Improvement for the Dynamics of Formula One Car Based on an Analysis of Center of Mass

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Abstract. Formula one has always been the culmination of motorsport, but not necessarily the end of it. As the rapid development of technology, the vehicle attains more advanced equipment and the F1 lap's times have improved year by year, meanwhile, the safety and stability of vehicle become even more significant. The center of gravity (CG) is a crucial factor when consider the stability. Rollover could be happened when vehicle turns too fast. When vehicle turns, the weight transfer happens and the location of CG will change according to the weight transfer, therefore torque been generated in order to attain better performance of stability, it is necessary to calculate the accurate CG location. But as mentioned the location CG could change when F1 moving, it is hard to acquire its value. In this study, the location of CG will be analyzed for improving the dynamics of F1.

Keywords: Center of mass, Wheelbase, Dynamics, F1 racing car.

1. INTRODUCTION

The Formula One racing car, is well known for its excellent performance in accelerating and cornering, represents the highest class of auto racing. The word "formula" in its name refers to the regulations the FIA imposes on all competitors and their race cars must obey [1]. To achieve higher speed in racing, spoilers are used to create extra aerodynamic points to ground, providing the vehicle better road grip and better traction to negotiate corners at a much higher speed than normal cars. With assumption of various of advanced technology, F1 car can going from 0 to 160 km/h (0 to 99 mph) and back to 0 in less than 5 seconds. Besides great maneuverability in straight line, it also has great cornering ability.

The center of mass is a kind of point that express the distribution of mass in space. It is a unique point where the weight relative position sums to zero in physics. This is the point where forces acting on can only cause a linear acceleration but without any torque.

In vehicles common see in life, the straight distance between front and rear tires is the wheelbase of the vehicle. For some trucks which has more tires, the distance between front steer angle and the center point is wheelbase of these kinds of vehicles [2].Wheelbase filtering is an effect which is caused by a time delay of front wheel input relative to rear one [3]. Qu Xiaofeng [4] found that lowering the front wheel cornering stiffness while higher the rear wheel cornering stiffness may reduce the yaw moment of inertia. Designing height of center of mass with reason can make sure that vehicle has a good handling stability and keep driving safety. The present study is focused on dynamic analysis based on forces variation and load transfer in accelerating, breaking, and cornering conditions.

2. METHOD

To analyze dynamic of vehicle, we need to use mathematical methods to calculate geometry conditions of vehicle, and conclude factors which may influence stability so we can optimize structure design of the vehicle. The center of mass can be showed as a point which can be represented the average position of an object or a system. And the center of mass cab be calculated by using the following equation [5].

$$x = \frac{(x_1 m_1 + x_2 \mathrm{m}_2 + x_3 m_3 + \cdots)}{(m_1 + m_2 + m_3 + \cdots)} \tag{1}$$

If the weight is carried up high, for example a panoramic sunroof will lead to higher vehicle CG. While placing heavy part low in the vehicle, such as the battery pack, is helpful to lower it. From the handling standpoint, lower height is better to reduce possibility to roll over. And weight transfer during cornering and breaking can be improved. Unfortunately, automakers don't publish CG-height data, one more equation is needed to calculate the data:

$$h_{cg} = R_F\left(\frac{a}{l}\right) + R_R\left(\frac{b}{l}\right) + \left(\frac{W_F - Wa}{Wtan\theta}\right) \tag{2}$$

First [6], we need to start by measuring the tire heights $\mathbf{R_F}$ and $\mathbf{R_R}$, and **W** is weight of vehicle. Considering the wheelbase **l** and weight distribution, the length **a** and **b** of the center of mass can be calculated. $\mathbf{W_F}$ is weight of the front axle. The angle **θ** can be computed by using the triangle which formed by the wheelbase and the height the rear wheels are raised. Plugging all of this into the formula can get height of center of mass.

