

Prediction of Shock Wave Produced by Small Bumps in Pipe Based on Small Disturbance Equation

Zheng Luo^{1,* ,†}, Wenna Wu^{2,†}

¹ School of Aerospace Engineering, Beijing Institute of Technology, Beijing, China

² School of Science, Chang'an University, Xi'an, China

* Corresponding Author Email: 1120193165@bit.edu.cn

[†]These authors contributed equally

Abstract. The study of shock wave structure generated by small bumps in pipe has great significance in engineering applications such as ramjet design and maintenance. In this paper, the transonic and supersonic inviscid steady flow in a two-dimensional pipe with a bump on one side of the wall or symmetrical bumps on both sides is solved based on a simplified form of modified small disturbance equation (MSD). Gauss-Seidel method is used for iteration. Then the shock wave structure in transonic and supersonic case is analyzed. The paper points out the typical characteristics of shock waves generated in pipe such as normal shock wave, oblique shock wave, λ -shaped shock wave and Mach disk. The calculation method is efficient and simple to implement, enabling rapid prediction of shock wave structure and providing reference for the analysis of flow in pipes.

Keywords: Two-dimensional pipe; Wall bump; Small disturbance equation; Transonic flow; Supersonic flow.

1. Introduction

The presence of flaws such as bumps, on the inner surface of supersonic pipes caused by wear or machining error, will inevitably generate shock wave that can affect the flow quality and even damage the pipe. Therefore, study of shock wave structure generated by bumps is important for the design and maintenance of ramjet engines or supersonic wind tunnels. Luis Eca et al. [1] studied the inviscid flow in a 2D pipe with a bump on one side of the wall in subsonic, transonic and supersonic cases. Z. Z. Sun et al. [2] studied the interaction between transonic shock wave and boundary layer. Y. Wu et al. [3] used the GAO-YONG compressible turbulence equation to simulate the turbulent boundary layer disturbances in the transonic flow field of the Delery pipe. The results showed that the typical phenomena caused by interaction between turbulent boundary layer and shock wave such as the " λ " shock wave was well simulated.

Solving flow in the 2D pipe with bumps based on small disturbance equation can quickly predict the location and structure of shock wave. It ignores the interaction between turbulent boundary layer and shock wave. In transonic and supersonic flow, both the leading and trailing edge points of a bump can generate shock waves. The small disturbance equation can be simplified to the modified small disturbance equation (MSD).[4] Due to the large shock angle in transonic flow, the reflection of shock wave generated by the leading point on the other side of the wall creates a typical " λ " structure and interacts with the shock wave generated by the trailing edge point. Therefore, the transonic flow field is very complex. For supersonic flow, the shock waves generated by the two points are parallel to each other and do not interfere with each other, so the flow field is relatively simple.

This paper presents a numerical simulation of transonic and supersonic inviscid steady flow in a 2D pipe with a bump on one side of the wall or symmetrical bumps on both sides. Calculation is based on a simplified form of modified small disturbance equation and the iteration method is Gauss-Seidel method, then analyze the structure of the shock waves. The results show the variation of the flow field and shock wave structure as the incoming Mach number increases. Our method is simple

and easy to implement and can quickly predict the shock wave structure, providing a reference for the analysis of flow in pipes.

2. Methods

2.1. Solution method

A large number of numerical simulations have confirmed the superiority of the modified small disturbance equation (MSD) whether it is conservative form not.[5] Wang Z Q's results confirmed the superior performance of MSD equation in capturing shock waves.[4] When the incoming Mach number is in the X direction, the two-dimensional MSD equation is

$$\left[1 - M_\infty^2 - \frac{\gamma + 1}{U_\infty} M_\infty^2 \frac{\partial \varphi}{\partial x}\right] \frac{\partial^2 \varphi}{\partial x^2} + \left[1 - \frac{\gamma - 1}{U_\infty} M_\infty^2 \frac{\partial \varphi}{\partial x}\right] \frac{\partial^2 \varphi}{\partial y^2} - \frac{2}{U_\infty} M_\infty^2 \frac{\partial \varphi}{\partial y} \frac{\partial^2 \varphi}{\partial x \partial y} = 0 \quad (1)$$

where φ is the disturbance velocity potential, M_∞ is the incoming Mach number, U_∞ is the incoming velocity and γ is the specific heat ratio. If a fast and approximate solution is required, MSD equation can be simplified as

$$\left[1 - M_\infty^2 - \frac{\gamma + 1}{U_\infty} M_\infty^2 \frac{\partial \varphi}{\partial x}\right] \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \quad (2)$$

Equation (2) can be written as the following form.

$$(1 - M^2) \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \quad (3)$$

where $M^2 = M_\infty^2 + \frac{\gamma + 1}{U_\infty} M_\infty^2 \frac{\partial \varphi}{\partial x}$

The flow field studied in this paper is transonic and supersonic inviscid steady flow in a pipe with small bumps. In order to quickly predict the characteristics of flow field and shock wave structure, we use Equation (2) to solve the flow field.

