Nonlinear vibration study of cylindrical shells

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Abstract. Shells are common structures and models in our life. With the continuous development of research on various types of shells, shells have been very widely used in daily life, industrial buildings, aerospace, space technology and other fields. In this paper, firstly, the expressions of each energy variant of the shell under the joint action of elasticity and piezoelectric layer electric field are given, and the physical model is established based on Hamilton's principle. Secondly, a nonlinear multi-degree-of-freedom dynamics model of the vibration of the cylindrical shell is developed by considering the asymmetric modes of the shell. Considering the boundary conditions and separating the displacement function from the independent time variables, the specific expressions of the stiffness matrix are calculated according to the energy variations as well as the internal forces and moments, and the generalized coordinate vectors are solved and analyzed by giving specific examples.

Keywords: Cylindrical shells; Vibration response curves; Nonlinear multiple-degree-of-freedom equations of motion; Hamilton's principle.

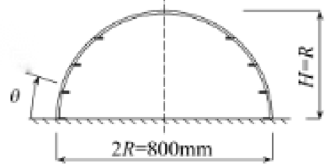
1. Introduction

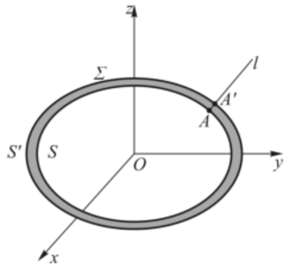
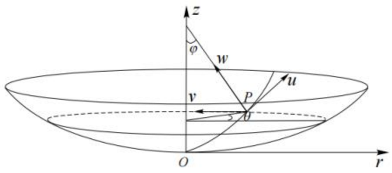
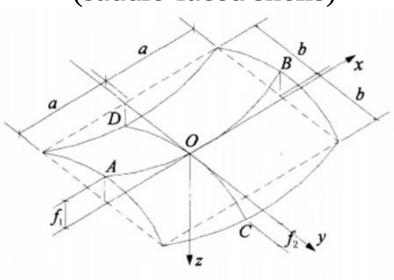
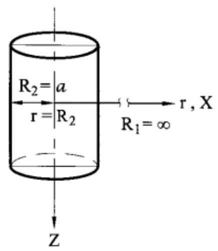
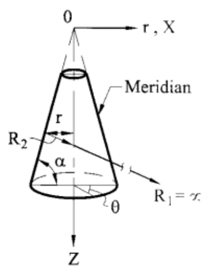
1.1 Types of housings and their applications

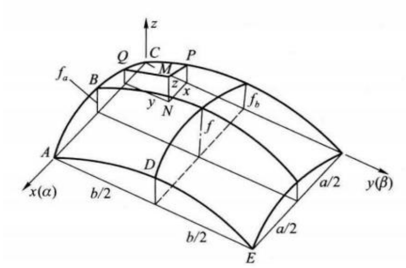
An object bounded by two surfaces is called a shell, and the surface formed at a point equidistant from the two shell surfaces is called the intermediate surface, or simply the mid-surface, and the length of the mid-surface normal to the two shell surfaces, truncated by the two shell surfaces, is called the thickness of the shell. The shell may be of equal or variable thickness. If the thickness δ of the shell is much less than the minimum radius of curvature R of the midplane of the shell, the shell is called a thin shell. Conversely, it is called a thick shell.

Different types of shells have been used in a very wide range of applications, whether in daily life, industrial construction or aerospace. Different types of shells have different physical models and geometric equations in practical problems, and according to the characteristics of these shells will have targeted modeling theory and simplification methods to help better solve practical problems. Shells are categorized in various ways, such as: by Gaussian curvature, by shell surface, by thickness, etc. The following is a common categorization method and the application of common shell types.

Table 1. Shell Type Summary

Categorization	Typology	Model theory	Applications
Positive Gaussian curvature shells	Spherical shell 	Donnell's simplified theory, moment-free theory of shells	Spherical Roof Structure

	Ellipsoidal shell 	Donnell's simplified theory, moment-free theory of shells	Large-span ellipsoidal shell double-tube planar truss structure, large stadiums, industrial buildings
	Elliptic parabolic shell 	Donnell simplification theory, moment-free theory of shells, flat shell approximation theory	Elliptic parabolic mesh shell structures, industrial buildings
Negative Gaussian curvature shells	Hyperbolic paraboloid (saddle-faced shells) 	Donnell's simplified theory, moment-free theory of shells	Hyperbolic parabolic thin-shell roof
Zero Gaussian curvature shells	Cylindric shell 	Donnell simplification theory, moment-free theory of shells, Flugge shell theory, Von K´arm´an theory	Solid fuel tanks for rockets, vibration damping bases for laser platforms, fall arrest buffers for elevators
	Conical shell 	Donnell's simplified theory, moment-free theory of shells	Large reinforced concrete silo structures, aerospace vehicles, submarines
Hybrid curvature shells	Flat shell	Flat shell approximation theory, moment-free theory of shells	Wire mesh cement hyperbolic flat shell structure, composite material