3. DYNAMIC ANALYSIS

3.1. Center of mass analysis

The location of center of mass and the wheelbase have played a significant role for the car dynamics, and a small change of it can cause huge difference of the car dynamic. For example, the F1 racing car has lower center of mass and larger wheelbase compared to the family-using car, because those two types of cars have different design purposes, and their requirement of car dynamics are different. F1 racing cars need to be faster and more stable when the car is changing the direction. In order to attain the requirements, there are many factors need to be considered: the air friction, the torque, the inertia, the engine placement and so on. After settling down all the factors, the F1 racing car can only be what it is looking like right now.

Long wheelbase can ensure the car`s stability and will not be easy to flip when the car turning the direction at a high speed.

3.2. Force analysis

As a common accident happens in every corner of the world, vehicles rollover accident brings amounts of loss and casualties every year, it's a risk to human safety and social stability. On 30 April 2022 on 22nd street, Sakastoon, a vehicle rollover for being crashed [6]. On December 9th, 2020, a vehicle rollover and engulfed with fire on highway for ran off the road [8]. Such dangerous accidents are quite common to see in our life and creates huge hidden danger. We need to pay attention to the origins that the vehicle rollover and try to take measures to avoid it.

Vehicle rollovers are divided into two categories: tripped and unstripped. Tripped rollovers are caused by forces from an external object, such as a curb or a collision with another vehicle, this is a condition when sudden load acting at the lower part of the vehicle. Unstripped rollovers are the result of cornering input, velocity, and ground friction. While the vehicle turns, the cornering forces from the tire push the vehicle towards the center of the curve. Ignoring the air drag, we mainly research on the forces come from tires.

3.2.1 Unstripped rollovers

Tires produce a force vector point to the center of round, is known as centripetal force, as shown in Figure 1. But as a result of this vector not going through the center of mass, it produces a torque which make the vehicle trends to rollover, which may create an angular acceleration trends the vehicle rollover. To analyze this problem easier, we can use the D'Alembert's principle. Which we are adding the so-called “inertial force” and “inertial torque”, points to the opposite site of the acceleration. So, we can analyze the dynamics problem under a statics condition.

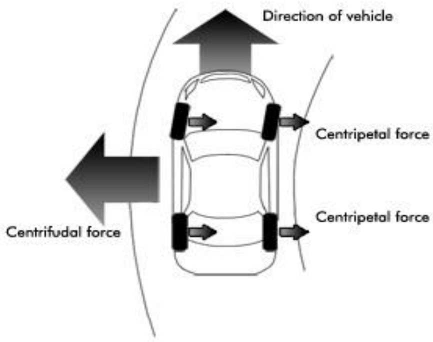

Figure 1. Forces condition

We can use the following formula to count centripetal force:

$$F_n = m\frac{v^2}{r} \tag{3}$$

where m stands for the mass of vehicle, r is the turning radius and v is velocity of the car. According to the formula (3.1) we can find that if the radius is smaller or velocity is higher, F_n will increase.

3.2.2 Rollover torque

The formula to calculate torque is:

$$M = \vec{F} \times \vec{r} \tag{4}$$

where r is a vector points to center of mass, and F is the inertial force. This torque gives the vehicle a trend to roll to the outside of round. We have another torque here to make sure vehicle is stable. Where F is the gravity of vehicle and r is half of width of vehicle. This torque is opposite to the one above, it avoids the vehicle to rollover. But as the weight of vehicle is roughly a constant, when the tire and inertial forces are enough to overcome the force of gravity, the vehicle starts to turn over.

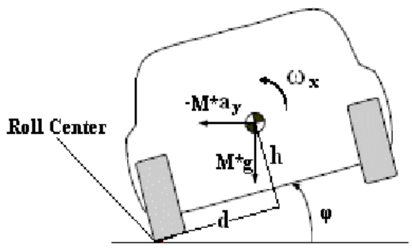

Figure 2. Forces acting on mass

According to analyze on geometry conditions, we define lean angle as φ, width of car as d, as shown in Figure 2 Rollover torque depends on d/2*sinφ and balance torque depends on d/2*cosφ, if vehicle lean to one side due to the rollover torque, rollover torque will keep increase while balance torque reduce, vehicle rollovers at a very fast speed. We judge the rollover happens once one side of tires leave the road.