Equation (2) is discretized using finite differential method[6]. If the local flow is subsonic ($1 - M^2 > 0$), Equation (2) is an elliptic equation, which is discretized by central differential scheme. If the local flow is supersonic ($1 - M^2 < 0$), Equation (2) is a hyperbolic equation and should be discretized by upwind differential scheme. The local marks used for central differential scheme and upwind differential scheme are shown in the Figure 1. If all the mesh elements are square elements and they are the same in size, the differential equations of subsonic flow and supersonic flow are Equations (4) and (5) respectively.

$$\left(1 - M_\infty^2 - \frac{\gamma + 1}{U_\infty} M_\infty^2 \frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta}\right) (\varphi_{i-1,j} - 2\varphi_{i,j} + \varphi_{i+1,j}) + \varphi_{i,j-1} - 2\varphi_{i,j} + \varphi_{i,j+1} = 0 \quad (4)$$

$$\left(1 - M_\infty^2 - \frac{\gamma + 1}{U_\infty} M_\infty^2 \frac{\varphi_{i,j} - \varphi_{i-2,j}}{2\Delta}\right) (\varphi_{i-2,j} - 2\varphi_{i-1,j} + \varphi_{i,j}) + \varphi_{i,j-1} - 2\varphi_{i,j} + \varphi_{i,j+1} = 0 \quad (5)$$

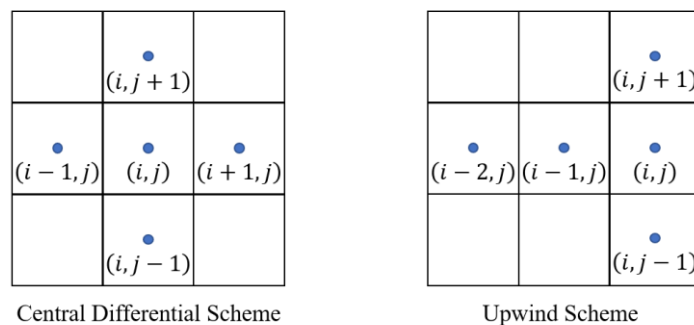


Figure 1. Local marks used for central differential scheme and upwind differential scheme

Gauss-Seidel method is used for iteration. Take Equation (4) as example for applying Gauss-Seidel method. We obtain the following formula from Equation (4).

$$\Delta\varphi_{i,j}^{(k)} = \varphi_{i,j}^{(k)} - \frac{\left(1 - M_\infty^2 - \frac{\gamma+1}{U_\infty} M_\infty^2 \frac{\varphi_{i+1,j}^{(k)} - \varphi_{i-1,j}^{(k)}}{2\Delta x}\right) (\varphi_{i-1,j}^{(k)} + \varphi_{i+1,j}^{(k)}) + \varphi_{i,j-1}^{(k)} + \varphi_{i,j+1}^{(k)}}{2 \left(1 - M_\infty^2 - \frac{\gamma+1}{U_\infty} M_\infty^2 \frac{\varphi_{i+1,j}^{(k)} - \varphi_{i-1,j}^{(k)}}{2\Delta x}\right) + 2} \quad (6)$$

The superscript (k) indicates the k th iteration value of $\varphi_{i,j}$. The iteration formula is

$$\varphi_{i,j}^{(k)} = \varphi_{i,j}^{(k-1)} + \Delta\varphi_{i,j}^{(k-1)} \quad (7)$$

The global calculation error $E^{(k)}$ can be expressed by (2.8). When $E^{(k)}$ is less than a given small value and never surpass it, the calculation is regarded as converged [7].

$$E^{(k)} = \sum_{j=1}^n \sum_{i=1}^n |\Delta\varphi_{i,j}^{(k)}| \quad (8)$$

2.2 Model and boundary conditions

The physical model is described as inviscid steady transonic or supersonic flow in a two-dimensional pipe with a bump on one side of the wall or symmetrical bumps on both sides. Two-dimensional pipe is the simplification of three-dimensional pipe. It represents a three-dimensional pipe that has infinite extension along the normal direction of the 2D plane [8].

