Study on Caster Angle and Steering Stability of Racing Car

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Abstract. The topic in this article is the influence of the caster angle of the automobile. High speed and large steering is a working condition that a racing car must experience, this paper is eager to find the law of the influence of caster angle of kingpin on steering instability under high speed and large steering condition. The caster angle of the Kingpin is of great significance to the steering of a vehicle. On the one hand, it determines the stability of a vehicle when it is running in a straight line, and on the other hand, it determines the return force of the steering wheel when it is turning. In contrast, a smaller caster angle makes steering easier, but less stable at high speeds, while a larger caster angle means a slower steering with a more pronounced return to normal but better stability at high speeds, the determination of the magnitude of the caster angle is very important for the ride comfort of the vehicle.

Keywords: formula racing car, caster angle, steering stability of a vehicle, pneumatic trail, virtual test, ADMAS.

1. Introduction

The topic in this article is the influence of the caster angle of the automobile on the balance of the automobile. High speed and large steering is a working condition that a racing car must experience, this paper is eager to find the law of the influence of caster angle on steering instability under high speed and large steering condition.

The caster angle of is of great significance to the steering of a vehicle. On one hand, it determines the stability of a vehicle when it is running in a straight line, and on the other hand, it determines the return force of the steering wheel when it is turning. In contrast, a smaller caster angle makes steering easier, but less stable at high speeds, while a larger caster angle means a slower steering with a more pronounced return to normal but better stability at high speeds, the determination of the magnitude of the caster angle is very important for the ride comfort of the vehicle.

To study the effect of front wheel kingpin alignment parameters on the vehicle steering stability, we simplified the vehicle as four degrees of freedom dynamic model, and established the state differential equation. Based on python, the yaw rate transient responses was simulated under force step input, the effect of front wheel kingpin alignment parameters on the vehicle steering stability was analyzed under different vehicle speed. The results show that the change of front wheel caster angle mainly affects vehicle stability at high speeds, both yaw rate over-shoot and reaction time tend to reduce with the increase of caster angle, while the change of front wheel inclination mainly affects vehicle stability at low speed, smaller inclination angle and shorter reaction time.

In 1993, LEE U et al. established the suspension model by applying the multi-body dynamics theory and solved the kingpin position parameters of the multi-link suspension[11]. In 2000, MATSCHINSKY W of Germany derived the relation of suspension wheel jump motion by using virtual work principle, and established the 6-dOF displacement model of the kingpin and wheel in the process of in-situ steering, providing a new theoretical method for identification of the position parameters of the main pin[2]. In 2001, Ragnar Ledesma and Shan Shih studied the influence of kingpin inroll Angle and kingpin offset distance on the rightward moment of medium truck, and compared the influence process of kingpin inroll Angle and kingpin offset distance on rightward moment based on Adams. Skip Essma in 2002 discussed the performance of steering force of racing cars on elliptical track based on Adams research, and analyzed the influence of camber and kingpin camber Angle changes on steering positive moment. It was found that the decrease of camber and kingpin camber Angle may lead to the decrease of steering force, but the decrease of kingpin camber
Angle may lead to front wheel yaw\cite{3}. In 2009, Y. G. Cho described the position relationship between the kingpin axis and the wheel in detail, analyzed the force of the tire under low-speed steering, and calculated the positive torque generated by the forces in each direction relative to the kingpin. However, when calculating the overroll lateral force, it did not consider the dynamic change of the inclination angle during the steering process\cite{5}. In 2012, Reza N Jazar et al. studied suspension kinematics with variable kingpin rear angle based on screw theory, and proposed the mathematical equation needed to calculate camber angle, which was used as a function of suspension mechanism parameters and other related variables\cite{5}. In 2016, Quoc V D et al. established a kingpin-tire motion model considering the instantaneous ground contact point of tires, and then calculated the kingpin shaft torque caused by tire forces according to the kinematic relationship, and compared the analysis results with the test results of the multi-body model for verification\cite{6}.

2. Background

Next, the author will introduce some physics concepts related to this study and the derivation of related formulas.

