{-j\omega_0} \]  

(1)

The second is time-domain equalization, which is to directly consider the time response so that the impulse response of the entire system including the equalizer meets the condition of no intersymbol interference. This paper mainly discusses related algorithms and implementations in time domain equalization. At present, the horizontal equalizer is widely used as the time-domain equalizer, which is implemented by a horizontal filter or an FIR filter, and can be adjusted according to changes in channel characteristics. The horizontal equalizer is composed of a multi-stage tapped delay line, a weighted coefficient multiplier and an adder, and the structure is shown in Fig. 1 [16].

![Fig. 1 Structure of Transverse Filter.](image-url)
The equalizer is designed according to the Nyquist criterion, and generally \( T \) is equal to the symbol width \( T_S \). The impulse response of the actual system is \( x(t) \). For a certain symbol, \( x(t) \) is no longer zero when divided by the sampling value of each sampling point relative to the symbol \( t=0 \), forming inter-symbol interference, which is defined as Eq. (2):
\[
    x(t) = \sum_{k=-\infty}^{\infty} x_k \tag{2}
\]

The impulse response of the transversal filter is implied as Eq. (3):
\[
    q(t) = \sum_{k=-N}^{N} C_K \delta(t - KT) \tag{3}
\]

The access of the transversal filter will make the output waveform \( y(t) \) of the system become the weighted sum of \( 2N+1 \) equalizer input waveforms \( x(t) \) with different time delays. The output response is shown in Eq. (4).
\[
    y(n) = \int_{0}^{t} x(t - \tau) \sum_{k=-N}^{N} C_K \delta(\tau - KT) d\tau = \sum_{k=-N}^{N} C_K x(t - KT) \tag{4}
\]

At \( t=nT \), the output is shown in Eq. (5):
\[
    y(nT) = \sum_{k=-N}^{N} C_K x[(n - K)T] \tag{5}
\]

Here, \( x(n-K) \) represents the inter-symbol interference caused by the \( K \)th symbol before and after \( n \) as the center at the sampling time \( t=nT \) to the \( n \)th symbol. Therefore, the function of the transversal filter is to adjust the tap gain coefficient \( C \), so that the sample values of the \( 2N \) symbols centered on \( n \) at the sampling time \( t=nT \) tend to zero, so as to eliminate their interference on the \( n \)th symbol.

2.2. Blind Adaptive Equalization

The mathematical model of blind equalization is illustrated in Fig. 2. It does not require reference signals to maintain normal work, so it fundamentally avoids the use of reference signals, and has a large convergence range and a wide range of applications.

As given in Fig. 2, \( x(n) \) is the sending sequence of the system. \( h(n) \) is the impulse response of the discrete-time transmission channel, \( n(n) \) is the Gaussian noise superimposed in the channel, \( y(n) \) is the input sequence of the equalizer, and its value is defined in Eq. (6). Here, \( w(n) \) is the impulse response of the blind equalizer. \( x'(n) \) is the output signal of the system, that is, the equalized sequence, and its value is shown in Eq. (7) [16]:
\[
    y(n) = h(n) * x(n) + n(n) \tag{6}
\]
\[
    x'(n) = w(n) * y(n) = w(n) * h(n) * x(n) \tag{7}
\]

From Eq. (6) and Eq. (7), one can see that \( y(n) \) is formed by convolving \( h(n) \) and \( x(n) \). \( x'(n) \) is formed by \( w(n) \) and \( y(n) \) is convolved. To recover \( x(n) \) from means to unconvolve factor \( h(n) \). Such process is called deconvolution. Blind equalization involves deconvolving \( h(n) \) when only \( y(n) \) is known. There are three main types of blind equalization algorithms commonly used today [17]:

- Smooth signal blind equalization. Its basic feature is to sample the received signal at the baud rate, that is, to take a sample value for each symbol. This technique can be divided into Bussgang type blind equalization and blind equalization based on high-order statistics. For this method, in the future, one should mainly look for an algorithm with a smaller amount of calculation to make it practical.
• Cyclostationary signal blind equalization. Its core is oversampling, and its sampling frequency is an integer multiple of the Nyquist sampling frequency. Identification and equalization can be accomplished using second-order cyclostationary statistics for time-invariant non-minimum phase systems. However, if it is a time-varying system, then you need to use higher-order cyclostationary statistics. Its advantage is that it can separate stationary and non-stationary signals and recover the phase information of time-varying systems.

• Blind equalization based on neural network and fuzzy theory. Channel equalization can also be regarded as a classification problem, that is, the equalizer is regarded as a decision device in order to reconstruct the transmission sequence as accurately as possible. The neural network can have a strong classification function after algorithm training, and then overcome the processing difficulties caused by the uncertainty in the problem description process through fuzzy theory.