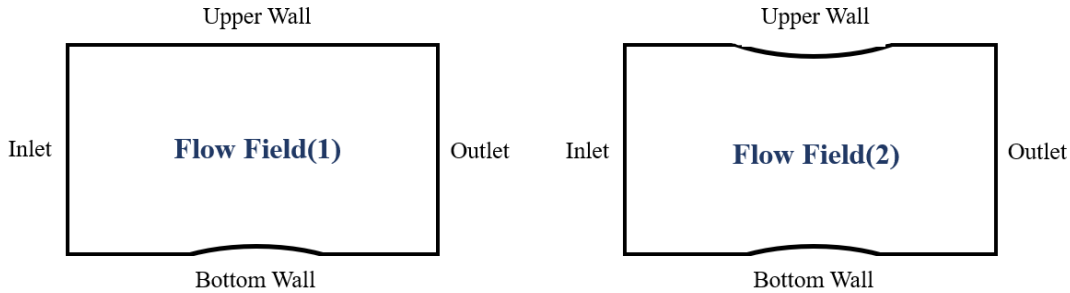


Figure 2. Schematic diagram of boundaries

Both pipes have four boundaries: inlet, outlet, upper wall and bottom wall. The schematic diagram of boundaries is shown in Figure 2.

The inlet boundary condition of the pipe is defined as a uniform incoming velocity U_∞ and Mach number M_∞ perpendicular to the inlet cross-section. The outlet boundary is set as undisturbed boundary with the partial derivatives of disturbance velocity potential to x and y equal to zero ($\frac{\partial\varphi}{\partial x} = 0, \frac{\partial\varphi}{\partial y} = 0$). Neumann boundary condition is applied for wall boundary denoted as $\frac{\partial\varphi}{\partial n} = 0$, where n is the normal direction of the wall. If the shape function of the wall is $y = f(x)$. The wall boundary condition can be simplified as the following form assuming that $f(x)$ is very small compared with the size of the flow field.

$$\frac{\partial\varphi}{\partial y} = U_\infty f'(x) \quad (9)$$

As shown in Figure 3, when applying (2.4) or (2.5) to the space close to the wall, we need the information at the ghost point. We discretize (2.9) as the following formula [9].

$$\frac{(\varphi_{i,j+1} - \varphi_{i,j-1})}{2\Delta} = U_\infty f'(x)_{i,j} \quad (10)$$

According to (2.10), we obtain the disturbance velocity potential at the ghost point.

$$\varphi_{i,j-1} = \varphi_{i,j+1} - 2\Delta \cdot U_{\infty} f'(x)_{i,j} \quad (11)$$

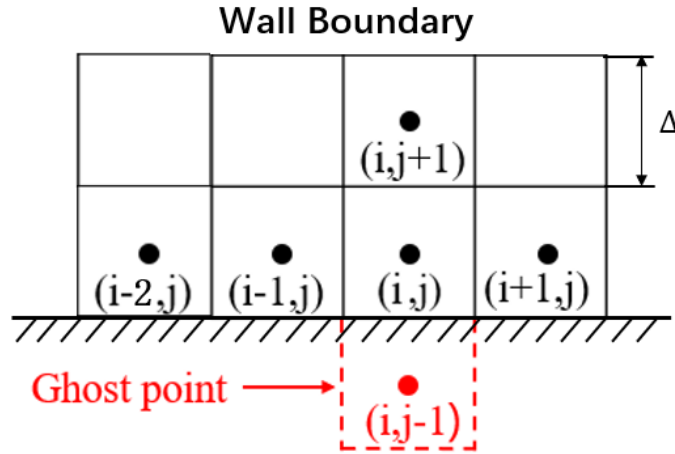


Figure 3. Discretization close to the wall

3. Results and Discussion

The dimensions of the pipe are shown in Figure 4. The function expression of the bump at the bottom wall is given as below [10].