sandwich flat shell structure

1.2 Introduction to Modeling Theory

For the shell model used in this paper, here are some theories. As we know, from the beam model expanded to two dimensions there is a plate model, and then after bending the shell is obtained. 1874 German H. Aron extended the Kirchhoff assumption in the thin plate theory to the shell, and the thin shell theory was established at the end of the 19th century on the basis of the Kirchhoff-Leffe assumption. After entering the 20th century, various simplified theories for different types of shells began to be studied and applied in practice, driven by production technology. In the 1950s, physicists began to revise the Kirchhoff-Leffe assumption to make the thin-shell theory precise. The geometries and deformations of thin shells are usually complex, and a series of simplifying assumptions must be introduced in order to study them. The most commonly used of these is Donnell's simplified theory (1934), which was originally used for cylindrical shells, i.e., the effect of circumferential curvature on the transverse shear N2 is neglected in the equations of circumferential equilibrium for the faces in the shell and the terms containing tangential displacement components u and v are omitted from the geometrical equations for the deformation components, and which is equally applicable to other shells, and which, because of its relative simplicity and practical accuracy, has been widely used Sanders (1963) developed a more accurate nonlinear theory of shells than Donnell's theory, expressed in tensor form, and his theory gives accurate results for the case of thin shells where the amplitude of vibration is significantly greater than the thickness of the shell.

In most of the practical problems, we have approximated the shell as a thin shell to study, the theory for thin shells is called the momentless theory of shells, also known as the thin film theory, refers to the thin shell like a membrane can only withstand compressive and tensile stresses and can not be bent, momentless that is, ignoring bending moments and bending stresses, in the actual model, that is, the bending moment components of $M1 = M2 = M12 = 0$, used to simplify the equilibrium equations. Another type of problem is the flat shell, when the ratio of the vector height of the parabolic film to its aperture is less than 1/8, it will be called a flat parabolic film. In some practical problems, because the vector height of the parabolic film and its aperture are generally smaller, geometrically similar to the rotating flat shell, you can use the flat shell approximation theory to simplify the equations for its geometric properties, the use of the shell in the middle of the face of the flat features of the Gaussian curvature approximation to zero, in addition to the geometric relationship between the middle of the face of the strain component of the equation and retain the curvature effect of the normal equilibrium equations, and other approximations to the plate The expression of the equation can thus also be applied to shells with zero Gaussian curvature (e.g., cylindrical shells, conical shells).

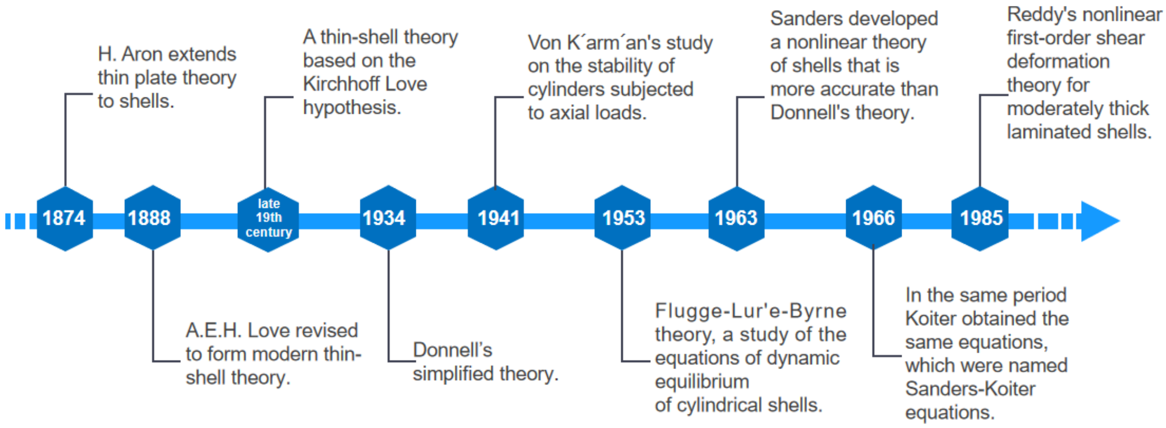


Fig.1. timeline

2. Piezoelectric sheet modeling

A piezoelectric patch is added to a thin shell, and a physical model based on Hamilton's principle is the most basic equation for the dynamics of a piezoelectric laminated shell under the combined effect of elastic

and electric fields:

$$\delta \int_{t_0}^{t_1} (K_E - U_E + W)\,\mathrm{dt} = 0$$

This is where K_E is the kinetic energy of the system, U_E is the deformation energy of the system, i.e., the elastic strain energy and piezoelectric strain energy, and W is the work of the system, i.e., the boundary force work, the applied surface load work, and the piezoelectric potential work. After calculation, each energy variable has the following form respectively:

$$\int_{t_\theta}^{t_1} \delta K_E dt = -\rho h \int_{t_\theta}^{t_1} \int_{\alpha_1} \int_{\alpha_2} (\ddot{u}_1 \delta u_1 + \ddot{u}_2 \delta u_2 + \ddot{u}_3 \delta u_3) A_1 A_2 \, d\alpha_1 \, d\alpha_2 \, dt$$

$$\int_{t_0}^{t_1} \delta U_E dt \int_{t_0}^{t_1} \int_{\alpha_1} \int_{\alpha_2} \int_{\alpha_3} [(\sigma_{11}^m - d_{31} E_3)\delta S_{11} + (\sigma_{22}^m - d_{32} E_3)\delta S_{22} + \sigma_{12}^m \delta S_{12}$$

$$+\sigma_{13}^m \delta S_{13} + \sigma_{23}^m \delta S_{23}] A_1 A_2 \left(1 + \frac{\alpha_3}{R_1}\right)\left(1 + \frac{\alpha_3}{R_2}\right) d\alpha_1 d\alpha_2 d\alpha_3 dt$$

$$\int_{t_0}^{t_1} \delta W_{\tilde{Q}} \quad dt = \int_{t_0}^{t_1} \left[\left[\int_{\alpha_1} \int_{\alpha_2} \left(d_{31} S_{11} + d_{32} S_{22} + d_{33} S_{33} + \varepsilon_{33} E_3 + \tilde{Q}_3\right) A_1 A_2 \delta\varphi d\alpha_1 d\alpha_2\right.\right.$$

$$\left. - \int_{\alpha_1} \int_{\alpha_2} \int_{\alpha_3} \frac{\partial[(d_{31} S_{11} + d_{32} S_{22} + d_{33} S_{33} + \varepsilon_{33} E_3) A_1 A_2]}{\partial \alpha_3} \delta\varphi d\alpha_1 d\alpha_2 d\alpha_3 \right] dt$$

Adding the boundary force work, the applied surface loading work, and the piezoelectric potential work yields the variational component of the total system work, and the expression is brought into Hamilton's equation to give the final form of the piezoelectric layer hermetic shell dynamics

equation.

3. Nonlinear multi-degree-of-freedom dynamics model

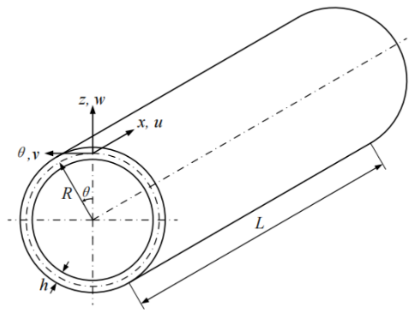


Fig 2. Cylindrical Shell Legend

For a cylindrical shell, we have its geometric equations:

$$u_1(x,\theta,z,t) = u(x,\theta,t) + z\Phi_x(x,\theta,t)$$
$$v_1(x,\theta,z,t) = v(x,\theta,t) + z\Phi_\theta(x,\theta,t)$$
$$w_1(x,\theta,z,t) = w(x,\theta,t)$$

This equation expresses the displacements at each point in the shell in terms of the axial, circumferential and radial displacements u, v and

w of the midplane, $\phi_x(x,\theta,t)$and $\phi_\theta(x,\theta,t)$are the rotational displacements of the transverse normal about the x and θ axes, respectively. According to Donnell's theory and Von Kármán's geometrically nonlinear theory, the strain-displacement relation for the midplane of a cylindrical shell can be expressed as:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \varepsilon_{x\theta}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{w}{R} + \frac{1}{2}\left(\frac{1}{R}\frac{\partial w}{\partial \theta}\right)^2 \\ \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta} \end{Bmatrix}, \begin{Bmatrix} \varepsilon_{xz}^0 \\ \varepsilon_{\theta z}^0 \end{Bmatrix} = \begin{Bmatrix} \varphi_x + \frac{\partial w}{\partial x} \\ \varphi_\theta + \frac{\partial w}{R\partial \theta} \end{Bmatrix}$$