2.3. Adaptive Equalization Algorithm Based on DNN Neural Network

In recent years, with the continuous increase of channel rate, neural network has also been applied in adaptive equalization. Jian Jie proposed an equalizer structure based on DNN structure. This structure uses the backpropagation algorithm (BP) based on stochastic gradient descent (SGD) to train the parameters of the deep neural network. Since there is no feedback loop, the equalizer solves the time-limited problem, and the theoretically supported channel rate is not limited.

The data of all hidden layers and output layers can be considered as arrays related to neurons, and the output is shown as Eq. (8):

$$y = \sigma(\sum_i w_i x_i - b) = \sigma(W^T X - b)$$  

Here, $W=\{w_i\}$ is the weight vector, $X=\{x_i\}$ is the input value, and $b$ is the threshold. The neural network structure is completely determined by the number of network layers and the number of neurons in each layer. The network can obtain the maximum balance effect by adjusting the above parameters and taking the minimum network resources.

3. Comparative Study on Adaptive Equalization Algorithms

3.1. Adaptive Equalization Algorithm Based on LMS Class

The least mean square algorithm is an adaptive filtering algorithm often used in signal processing, communication systems, control systems, and other fields. The core idea of this algorithm is to adjust the weight of the filter according to the error signal, so as to gradually reduce the error and finally make the output signal approach the desired signal [16].

3.1.1 Implementation steps

The structure of the FIR transversal filter using the LMS algorithm is shown in Fig. 3, where $W(n)$ is the weight vector of the filter at time $n$, $x(n)$ is the input signal vector at time $n$, $y(n)$ is the equalized signal, $d(n)$ is the reference signal, $e(n)$ is the error signal. Through the LMS algorithm, the weight vector $W(n+1)$ at the next moment is equal to the weight vector $W(n)$ at the previous moment plus a correction amount, which is the weighted value of $e(n)$, and its weighting coefficient is proportional to the input signal $X(n)$. 

\[ y = \sigma(\sum_i w_i x_i - b) = \sigma(W^T X - b) \]
Since the LMS algorithm was proposed earlier, there are still many areas that can be improved. This section analyzes an NLMS algorithm that normalizes the iteration step size. As long as the initial step $0 < \mu < 1$ can ensure that after iterations, the algorithm can converge stably, which ensures the stability of the convergence of the adaptive equalization algorithm.

### 3.1.2 Result analysis

First of all, this paper analyzes the performance curves of the LMS adaptive equalization algorithm under different iteration step sizes. The system uses a 15-order LMS filter with a step size of 0.0005, 0.001, 0.005, and 0.01, and the noise is single-frequency white noise. Fig. 4 simulates the LMS equalization for these step sizes. The abscissa in the figure above is the number of iterations, and the ordinate is the convergence performance. When the curve is closer to the upper left corner, the equalization effect is better. The figure below shows the convergence error of the LMS algorithm under different step lengths. The horizontal axis is the number of iterations, and the vertical axis is the size of the error oscillation. The smaller the size of the shock, the smaller the error, and the better the performance of the LMS algorithm.
Fig. 5 Performance Comparison between LMS and NLMS.

By comparison, it can be found that when the iteration step size is 0.0005, the convergence speed of the algorithm is slow, and it does not basically converge until 6000 iterations. When the step size is 0.01, the algorithm converges faster, and the convergence is completed in about 500 times. However, when the step size is 0.01, there is a certain gap between the system and the optimal solution after entering the steady state, that is, the difference between the inflection point value and the y-axis data after stabilization is 0.07. When the step size is 0.0005, the error between the two is only 0.004, reflecting good stability. Meanwhile, it can be seen from the Fig. 4 that the oscillation is ±0.14 when the step size is 0.01, and only ±0.11 when the step size is 0.0005. It can be concluded that the LMS algorithm with a fixed step size cannot have both the convergence speed and the steady-state error.

Then, this paper analyzes the improved NLMS algorithm and compares it with the LMS algorithm. The definitions of the x and y axes in Fig. 6 are the same as those in Fig. 5. For better performance comparison, the initial step size of NLMS is 0.01, while the initial step size of LMS is 0.001. It can be seen from Figure 6 that although the step size of NLMS algorithm is larger than that of the LMS algorithm, its curve is closer to the upper left corner, representing it has better convergence speed and performance. At the same time, the steady-state error of the two algorithms is also close to 0.01. As can also be seen from the figure below, the convergence error of LMS is greater than that of NLMS. Therefore, the NLMS algorithm is better than the traditional LMS algorithm in terms of convergence speed and minimum mean square error, which verifies the principle of the above-mentioned normalization algorithm.