$$f(x) = \beta \sin[\pi(x - 1)] \quad 1 \leq x \leq 2 \quad (12)$$

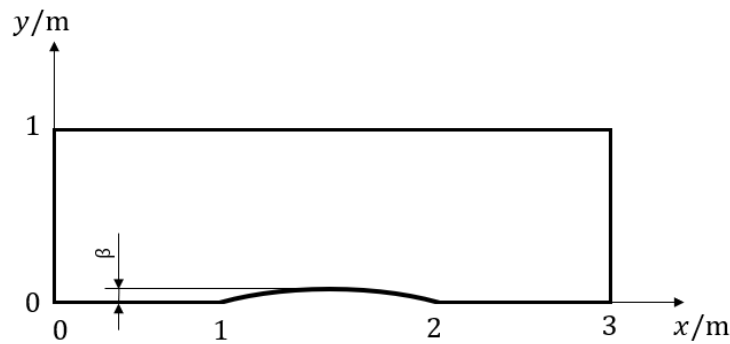


Figure 4. Schematic diagram of the pipe

For the pipe with symmetrical bumps on both sides of the wall, the upper bump is symmetrical to the bottom bump about $y=0.5$. So the function expression of the upper bump is

$$f(x) = -\beta \sin[\pi(x - 1)] + 1 \quad 1 \leq x \leq 2 \quad (13)$$

The flow field is solved based on small disturbance equation. And given that the bump due to abrasion or machining error is usually small, β should be much smaller than the dimension of the pipe. So we take $\beta = 0.03m$. The computational domain is structured mesh with square elements. The number of the elements is 30000.

3.1. Bump on one side of the wall

The structure of the shock wave in case of transonic and supersonic flow is studied with the incoming Mach number M_{∞} as the independent variable.

When $M_{\infty} = 1.3$, there are subsonic and supersonic regions in the flow field. Therefore, the flow field is transonic. It takes a relatively long time for transonic flow field to converge. Because the Mach number is close to 1 and the shock wave angle is large, Mach reflection will occur when the shock waves generated by the front and rear disturbance points meet the upper wall. The interaction

between shock waves and wall leads to the formation of λ -shaped shock waves. The cloud chart of local Mach number is shown in Figure 5, from which we can observe the shock wave structure more clearly.

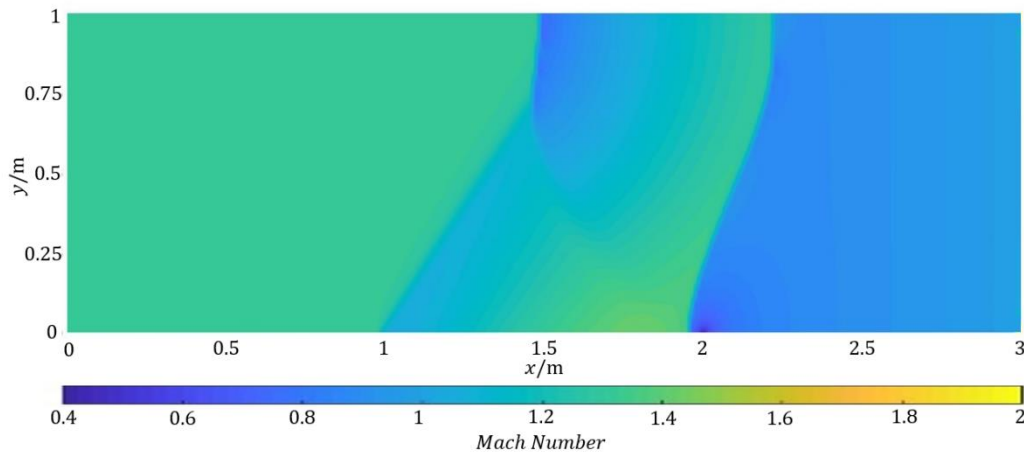


Figure 5. Cloud chart of local Mach number ($M_\infty = 1.3$)