R represents its radius of curvature, and the strain component at any position on the non-central plane is denoted as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \varepsilon_{x\theta}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{Bmatrix}, \begin{Bmatrix} \varepsilon_{xz} \\ \varepsilon_{\theta z} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xz}^0 \\ \varepsilon_{\theta z}^0 \end{Bmatrix}$$

z denotes the distance of any point from the midplane, $\varepsilon_x$and ε_θ is the positive strain, $\varepsilon_{x\theta}$, ε_{xz} and $\varepsilon_{\theta z}$is the shear strain in each plane. k_x,k_θ and $k_{x\theta}$ are the midplane curvature and torsion of the cylindrical

shell with the following relation:

$$\begin{Bmatrix} \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \varphi_x}{\partial x} \\ \frac{\partial \varphi_\theta}{R\partial \theta} \\ \frac{1}{R}\frac{\partial \varphi_x}{\partial \theta} + \frac{\partial \varphi_\theta}{\partial x} \end{Bmatrix}$$

The stress-strain intrinsic relationship of the cylindrical shell is:

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{\theta z} \\ \sigma_{xz} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_{\theta z} \\ \varepsilon_{xz} \\ \varepsilon_{x\theta} \end{Bmatrix}$$

σ_x and σ_θ are the positive stresses, $\sigma_{\theta z}$, σ_{xz} and $\sigma_{x\theta}$ are the shear stresses in each plane, and Q_{ij} is the discounted stiffness matrix with a

stiffness factor of:

$$Q_{11} = \frac{E_{11}}{1-v_{12}v_{21}}, Q_{22} = \frac{E_{22}}{1-v_{12}v_{21}}, Q_{12} = \frac{v_{21}E_{11}}{1-v_{12}v_{21}},$$
$$Q_{44} = G_{23}, Q_{55} = G_{13}, Q_{66} = G_{12},$$

E_{11} and E_{22} is the modulus of elasticity, G_{12}、G_{13} and G_{23} is the shear modulus, ν_{12} and ν_{21} is the Poisson's ratio.

Integrating each stress component in the direction of the thickness of the cylindrical shell, the expressions for the internal forces and

moments of the cylindrical shell can be obtained:

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \varepsilon_{x\theta}^0 \\ \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{Bmatrix}, \begin{Bmatrix} Q_\theta \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{\theta z}^0 \\ \varepsilon_{xz}^0 \end{Bmatrix}$$

Where A_{ij}, B_{ij}, D_{ij} are the tensile, coupling and bending stiffnesses, respectively, and A_{44} and A_{55} are the transverse shear stiffnesses,

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\,(1, z, z^2)\,\mathrm{d}z(i, j = 1,2,6)$$
$$A_{44} = k_s \int_{-h/2}^{h/2} Q_{44}\ dz, A_{55} = k_s \int_{-h/2}^{h/2} Q_{55}\ dz$$

k_s is the shear correction factor, and the stiffness matrix can be derived by substituting the resulting internal forces and moments, as well as each of the strain components, into each of the energy variation

described in the previous section.

We consider asymmetric modes, whose displacement function satisfied by the cylindrical shell with the addition of boundary conditions can be

expressed as:

$$u(x,\theta,t) = \sum_{i=1}^{I}\sum_{j=1}^{J} u_{i,j}\,(t)\,cos(\,i\pi x)\,cos(\,jn\theta)$$
$$v(x,\theta,t) = \sum_{i=1}^{I}\sum_{j=1}^{J} v_{i,j}\,(t)\,sin(\,i\pi x)\,sin(\,jn\theta)$$
$$w(x,\theta,t) = \sum_{i=1}^{I}\sum_{j=1}^{J} w_{i,j}\,(t)\,sin(\,i\pi x)\,cos(\,jn\theta)$$
$$\varphi_x(x,\theta,t) = \sum_{i=1}^{I}\sum_{j=1}^{J} \varphi_{x_{ij}}\,(t)\,cos(\,i\pi x)\,cos(\,jn\theta)$$
$$\varphi_\theta(x,\theta,t) = \sum_{i=1}^{I}\sum_{j=1}^{J} \varphi_{\theta_{ij}}\,(t)\,sin(\,i\pi x)\,sin(\,jn\theta)$$

where n denotes the number of axial waves in the cylindrical shell, the displacement function is written in matrix form as:

$$\begin{aligned} u(x,\theta,t) &= U^T(x,\theta)p(t) \\ v(x,\theta,t) &= V^T(x,\theta)q(t) \\ w(x,\theta,t) &= W^T(x,\theta)r(t) \\ \Phi_x(x,\theta,t) &= \Phi_x^T(x,\theta)q_{\phi x}(t) \\ \Phi_\theta(x,\theta,t) &= \Phi_\theta^T(x,\theta)q_{\phi\theta}(t) \end{aligned}$$

where U, V, W, Φ_x, Φ_θ are the modal function matrices of the motions in each direction, and p、q、r、$q_{\phi x}$、$q_{\phi\theta}$ are the generalized coordinate

vectors of the displacements in each direction, respectively.