3.2.3 Steer angle and centripetal force

To avoid the problem of rollover torque bigger than balance torque, relation between the vehicle dynamics and centripetal force should be analyze, we need to get the relation between steering angle and centripetal force. From the following picture, we can assume the forward speed u approximately equals to V, as shown in Figure 3.

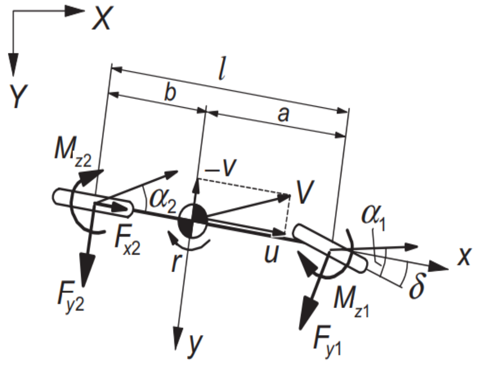

Figure 3. Relation between V and u

We can get the equation of motion by law of conservation of momentum:

$$\mathrm{m}(\dot{v}+ur)=F_{y1}+F_{y2} \tag{5}$$

$$I\dot{r}=aF_{y1}-bF_{y2} \tag{6}$$

where v denotes the lateral velocity of the center of gravity and r is the yaw velocity. I is the moment of inertia about the vertical axis through the center of gravity. The side forces are functions of the respective slip angles:

$$F_f=C_f\alpha_f \tag{7}$$

$$F_r=C_r\alpha_r \tag{8}$$

where Cf denotes the front tire cornering stiffness and Cr denotes rear tire cornering stiffness. The two linear first-order differential equations now read:

$$\begin{aligned}&m\dot{v}+\frac{1}{u}(C_1+C_2)v+\{mu+\frac{1}{u}(aC_1-bC_2)\}r=C_1\delta\\&I\dot{r}+\frac{1}{u}(a^2C_1-b^2C_2)r+\frac{1}{u}(aC_1-bC_2)v=aC_1\delta\end{aligned} \tag{9}$$

After elimination of the lateral velocity v, we obtain the second-order differential equation for the yaw rate r:

$$\begin{aligned}&\mathrm{Im}\,u\ddot{r}+\{I(C_1+C_2)+m(a^2C_1+b^2C_2)\}\dot{\mathrm{r}}\\&+\frac{1}{u}\{C_1C_2l^2-mu^2(aC_1-bC_2)\}r=muaC_1\dot{\delta}+C_1C_2l\delta\end{aligned} \tag{10}$$

We have the curvature formula:

$$\frac{1}{R}=\frac{r}{V}\approx\frac{r}{u} \tag{11}$$

$$\frac{1}{R}=\frac{C_1C_2l}{C_1C_2l^2-mV^2(aC_1-bC_2)}\delta \tag{12}$$

The expression for the steering angle required to negotiate a round with a given radius R is obtained:

$$R = \frac{1}{\delta}(l - mV^2 \frac{aC_1 - bC_2}{lC_1C_2}) \tag{13}$$

Combining the formula of centripetal force, we get relation between centripetal force and steering angle:

$$F_n = \frac{C_1C_2l}{C_1C_2l^2 - mV^2(aC_1 - bC_2)} mV^2\delta \tag{14}$$

3.2.4 Tripped rollovers

This is the most common type of rollover in our life. When vehicle skid for the reason of slippery road, or side strike occurs on the vehicle, tripped rollovers are very likely to happen for the vigorously side acceleration acting on vehicle. The physics condition is similar to cornering rollovers. In a 2003 report, this was the most common mechanism, accounting for 71% of single-vehicle rollovers.