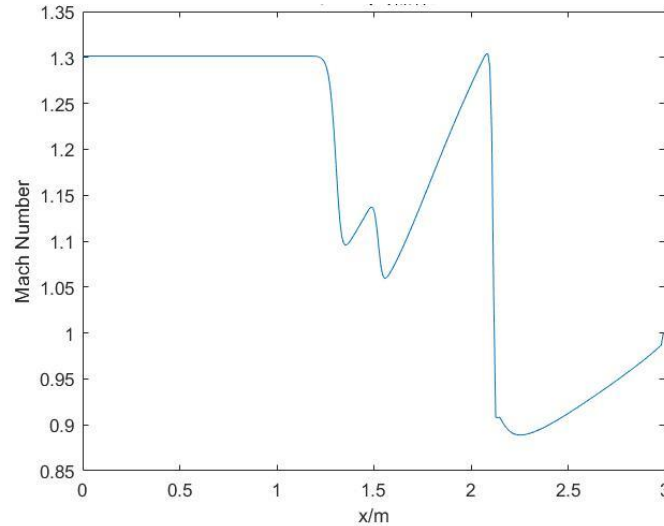


Figure 6. Distribution of local Mach number along $y=0.5\text{m}$

The distribution of local Mach number along $y=0.5\text{m}$ is plotted in Figure 6. When crossing the two arms of the front shock wave, the local Mach number decreases but is not less than 1. When crossing the rear shock wave, the local Mach number decreases sharply and is smaller than 1. Therefore, the rear shock wave is a strong shock wave. The two arms of the front shock wave are weak shock wave and the reflected arm is weaker than the incident arm. However, the Mach stem of the front shock wave is a strong shock wave.

Increase the incoming Mach number to 1.4, 1.6 and 1.8. The cloud charts of local Mach number are shown in Figure 7, 8 and 9 respectively. The angle of the incident shock wave generated by the front disturbance point decreases and the reflected shock wave overlaps with the shock wave generated by the rear disturbance point. The two shock waves interact with each other to form a single λ -shaped shock wave. As the Mach number increases, the interaction region shifts downwards and the whole λ -shaped shock wave moves towards the outlet.

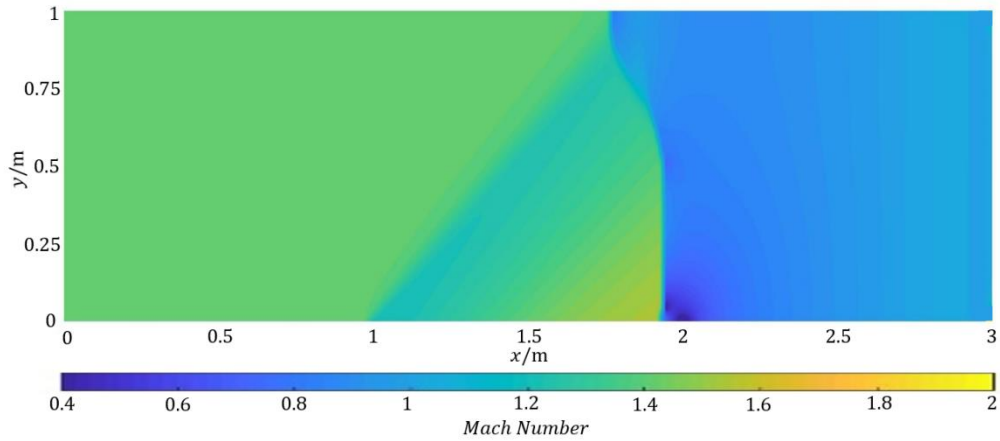


Figure 7. Cloud chart of local Mach number ($M_\infty = 1.4$)

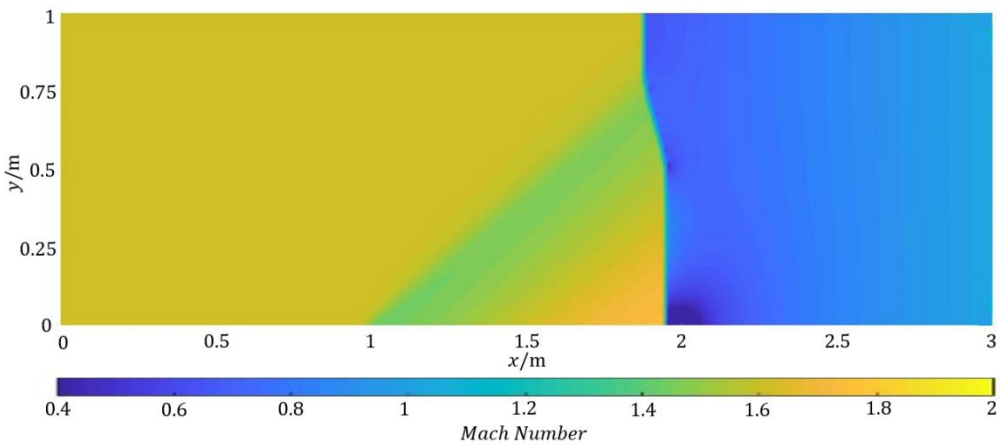


Figure 8. Cloud chart of local Mach number ($M_\infty = 1.6$)