Therefore, using Galerkin weighted integrals, the nonlinear ordinary differential equations of motion for the cylindrical shell are written

in the form of matrices as follows:

$$M\ddot{X} + (K + K^{(2)} + K^{(3)})X = F\cos(\Omega t)$$

where $X = [p^T, q^T, r^T, q_{\phi x}{}^T, q_{\phi\theta}{}^T]^T$ is a generalized coordinate vector, $F = [0,0,F_w,0,0]^T$ is a generalized force vector, M,K, $K^{(2)}$and $K^{(3)}$are the mass, linear stiffness, quadratic nonlinear and cubic nonlinear stiffness

matrices, respectively, which have the following forms:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{14} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{25} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{33} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{M}_{41} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{44} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_{52} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{55} \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{uv} & 0 & \boldsymbol{K}_{ux} & \boldsymbol{K}_{u\theta} \\ \boldsymbol{K}_{uv}^T & \boldsymbol{K}_{vv} & 0 & \boldsymbol{K}_{vx} & \boldsymbol{K}_{v\theta} \\ 0 & 0 & \boldsymbol{K}_{ww} & \boldsymbol{K}_{wx} & \boldsymbol{K}_{w\theta} \\ \boldsymbol{K}_{ux}^T & \boldsymbol{K}_{vx}^T & \boldsymbol{K}_{wx}^T & \boldsymbol{K}_{xx} & \boldsymbol{K}_{x\theta} \\ \boldsymbol{K}_{u\theta}^T & \boldsymbol{K}_{v\theta}^T & \boldsymbol{K}_{w\theta}^T & \boldsymbol{K}_{x\theta}^T & \boldsymbol{K}_{\theta\theta} \end{bmatrix}$$

$$\boldsymbol{K}^{(2)} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{uw}^{(2)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{vw}^{(2)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K}_{wu}^{(2)} & \boldsymbol{K}_{wv}^{(2)} & \boldsymbol{K}_{ww}^{(2)} & \boldsymbol{K}_{wx}^{(2)} & \boldsymbol{K}_{w\theta}^{(2)} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{xw}^{(2)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{\theta w}^{(2)} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{K}^{(3)} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{ww}^{(3)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

Define: $\lambda_1 = \frac{L}{R}, \lambda_2 = \frac{h}{R}, \lambda_3 = \frac{h}{L},$ I_1、I_2、I_3 are inertia coefficients,

$$\{I_1, I_2, I_3\} = \int_{-h/2}^{h/2} \rho(z)\{1, \quad z, \quad z^2\}dz$$

4. Numerical calculation

A specific calculation example is given here, so that L=410mm, R=301.5mm, h=1mm, E=210Gpa, ν=0.3, ρ=7850 kg/m3, then there are $\lambda_1 = 1.3599$, $\lambda_2 = 0.0033$, $\lambda_3 = 0.0024$, $I_1 = 7.85$, I_2 =0, I_3 =6.54167*10^{-7}, we can calculate each bending stiffness, shear stiffness, take I、J = 3, then there are:

$$U = [cos(\pi x)\,cos(\theta), cos(\pi x)\,cos(2\theta), cos(\pi x)\,cos(3\theta), cos(2\pi x)\,cos(\theta), cos(2\pi x)\,cos(2\theta), cos(2$$
$$cos(3\theta), cos(3\pi x)\,cos(\theta), cos(3\pi x)\,cos(2\theta), cos(3\pi x)\,cos(3\theta)]$$
$$V = [sin(\pi x)\,sin(\theta), sin(\pi x)\,sin(2\theta), sin(\pi x)\,sin(3\theta), sin(2\pi x)\,sin(\theta), sin(2\pi x)\,sin(2\theta), sin(2\pi x)\,s$$
$$, sin(3\pi x)\,sin(\theta), sin(3\pi x)\,sin(2\theta), sin(3\pi x)\,sin(3\theta)]\,;$$
$$W = [sin(\pi x)\,cos(\theta), sin(\pi x)\,cos(2\theta), sin(\pi x)\,cos(3\theta), sin(2\pi x)\,cos(\theta), sin(2\pi x)\,cos(2\theta), sin(2\pi x$$
$$s(3\theta), sin(3\pi x)\,cos(\theta), sin(3\pi x)\,cos(2\theta), sin(3\pi x)\,cos(3\theta)]$$
$$\varphi_x = [cos(\pi x)\,cos(\theta), cos(\pi x)\,cos(2\theta), cos(\pi x)\,cos(3\theta), cos(2\pi x)\,cos(\theta), cos(2\pi x)\,cos(2\theta), cos($$
$$cos(3\theta), cos(3\pi x)\,cos(\theta), cos(3\pi x)\,cos(2\theta), cos(3\pi x)\,cos(3\theta)]\,;$$
$$\varphi_\theta = [sin(\pi x)\,sin(\theta), sin(\pi x)\,sin(2\theta), sin(\pi x)\,sin(3\theta), sin(2\pi x)\,sin(\theta), sin(2\pi x)\,sin(2\theta), sin(2\pi x)$$
$$\theta), sin(3\pi x)\,sin(\theta), sin(3\pi x)\,sin(2\theta), sin(3\pi x)\,sin(3\theta)]$$