1. Camber angle\cite{3,5}: camber angle is the angle between the vertical plane and the plane of the wheel when the wheel is installed. Because the direction of force is not exactly the same as the rolling direction of the tire, Figure 1 is a schematic diagram for camber force.

![Figure 1. Sketch of vehicle subjected to ground force](image)

2. Kingpin caster angle\cite{3}: in the vertical plane, the angle between the kingpin axis and the vertical line is called Kingpin caster angle.

3. Righting Moment\cite{3}: Righting Moment is the moment of force acting on the OZ axis of the tire when the tire is cornering. When driving in a circle, the righting moment is one of the main torques to restore the wheel to its straight running position. The righting moment is produced by the lateral reaction of micro elements distributed in the ground plane.

4. Vehicle handling stability\cite{5}: refers to the driver does not feel too nervous, fatigue conditions, the car can follow the driver through the steering system and steering wheel given direction (straight or turn); And when the external interference (road uneven, crosswind, goods or passengers partial load), the car can resist interference and maintain stable performance.

5. Vehicle Ride\cite{4}: refers to the vehicle in the general driving speed range, to avoid the vibration and impact caused by the vehicle in the course of driving, making people feel uncomfortable, fatigue, and even damage to health, or cause damage to the goods. As ride comfort is mainly based on the evaluation of the degree of comfort of passengers, it is also known as ride comfort, it is one of the main performance of modern high-speed vehicles.

The model is used in the virtual experiment under the special track condition, and the fitting effect is good.
However, the model has its own limitations, such as the error in the semi-empirical formula, the car is equivalent to a two-degree-of-freedom system, and the existence of the lateral force caused by the camber angle of the car tire is not taken into account, the tire load transfer is also not taken into account in the model. Therefore, by modifying the two-degree-of-freedom model of tire, a more accurate formula for the relationship between caster angle and steering speed can be derived.

$$m_F a_y H = \frac{1}{2} \left( m_{FL} - m_{FR} \right) g L$$

$$m_R a_y H = \frac{1}{2} \left( m_{RL} - m_{RR} \right) g L$$

A modified two-degree-of-freedom model is obtained by introducing the lateral force of camber:

$$k_1 (m_T - m_R) \gamma + k_2 (m_T - m_R) \beta + \frac{1}{u} (a k_1 - b k_2) \omega - k_3 \delta = m v^* + u \omega$$

$$a k_1 (m_T - m_R) \gamma - b k_2 (m_T - m_R) \beta + \frac{1}{u} (a' k_1 + b' k_2) \omega - a k_3 \delta = I_1 \omega'$$

The axle load relation is introduced when the vehicle is tilting, and the lateral force of the four wheels is brought in. Through this modified two-degree-of-freedom model, we propose a more accurate formula for the relationship between caster angle and steering speed.

To determine the relationship between the caster angle of the Kingpin and the maximum stable steering speed, a two-degree-of-freedom linear vehicle dynamic equation with lateral and yaw motion around the center of mass is established:

$$(k_1 + k_2) \beta + \frac{1}{v} (a k_1 - b k_2) \omega - k_3 \delta = m \frac{d \beta}{dt} + \omega$$

$$(a k_1 + b k_2) \beta + \frac{1}{v} (a' k_1 + b' k_2) \omega - a k_3 \delta = m a b \frac{d \omega}{dt}$$

The steering system is simplified as the rotation around the front wheel kingpin, and the friction and viscous resistance moments of the system are ignored:

$$2 \xi k_1 \left( \beta + \frac{a \omega}{v} \right) - \dot{\delta} + I_\delta \frac{d^2 \delta}{dt^2} = T_\omega$$

The front wheel drag distance is composed of the front pneumatic trail distance and the caster drag distance. The front wheel drag distance $\xi$ is
\[ \xi = \xi_i + \mathbf{R} \sin \alpha \]  
(5)

The expression (2.7) is substituted for the expression (2.6)\(^{[11]}\)

\[ 2 \xi \frac{\delta}{\xi + \mathbf{R} \sin \alpha} k_{1} (\beta - \delta) + 2 \frac{a k_{1}}{\xi + \mathbf{R} \sin \alpha} \phi + I_{1} \frac{d^{2} \delta}{d t^{2}} = T_{a} \]  
(6)