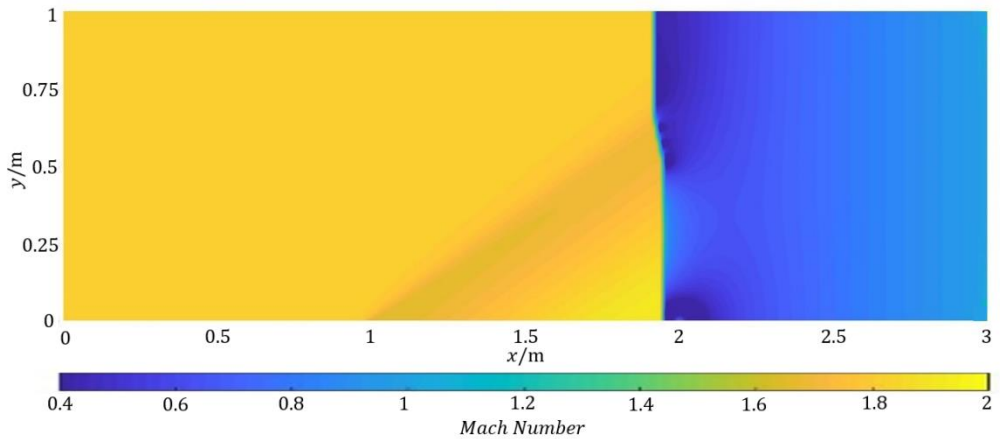


Figure 9 Cloud chart of local Mach number ($M_\infty = 1.8$)

When the flow field is completely supersonic, the shock waves generated by the front and rear disturbance points are parallel to each other and do not interfere with each other. They extend out of the flow field. If the pipe is long enough, the two shock waves will reflect at the upper wall. The cloud charts of local Mach number when $M_\infty = 3$ and 4 are shown in Figure 10 and 11 respectively. As the incoming Mach number increases, the shock wave angle decreases and satisfies the following formula.

$$\theta = \arcsin \frac{1}{M_\infty} \quad (14)$$

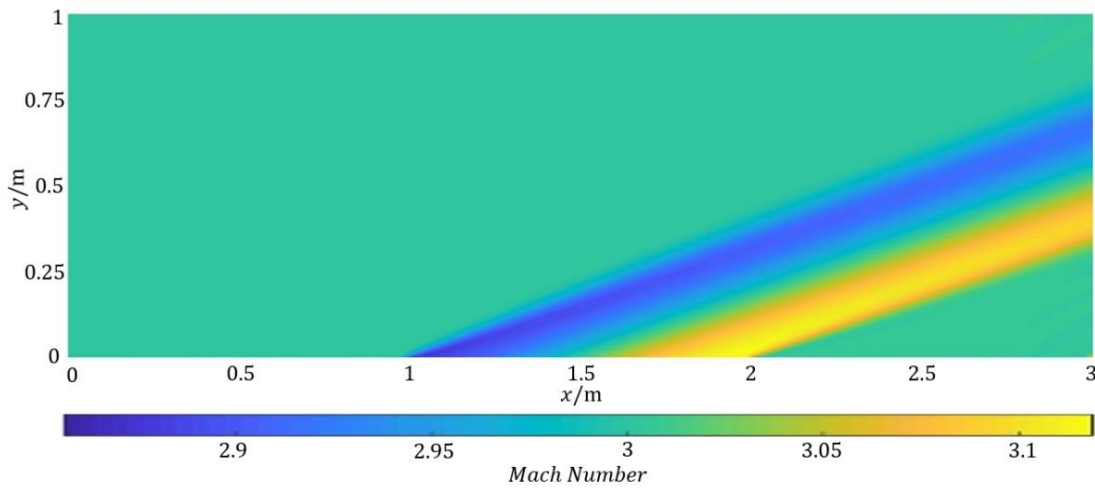


Figure 10. Cloud chart of local Mach number ($M_\infty = 3$)