By using matlab to solve, the images of the generalized coordinate vectors of the displacements in each direction of the nonlinear ordinary differential equation of motion $\mathrm{M}\ddot{\mathrm{X}} + (\mathrm{K} + \mathrm{K}^{(2)} + \mathrm{K}^{(3)})\mathrm{X} = \mathrm{F}\cos(\Omega\,\mathrm{t})$ are obtained for the linear and nonlinear cases, respectively:

Appendix

Quality matrix:

$$\boldsymbol{M} = \begin{bmatrix} \iint_V I_1\, UU^T dV & \mathbf{0} & \mathbf{0} & \iint_V I_2\, U\Phi_x^T dV & \mathbf{0} \\ \mathbf{0} & \iint_V I_1\, VV^T dV & \mathbf{0} & \mathbf{0} & \iint_V I_2\, V\Phi_\theta^T dV \\ \mathbf{0} & \mathbf{0} & \iint_V I_1\, WW^T dV & \mathbf{0} & \mathbf{0} \\ \iint_V I_2\, \Phi_x U^T dV & \mathbf{0} & \mathbf{0} & \iint_V I_3\, \Phi_x\Phi_x^T dV & \mathbf{0} \\ \mathbf{0} & \iint_V I_2\, \Phi_\theta V^T dV & \mathbf{0} & \mathbf{0} & \iint_V I_3\, \Phi_\theta\Phi_\theta^T dV \end{bmatrix},$$

$$K_{\mathrm{uu}} = -\iint (a_{11}U\frac{\partial^2 U^T}{\partial x^2} + a_{66}\lambda_1^2 U\frac{\partial^2 U^T}{\partial\theta^2})\,dxd\theta$$

$$K_{\mathrm{uv}} = -(a_{12} + a_{66})\lambda_1 \iint U\frac{\partial^2 V^T}{\partial x\partial\theta}\,dxd\theta$$

$$K_{\mathrm{ux}} = -\iint (b_{11}U\frac{\partial^2 \Phi_x^T}{\partial x^2} + b_{66}\lambda_1^2 U\frac{\partial^2 \Phi_x^T}{\partial\theta^2})\,dxd\theta$$

$$K_{\mathrm{u}\,\theta} = -(b_{12} + b_{66})\lambda_1 \iint U\frac{\partial^2 \Phi_\theta^T}{\partial x\partial\theta}\,dxd\theta$$

$$K_{\mathrm{vv}} = -\iint (a_{66}V\frac{\partial^2 V^T}{\partial x^2} + a_{22}\lambda_1^2 V\frac{\partial^2 V^T}{\partial\theta^2})\,dxd\theta$$

$$K_{\mathrm{vx}} = -(b_{12} + b_{66})\lambda_1 \iint V\frac{\partial^2 \Phi_x^T}{\partial x\partial\theta}\,dxd\theta$$

$$K_{\mathrm{v}\,\theta} = -\iint (b_{66}V\frac{\partial^2 \Phi_\theta^T}{\partial x^2} + b_{22}\lambda_1^2 V\frac{\partial^2 \Phi_\theta^T}{\partial\theta^2})\,dxd\theta$$

$$K_{\mathrm{ww}} = -\iint (a_{55}W\frac{\partial^2 W^T}{\partial x^2} + a_{44}\lambda_1^2 W\frac{\partial^2 W^T}{\partial\theta^2} - a_{22}\lambda_1^2 WW^T)\,dxd\theta$$

$$K_{\mathrm{wx}} = -(\frac{a_{55}}{\lambda_3} - b_{12}\lambda_1) \iint W\frac{\partial \Phi_x^T}{\partial x}\,dxd\theta$$

$$K_{\mathrm{w}\,\theta} = -(a_{44}\frac{\lambda_1}{\lambda_3} - b_{22}\lambda_1^2) \iint W\frac{\partial \Phi_\theta^T}{\partial\theta}\,dxd\theta$$

$$K_{\mathrm{xx}} = -\iint (d_{11}\Phi_x\frac{\partial^2 \Phi_x^T}{\partial x^2} + d_{66}\lambda_1^2 \Phi_x\frac{\partial^2 \Phi_x^T}{\partial\theta^2} - \frac{a_{55}}{\lambda_3^2}\Phi_x\Phi_x^T)\,dxd\theta$$

$$K_{\mathrm{x}\,\theta} = -(d_{12} + d_{66})\lambda_1 \iint \Phi_x\frac{\partial^2 \Phi_\theta^T}{\partial x\partial\theta}\,dxd\theta$$