Take the steering System dynamics equation as a complement to the vehicle dynamics equation, take the 1,4 equations as a set of equations, apply the Laplace transform to them to get the characteristic equation\(^{[11]}\):

\[ 2 \left( \frac{\xi}{\xi + \mathbf{R} \sin \alpha} \right) k_{1} (\beta - \delta) + 2 \frac{a k_{1}}{\xi + \mathbf{R} \sin \alpha} \phi + \frac{I_{1}}{2 (\xi + \mathbf{R} \sin \alpha)} \]  
(7)

3. Sample vehicle calculation and simulation analysis based on python

Using the Routh-hurwitz stability criterion, the matching expression between the caster angle of the Kingpin and the maximum steady speed of the vehicle steering is obtained:

\[ \alpha = \arcsin \left( \frac{I_{1} (k_{1} a + k_{2} b)}{k_{mab} \mathbf{R}} \left(2 - \frac{k_{1} I_{1}^2}{m_{a} v_{a}^2} \right) \frac{\xi}{\mathbf{R}} \right) \]  
(8)

<table>
<thead>
<tr>
<th>m/kg</th>
<th>a/mm</th>
<th>b/mm</th>
<th>k1/N*rad(^{-1})</th>
<th>k2/N*rad(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>1350</td>
<td>1300</td>
<td>44950</td>
<td>55060</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>l/mm</th>
<th>I_{\delta}</th>
<th>\xi /mm</th>
<th>R/mm</th>
<th>v/km*h(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2650</td>
<td>30</td>
<td>-5−20</td>
<td>370</td>
<td>40−140</td>
</tr>
</tbody>
</table>

The Table 1 shows the data related to this sample car. This formula is programmed in Python and plotted with the data of the vehicle. The changing image of the caster angle of the Kingpin and the maximum speed of the vehicle can be obtained:

**Figure 3. relationship between caster Angle and maximum steering steady speed**

From the above Figure 3 can be seen, the car kingpin tilt angle size and speed is positively correlated, but when the speed is low. When the speed is high, the change of caster angle of kingpin tends to smooth.

Taking the test data of this model as an example, the design value of the caster angle of the front wheel kingpin is 2.0° ± 20’. When the speed is 27M/s, that is, 97.2 km/h, the value of the caster angle reaches the upper limit of the design value. Therefore, it can be judged that the safety of steering can be ensured when the driving speed is below 97.2 km/h.

Therefore, through this model, we can bring in the relevant values of different models to judge the vehicle’s maximum steering stable speed under the Kingpin caster design specification. Here is the relevant python code in Figure 4.
For the car, it only runs on a fixed track and the working condition is fixed, so the design of the kingpin rear angle Angle of the car should only meet the steering stability of the car on this track. Taking a formula car as an example, the speed data collected by the on-board information system during the race on the track is shown in Figure 5.

According to the specific position of the track, the speed of each corner can be known, as shown in Table 2. Here, the deceleration before entering the corner and the acceleration at exiting the corner are ignored, and the average speed is taken as the speed when passing the corner.

<table>
<thead>
<tr>
<th>corner</th>
<th>velocity v/km*h⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
</tr>
</tbody>
</table>

By substituting the data of Table 1 and 2 into Equation (2.10), the kingpin dip Angle that meets the steering stability of each bend can be obtained, as shown in Table 3.

<table>
<thead>
<tr>
<th>corner</th>
<th>velocity v/km*h⁻¹</th>
<th>caster angle α/°</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>1.56</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>2.91</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>0.84</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>2.34</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>4.68</td>
</tr>
</tbody>
</table>

As shown in Table 3, the minimum caster angle required to ensure stable steering through each turn on the track is 4.68°. The car is designed to maintain optimum handling on the track. If the caster angle is designed to be less than 4.68°, it may result in erratic steering through turn 5. If the caster angle design is too large, although it can meet the requirements of steering stability, it will also cause the problem of heavy steering.
4. The establishment and verification of virtual sample car model

Due to the limitation of research conditions, it is of great practical significance to use virtual prototype method to model and simulate the racing car, and to predict and optimize its performance. Among many simulation software, ADAMS based on multi-rigid body dynamics simulation software is the most widely used engineering software. It is widely used in the research of automobile dynamics and crosswind stability.