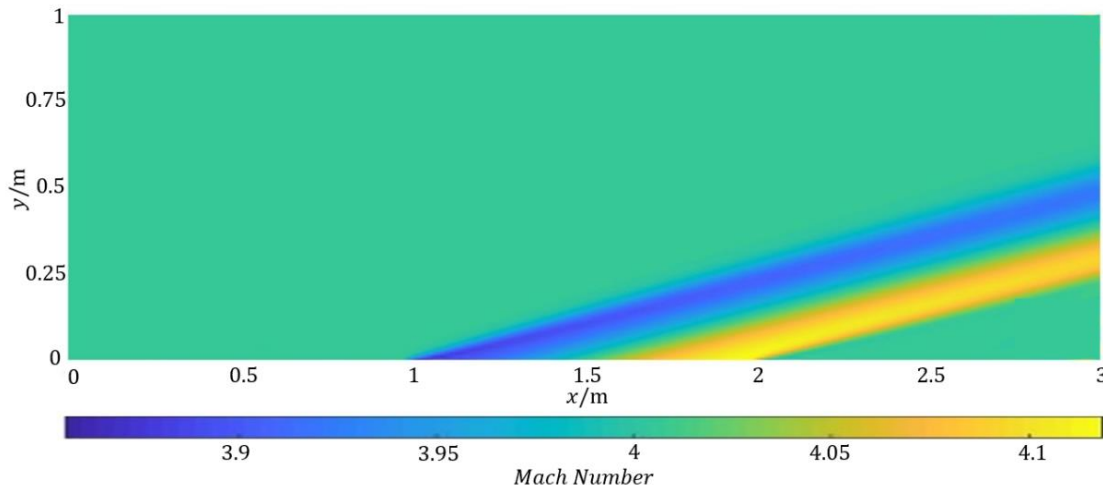


Figure 11. Cloud chart of local Mach number ($M_\infty = 4$)

3.2. Symmetrical bumps on both sides of the wall

For the case where there are symmetrical bumps on both sides of the wall, there are two front disturbance points and two rear disturbance points in the flow field.

If the incoming Mach number is 1.4, two λ -shaped shock waves are generated by the two front disturbance points respectively, interacting at the center of the flow field to produce a Mach disk. The λ -shaped shock waves generated at the two rear disturbance points interact with each other to form a bow-shaped shape wave with an approximate normal shock wave in the center. The cloud chart of local Mach number is shown in Figure 12. And the shock wave structure is shown in Figure 13.

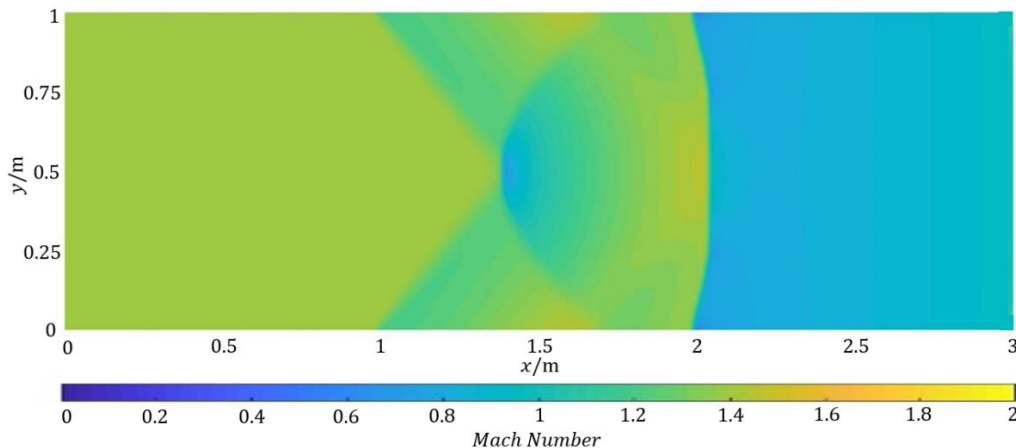


Figure 12. Cloud chart of local Mach number ($M_\infty = 1.4$)