3.3. WEIGHT TRANSFER

The extreme performance of the F1 racing cars is also derived from its low susceptibility towards weight or load transfer. When cornering, weight transfers from the inside to the outside; when braking, weight transfers from the rear to the front; and when accelerating, weight transfers from the front to the rear. Weight transfer affects vehicles' general performance and, in severity, damages the vehicle.

When cornering, vertical load changes on all four tires [9]. We call this type of weight transfer lateral weight transfer. The tire on the inside loses vertical load while the outside tire gains vertical load. Traction on the inside tire decreases while the traction on the outside tire increases. Experiments have found that the relationship between load transfer on tires and the traction force of the tires is not linear. The weight decreased on the inside tires leads to the loss of traction, faster than the traction gaining on the outside tires, which means the total traction of the tires decreases after the weight transfer happens. It is impossible to eliminate weight transfer on any vehicle, but there are ways to minimize it so that vehicles especially racing ones may have as much traction as possible.

When braking, there happens another type of weight transfer called longitudinal weight transfer. When a car is accelerating, especially a rear-wheel-drive car, longitudinal weight transfer helps the acceleration for the traction gain on the drive wheels and the traction loss on front wheels, which are the ones not driving the car [10]. Conversely, for a front-wheel-drive, longitudinal weight transfer works against this principle. However, today's automobile production companies are keen to making all-wheel-drive or four-wheel-drive (typically designed for off-road vehicles for its ability to switch it on and off) models rather than RWD or FWD ones for the reason that even though RWD cars gain acceleration traction from longitudinal weight transfer, the car is only gaining slight advantage going straight at the cost of much more disadvantages around corners, which makes RWD cars not ideal when competing in lap races. AWD and 4WD vehicles perform much better in finishing time and cornering speed because traction is distributed evenly on every wheel [11]. Even when cornering while braking, the car does not lose much traction because the outside wheel that experience most weight transfer also has power from the engine due to the AWD system.

Now, cars lose traction in any non-static situation have been established, what are things we could do to avoid it? There are four factors that affect the amount of weight transferred. Nothing else affects the amount of weight transfer.

1. The total weight of the vehicle.
2. The height of the center of gravity above ground.
3. The track width and the wheelbase.

3.3.1 Total Weight

Heavier the car, the more weight transfer occurs on the tires. Although the ratio of the weight transferred on the tires to the total weight of a heavy car does not vary much from the ratio of that of a light car, the tire specs and maximum friction that it can provide have limits. Theoretically speaking, to guarantee the same cornering speed and braking distance, heavier cars require much wider and softer or more sticky tires so that they provide enough traction forces. Nevertheless, these tires on these heavy cars do more harm than good for its own weight and lack of durability. Commercial models like the PORSCHE 911 weighting as light as 3126 lbs with a set of 305 rear tires are much better at handling weight transfer than the Bentley Continental GT, which weights around 5000 lbs with also a set of 305 rear tires. Admittedly, other factors can also contribute to such performance difference.

3.3.2 Track and wheelbase length

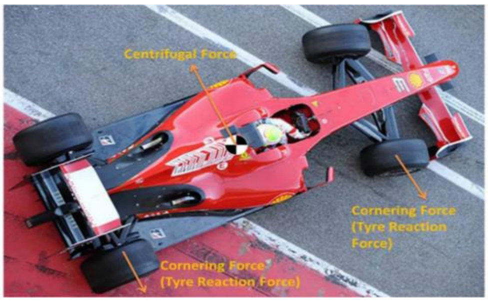

Figure 4. Forces Exerting on the Formula 1 vehicle while turning

Figure 4 shows the forces the F1 car experiences when cornering. The cornering forces exert on the tires while the centrifugal force exerts on the center of mass.