3.1.3 Discussion

As a classic algorithm in the field of adaptive equalization, LMS algorithm has been widely used in the field of signal processing because of its simple and efficient characteristics. The LMS algorithm represents the essence of adaptive balancing, i.e., dynamically adjusting resource allocation according to changes in system state and demand to optimize system performance. In recent years, with the continuous development of communication technology, there have been many improvements to the basic LMS algorithm. Through in-depth analysis of the mathematical principle, scope of application and practical application of LMS algorithm, the value and limitations of adaptive equalization algorithm in different fields can be deeply discussed.

3.2. Constant Modulus Algorithm Based on Bussgang

The constant modulus algorithm is an adaptive filtering algorithm widely used in the field of digital signal processing. Its main feature is to keep the constant modulus of the output signal near a preset constant value by adjusting the filter weight [18, 19].

300
3.2.1 Implementation steps

The constant modulus algorithm is the most commonly used one of the Bussgang blind equalization algorithms, which is the Godard algorithm when the parameter $P=2$. The equalizer structure using the CMA algorithm is shown in Fig 6, where $x(n)$ is the input signal, $e(n)$ is the compensation function, and $y(n)$ is the output after equalization. It can be seen from the figure that the algorithm does not need a reference signal, and only uses the amplitude statistical characteristics of the transmitted signal to adjust the weight coefficient, so that the amplitude of the output signal remains constant.

![Fig. 6 CMA Equalization Structure.](image)

3.2.2 Result analysis

Primarily, this paragraph first verifies the feasibility of the CMA algorithm. Taking symbols, the number of sampling points for a single symbol is 8, and the symbol period is $10^{-9}$. First, one needs to generate a random [0,1] sequence, then sample it 8 times, and then add noise. At this time, the waveform can be seen to vibrate obviously. Then the waveform with noise is subjected to the CMA equalization algorithm, and 0.5 is used as the threshold to distinguish [0,1] values. It can be seen from Fig. 7 that after shifting the post-judgment sequence to the right by $0.3\times10^{-6}$, the coincidence rate of the decision sequence and the original sequence can reach more than 95%, which verifies the high performance of the CMA algorithm in the field of blind equalization.

After verifying the effectiveness of the CMA algorithm, this paper also constructs an equalizer based on the LMS algorithm for comparison. Fig. 8 depicts the constellation diagrams of the CMA algorithm and the LMS algorithm modulated by 4-QAM. The dispersion range of the CMA algorithm after equalization is 0.7, while the dispersion range of the LMS algorithm is 1.3 after equalization, which is significantly larger than that of the CMA algorithm. Therefore, the CMA algorithm is better than the LMS algorithm in terms of signal equalization, and has a better discrimination rate. This experiment also proves that the CMA algorithm is suitable for the problem of blind source separation.
3.2.3 Discussion

Traditional adaptive equalization systems require a large number of training sequences to be preprocessed, which wastes a lot of valuable spectrum resources. Blind equalization is thus created, which can adaptively equalize the channel without the help of training sequences, but only with the received signal sequence. CMA is one of the most commonly used blind equalization algorithms, with high immunity to narrowband interference, low time synchronization requirements, and insensitivity to frequency offset. Through the study of CMA algorithm, this study summarizes the use of blind equilibrium and its equilibrium effect. It not only introduces an indispensable part of adaptive equalization, but also delves into the importance of CMA algorithms in nonlinear communication systems and signal processing.

3.3. Adaptive equalization algorithm based on SGD-BP

In mobile communication, adaptive neural network equalizers perform a nonlinear mapping and are used for adaptive equalization of nonlinear and time-varying channels.

3.3.1 Implementation steps

The neural network can achieve better balanced performance by optimizing the network parameters and making the network better. In order to minimize the value of the compensation function $C$, Hinton et al. used the backpropagation algorithm to train the network, and its basic principle comes from the chain derivation rule of multivariate functions [20]. First, the sample set is divided into several batches, each batch contains groups, and each group contains a signal value; then the error value of the output layer is calculated, and the gradient value of all network parameters is calculated using the back propagation algorithm; the group input signal of each batch is generated. The gradient value of is averaged; then, the weight vector and offset vector of the network are updated. Finally, using a set of independent verification samples as the input of the neural network, the signal error rate after passing through the neural network is calculated. The above process is iteratively repeated until the SER value is lower than a certain threshold [18].

3.3.2 Result analysis

In order to facilitate the comparison of the functions of the neural network and the traditional method, this paper constructs a traditional 15-order FFE+2-order DFE equalizer (structure shown in Fig. 9) for comparison, and uses the LMS algorithm to calculate the weight parameters. Since the neural network is mainly used in the future high-speed channel field, the input signal of this experiment uses a 28GbPAM4 signal with a transmission loss of 15dB and a 56GbPAM4 signal with a transmission loss of 30dB. It can be seen from Fig. 10 that under the 28GbPAM4 signal and 15dB transmission loss, the BER value of the LMS algorithm is $10^{-3}$. The DNN equalizer can achieve $7 \times 10^{-4}$ accuracy with only one hidden layer, far exceeding the FFE+DFE structure. As
the quality of the input signal deteriorates, both the FFE+DFE equalizer and the DNN equalizer can achieve an accuracy of $8 \times 10^{-3}$ under a 56GBd PAM4 signal and a transmission loss of 30dB. It can also be found from the figure that the neural network algorithm has better work efficiency. No matter how the FFE and DFE structure is configured, its convergence time will not be less than 200\(\mu\)s. In the DNN structure, when $n_{main} = 5$ and the channel rate is 28GBd, the convergence time is less than 100\(\mu\)s after algorithm iteration. When the channel rate reaches 56GBd, its convergence time is less than 50\(\mu\)s, reflecting good functionality.