$$K_{\theta\theta} = -\iint (d_{66}\Phi_\theta \frac{\partial^2\Phi_\theta^T}{\partial x^2} + d_{22}\lambda_1^2\Phi_\theta \frac{\partial^2\Phi_\theta^T}{\partial\theta^2} - \frac{a_{44}}{\lambda_3^2}\Phi_\theta\Phi_\theta^T)\, dxd\theta$$

$$K_{\mathrm{uw}}^{(2)} = -\iint U \frac{\partial W^T}{\partial x} r(a_{11}\lambda_3 \frac{\partial^2 W^T}{\partial x^2} + a_{66}\lambda_1\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2})\, dxd\theta - (a_{12}+a_{66})\lambda_1\lambda_2 \iint U \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial^2 W^T}{\partial x\partial\theta} dxd\theta$$

$$K_{\mathrm{vw}}^{(2)} = -(a_{12}+a_{66})\lambda_2 \iint V \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial^2 W^T}{\partial x\partial\theta} dxd\theta - \iint V \frac{\partial W^T}{\partial\theta} r(a_{66}\lambda_2 \frac{\partial^2 W^T}{\partial x^2} + a_{22}\lambda_1^2\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2})\, dxd\theta$$

$$K_{\mathrm{wu}}^{(2)} = -\iint W \frac{\partial W^T}{\partial x} r(a_{11}\lambda_3 \frac{\partial^2 U^T}{\partial x^2} + a_{66}\lambda_1\lambda_2 \frac{\partial^2 U^T}{\partial\theta^2}) dxd\theta - (a_{12}+a_{66})\lambda_1\lambda_2 \iint W \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial^2 U^T}{\partial x\partial\theta} dxd\theta$$
$$- \iint W(a_{11}\lambda_3 \frac{\partial^2 W^T}{\partial x^2} + a_{12}\lambda_1\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} \frac{\partial U^T}{\partial x}\, dxd\theta - 2a_{66}\lambda_1\lambda_2 \iint W \frac{\partial^2 W^T}{\partial x\partial\theta} \mathrm{r} \frac{\partial U^T}{\partial\theta} dxd\theta$$

$$K_{\mathrm{wv}}^{(2)} = -(a_{12}+a_{66})\lambda_2 \iint W \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial^2 V^T}{\partial x\partial\theta} dxd\theta - \iint W \frac{\partial W^T}{\partial\theta} r(a_{66}\lambda_2 \frac{\partial^2 V^T}{\partial x^2} + a_{22}\lambda_1^2\lambda_2 \frac{\partial^2 V^T}{\partial\theta^2}) dxd\theta$$
$$-2a_{66}\lambda_2 \iint W \frac{\partial^2 W^T}{\partial x\partial\theta} \mathrm{r} \frac{\partial V^T}{\partial x}\, dxd\theta - \iint W(a_{12}\lambda_2 \frac{\partial^2 W^T}{\partial x^2} + a_{22}\lambda_1^2\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} \frac{\partial V^T}{\partial\theta} dxd\theta$$

$$K_{\mathrm{ww}}^{(2)} = -\frac{a_{12}}{2}\lambda_2 \iint W \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial W^T}{\partial x}\, dxd\theta - \frac{a_{22}}{2}\lambda_1^2\lambda_2 \iint W \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial W^T}{\partial\theta} dxd\theta$$
$$- \iint W(a_{12}\lambda_2 \frac{\partial^2 W^T}{\partial x^2} + a_{22}\lambda_1^2\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} W^T\, dxd\theta$$

$$K_{\mathrm{wx}}^{(2)} = -\iint W \frac{\partial W^T}{\partial x} r(b_{11}\lambda_3 \frac{\partial^2 \Phi_x^T}{\partial x^2} + b_{66}\lambda_1\lambda_2 \frac{\partial^2 \Phi_x^T}{\partial\theta^2})\, dxd\theta - (b_{12}+b_{66})\lambda_1\lambda_2 \iint W \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial^2 \Phi_x^T}{\partial x\partial\theta} dxd\theta$$
$$- \iint W(b_{11}\lambda_3 \frac{\partial^2 W^T}{\partial x^2} + b_{12}\lambda_1\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} \frac{\partial \Phi_x^T}{\partial x}\, dxd\theta - 2b_{66}\lambda_1\lambda_2 \iint W \frac{\partial^2 W^T}{\partial x\partial\theta} \mathrm{r} \frac{\partial \Phi_x^T}{\partial\theta} dxd\theta$$