Maximum track width is one of the most important factors when considering lateral weight transfer. It is optimal to run the widest track width possible so the lateral weight transfer can be decreased. Figure 4 shows that the cornering forces from the tires causes the lateral acceleration, (Ay). The units for “Ay” are in m/sec2. For simplification, we convert it to units of g. So:

$$Ay(g) = \frac{A_y}{9.81} \tag{15}$$

From Newton’s Second Law, we can assume:

$$CF \ \left(Centrifugal\ Force\right) = -WAy \tag{16}$$

where W is the total vehicle mass and Ay is the lateral acceleration in units of g.

Figure 5. Lateral labels of A Car

When the car is static, the product of the mass on the left wheel and the track length should equal to the product of the whole vehicle weight and half the track length. In motion, however, the two values differ when the lateral weight transfer is taking place. Figure 5 shows the forces and accelerations acting on the vehicle along with length labels representing height of the center of mass and its track length.

By analyzing the dynamics of the vehicle in Figure 5, we obtain:

$$WL \times t = \left(W \times \frac{t}{2}\right) + \left(WAyh\right) \tag{17}$$

where WL is the static mass on left wheel, t is the track Width, W is the vehicle total mass (kg), Ay is the lateral acceleration in g, and h is the height of center of gravity. Simplifying eq. (15), we obtain:

$$\Delta W = WL - \frac{W}{2} = \frac{(WAYh)}{t} \tag{18}$$

Then, we use LLT to express the weight transfer ΔW

$$W:\ LLT = \frac{(Ay \times H)}{t} \tag{19}$$

This tells us that the longer the track length t, the smaller the lateral weight transfer LLT is. So, it is optimal for car designers to enlarge track length as much as possible.The longitudinal Weight transfer happens when the car accelerates or decelerates. The most effective way to reduce longitudinal weight transfer is to increase the wheelbase length.

Figure 6. Longitudinal labels of A Car

Figure 6 labels all the components we need to analyze longitudinal weight transfer while braking or accelerating.Taking moments about the front wheel we can generate the following equation:

$$\Delta Wx \times L = h \times W \times Ax \tag{20}$$

where ΔWx is the increase in the rear axle downward load and the decrease in the front axle load, L is the wheelbase of the car in meters, h is the height of the center of gravity from the ground in meters, W is total mass of the car, and Ax is the longitudinal acceleration in g force.
Simplifying, we get:

$$\Delta WX = \frac{\mathrm{h}}{L} \times W \times Ax \tag{21}$$

From eq. (19), we conclude that the longer the wheelbase, the smaller the longitudinal weight transfer is. From eq. (17), we conclude that if the track length stays constant, the lower the center of mass is, the smaller the lateral weight transfer is. From eq. (19), we also conclude the same for the longitudinal weight transfer.

4. Conclusions

We calculated a numerical solution on the center of mass of Formula one racing car, and we derived methods to improve the vehicle dynamic stability in cornering, breaking and accelerating condition. The detailed numerical results suggested the following:

Center of mass of the vehicle may significantly affect dynamic feature. This is a unique point where any force can equivalently act on, and it can be simplified as combination of a force vector and a torque vector. A low center of mass facilitates a smoother acceleration and a larger cornering speed.

Lowering the height of center of mass will reduce the torque generated by cornering or acceleration changing. And reduce possibility for vehicle to rollover in both tripped and unstripped condition. It is also helpful to reduce load transfer for better performance on accelerating, braking, and cornering, transferring less load on the front tires that do not drive the vehicle, resulting in better power to speed efficiency.

Increasing track and wheelbase length is also one of the ameliorated measures to optimize dynamic feature in a high-speed scenario. Track length is helpful for strengthening the balance torque while cornering, the longer the track length is, the smaller possibility to rollover. Weight transfer will be improved with longer track and wheelbase length for the same reason that the torque on each wheel has less variation, thus more power-speed or power-handling efficiency. Vehicle has better ground grip while cornering at high speed.

Conclusions derived from research above gives basic methods on improving dynamic reaction and racing performance of Formula one racing car. It would be helpful to design better mechanical structure and optimize mass distribute. Using such conclusion on cargo truck can significantly reduce possibility on taking place of such rollover or side slip accidents.

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