![Fig. 9 15-order FFE+2-order DFE Equalizer Structure.](image1)

![Fig. 10 Comparison of FFE-DFE and DNN Equilibrium.](image2)

3.3.3 Discussion

Adaptive equalization algorithms based on neural networks have been the hottest topic in recent years. Through its nonlinear modeling capabilities, neural networks can learn from massive amounts of data and extract features to adapt to changes in different environments. This capability coincides with adaptive equilibrium. Through the iterative training of multiple rounds of datasets, neural networks have shown excellent performance in the field of adaptive equilibrium. Through the above analysis, this paragraph illustrates the high accuracy of neural networks and the ability to process information quickly, revealing the potential development direction of the future equilibrium field.

3.4. Comparative Study

By comparing the above three commonly used adaptive equalization methods (seen from Table 1), one can obtain the functional evaluation of each algorithm. As one of the most classic algorithms, the LMS algorithm is simple to implement and has high operational efficiency. However, the signal
is poor when the correlation between the noise and the original signal is strong or there is multipath propagation. At the same time, it cannot handle nonlinear signals, so it can be used when the noise is small and the communication quality is not high. The CMA algorithm has a better processing function for nonlinear problems. At the same time, because the CMA algorithm has the function of blind equalization, it can automatically adjust parameters according to the input. However, when the signal is weak and the signal direction is less, it does not handle the performance well. Therefore, the CMA algorithm is widely used in related fields such as blind source separation.

Algorithms based on DNN neural network can handle a large number of features and inputs. Its biggest advantage is that it can process a large amount of data at high speed, and it has a good application prospect in the environment where the communication rate is getting faster and faster in the future. However, all neural network algorithms need to be trained in advance, and have high requirements for data sets and computing resources. At the same time, their fitting algorithms also need to be continuously improved.

Based on the analysis, the adaptive equalization algorithm based on neural network has great potential and significant advantages in suppressing intersymbol interference in the future. By making full use of the learning and adaptability of the neural network, this algorithm is expected to effectively reduce or even eliminate inter-symbol interference, thereby significantly improving the performance and stability of the communication system. Traditional methods may be limited to specific environments or scenarios, while neural network algorithms can automatically adjust to adapt to different situations, thereby enhancing the effect of suppressing interference. In addition, with the development of technology, the improvement of hardware computing power also provides more possibilities for the application of neural networks, making this adaptive equalization algorithm run more efficiently.

### Table 1. Comparison of different algorithm functions

<table>
<thead>
<tr>
<th>Algorithm type</th>
<th>Astringent effect</th>
<th>channel rate</th>
<th>algorithmic complexity</th>
<th>Application Environment</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>Low</td>
<td>Slow</td>
<td>Low</td>
<td>Environment with low noise and low communication quality requirements</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No training sequence, suitable for blind source separation problems</td>
</tr>
<tr>
<td>CMA</td>
<td>Mid</td>
<td>Slow</td>
<td>Mid</td>
<td>Requires pre-training and can be used in most environments</td>
</tr>
<tr>
<td>SGD-BP</td>
<td>High</td>
<td>Fast</td>
<td>High</td>
<td></td>
</tr>
</tbody>
</table>

### 4. Conclusion

To sum up, this study analyzes adaptive equalization algorithms in general. In various practical applications, noise affects system performance. LMS algorithms are superior in some problems with less noise because of their simplicity and efficiency but low accuracy. The CMA algorithm is better at dealing with states without training sequences and performs well when dealing with complex signal environments and nonlinear problems. The neural network algorithm based on SGD-BP has the ability of iterative upgrading and adaptability to complex nonlinear problems and is suitable for high-speed data. In summary, DNN-based neural network algorithms will have excellent applications in future high-speed communication scenarios. As long as the neural network is preprocessed, it has better potential in all aspects and can be widely used in signal processing and other fields. Noise significantly impacts system performance and signal quality in various applications. Adaptive equalization algorithms effectively reduce noise, enhancing signal clarity and reliability in wireless communication, healthcare, and more. As AI and IoT advance, these algorithms are poised to play a vital role in areas like environmental monitoring and industrial distribution.
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