Virtual prototype model of racing car was established based on ADAMS, including subsystems such as front and rear suspension, steering, tire, body and power system, etc. Structure and position parameters were obtained from 3d design model, suspension shock absorber and tire characteristics were determined by test data provided by manufacturers. Nonlinear mechanical properties of tires are expressed by magic formula widely used in engineering [10]. Due to space limitations, the specific modeling process will not be described. The completed vehicle model is shown in Figure 6.

![Figure 6. Racing car virtual prototype vehicle model](image)

Based on the established virtual prototype model in the figure 6, the steering wheel Angle step simulation is carried out. The test speed was 100 km/h, the final steering Angle was 30°, the starting time was 1 s, and the step action time was 0.1 s. The comparison between simulation results and test results is shown in Figure 7.

![Figure 7. Comparison of experimental result and simulation result](image)

As shown in the Figure 7, the simulation results are in good agreement with the test results, which can meet the engineering requirements.

5. Optimization design of kingpin back dip Angle

Taking turning 5 as an example, the speed of the car is 80km/h at this moment. It can be seen from Table 3 that the kingpin inclination Angle to maintain steering stability at this moment is at least 4.68°, while the original kingpin inclination Angle of the car is 3.82°. According to theoretical calculation, the car is in an unstable steering state at this moment.

Based on the vehicle simulation prototype model, the steering wheel Angle step simulation was carried out with the original car model with the kingpin Angle of 3.82° and the racing car model with the kingpin Angle of 5° respectively to verify the accuracy of the theoretical calculation. The driving speed is 80km/h, and the steering wheel jump time is 1s. The comparison figure of simulation results is shown in Figure 5, 6 and 7.
Figure 8. Yaw velocity comparison

Figure 8 shows that the yaw velocity response overshoot of the original car model is 127.6%, while the overshoot of the modified racing car model is only 100.6%, which verifies the accuracy of the theoretical calculation.

Figure 9. Roll angle comparison

As can be seen from Figure 9, the maximum response of the original vehicle's roll angle during step steering is 2.5%, and the maximum response of the modified vehicle's roll angle is 1.8°, indicating that the original vehicle's roll angle response is larger, which also verifies the accuracy of the theoretical calculation.

Figure 10. Lateral force comparison

Figure 10 shows that the tire lateral force fluctuates greatly during the step steering of 80km/h, resulting in the instability of steering. However, the tire lateral force of the modified racing model changes steadily and steering is stable.

6. Conclusion

In this paper, the author uses a linear 2-DOF vehicle model to obtain the matching mathematical relation between kingpin angle and steering speed. Based on the difference between the car and the civil car, aiming at ensuring the steering stability of the car on this track, through analyzing the speed data collected by the on-board information system in the race, the design method of the kingpin tilt angle of the car for a specific track was proposed.

The accuracy of theoretical analysis was verified by virtual prototype simulation. The simulation results show that when the racing car turns at the speed of 80 km/h, the steering instability will occur if the kingpin tilt A angle is less than 4.68°, and the steering stability is good when the kingpin tilt A angle is greater than 4.68°, which verifies the correctness of the theoretical analysis.

In the design of civil car, the performance of the car should be guaranteed under various working conditions, while the car only runs on a specific track, and the working conditions are relatively fixed, so the design of the car should be based on the performance on a specific track, this paper provides a reference.

Of course, the following points in the article need to be improved:
1. The two-degree-of-freedom model neglects the role of the suspension, and the vehicle is simplified as two wheels. It considers that the tire cornering characteristics are linear, and it neglects the longitudinal drive or resistance, and it considers that the longitudinal speed remains constant, the error of the empirical formula will affect the accuracy of the calculation.

2. Ignoring the suspension function, it is considered that the car body only moves parallel to the ground, that is, the displacement of the car along the z axis, the pitch angle around the y axis and the roll angle around the x axis are both zero.

3. The lateral acceleration is limited below 0.4 g to ensure that the tire cornering characteristics are in a linear range.

REFERENCES