$$K_{\mathrm{w}\theta}^{(2)} = -(b_{12}+b_{66})\lambda_2 \iint W \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial^2 \Phi_\theta^T}{\partial x\partial\theta} dxd\theta - \iint W \frac{\partial W^T}{\partial\theta} \mathrm{r}\, (b_{66}\lambda_2 \frac{\partial^2 \Phi_\theta^T}{\partial x^2} + b_{22}\lambda_1^2\lambda_2 \frac{\partial^2 \Phi_\theta^T}{\partial\theta^2}) dxd\theta$$
$$-2b_{66}\lambda_2 \iint W \frac{\partial^2 W^T}{\partial x\partial\theta} \mathrm{r} \frac{\partial \Phi_\theta^T}{\partial x}\, dxd\theta - \iint W(b_{12}\lambda_2 \frac{\partial^2 W^T}{\partial x^2} + b_{22}\lambda_1^2\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} \frac{\partial \Phi_\theta^T}{\partial\theta} dxd\theta$$

$$K_{\mathrm{xw}}^{(2)} = -\iint \Phi_x \frac{\partial W^T}{\partial x} r(b_{11}\lambda_3 \frac{\partial^2 W^T}{\partial x^2} + b_{66}\lambda_1\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) dxd\theta - (b_{12}+b_{66})\lambda_1\lambda_2 \iint \Phi_x \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial^2 W^T}{\partial x\partial\theta} dxd\theta$$

$$K_{\theta\mathrm{w}}^{(2)} = -(b_{12}+b_{66})\lambda_2 \iint \Phi_\theta \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial^2 W^T}{\partial x\partial\theta} dxd\theta - \iint \Phi_\theta \frac{\partial W^T}{\partial\theta} r(b_{66}\lambda_2 \frac{\partial^2 W^T}{\partial x^2} + b_{22}\lambda_1^2\lambda_2 \frac{\partial^2 W^T}{\partial\theta^2}) dxd\theta$$

$$K_{\mathrm{ww}}^{(3)} = -\iint W(\frac{3a_{11}}{2}\lambda_3^2 \frac{\partial^2 W^T}{\partial x^2} + \frac{a_{12}+2a_{66}}{2}\lambda_2^2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial W^T}{\partial x}\, dxd\theta$$
$$- \iint W(\frac{a_{12}+2a_{66}}{2}\lambda_2^2 \frac{\partial^2 W^T}{\partial x^2} + \frac{3a_{22}}{2}\lambda_1^2\lambda_2^2 \frac{\partial^2 W^T}{\partial\theta^2}) \mathrm{r} \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial W^T}{\partial\theta} dxd\theta$$
$$-(2a_{12}+4a_{66})\lambda_2^2 \iint W \frac{\partial W^T}{\partial x} \mathrm{r} \frac{\partial W^T}{\partial\theta} \mathrm{r} \frac{\partial^2 W^T}{\partial x\partial\theta} dxd\theta$$

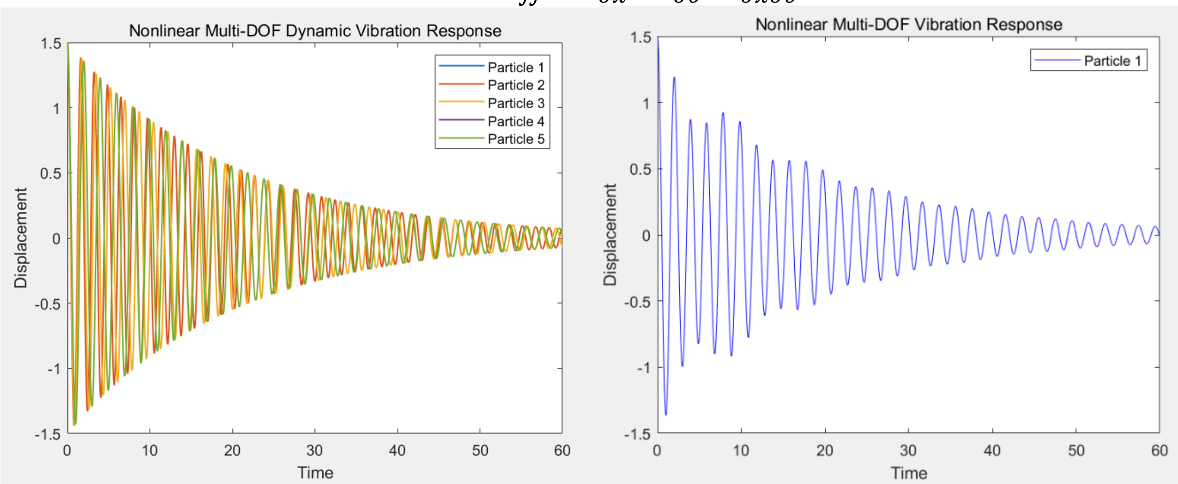


Fig.3.Nonlinear Multi-DOF Dynamic Vibration Response

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