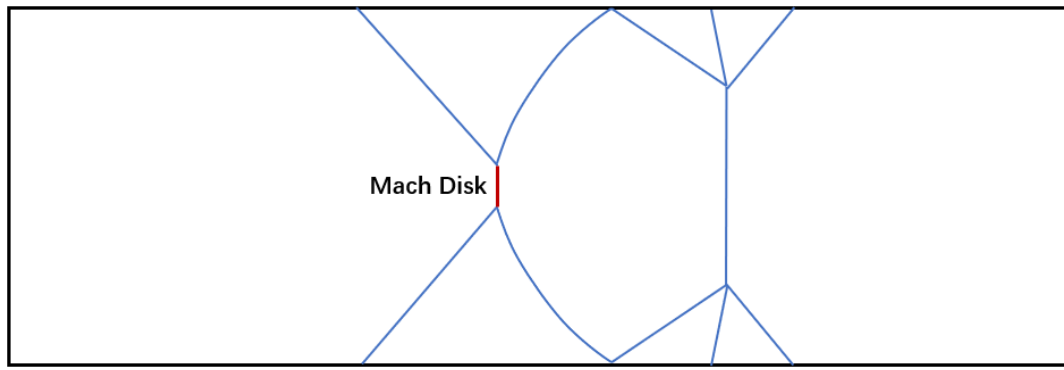


Figure 13. Shock wave structure in the flow field ($M_\infty = 1.4$)

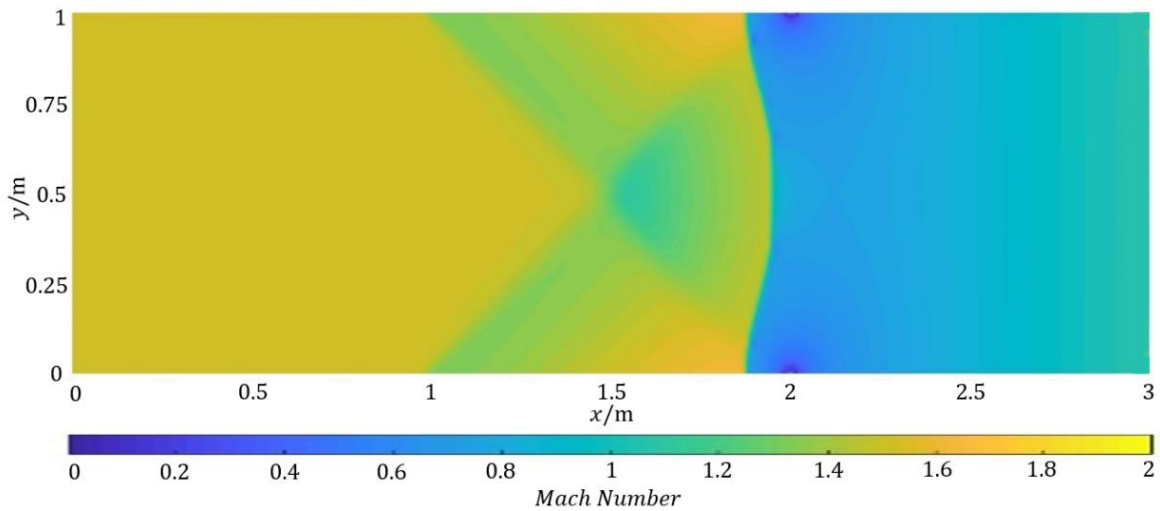


Figure 14. Cloud chart of local Mach number ($M_\infty = 1.5$)

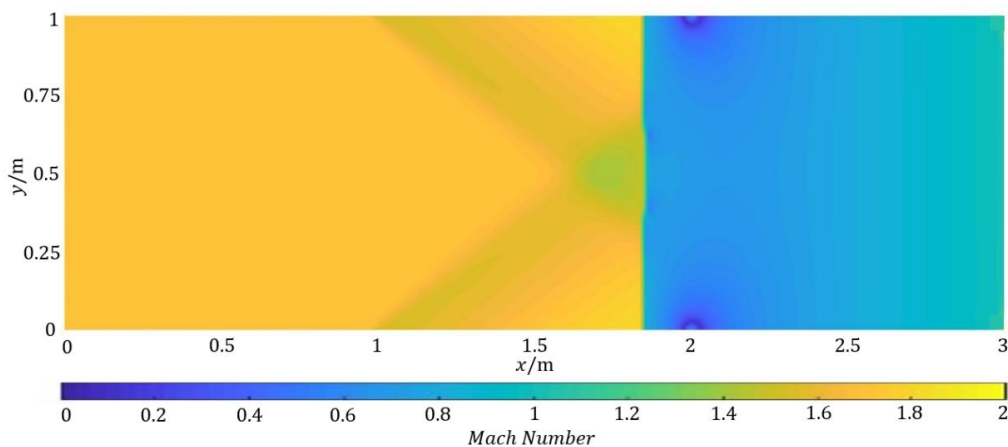


Figure 15. Cloud chart of local Mach number ($M_\infty = 1.7$)

Increase the incoming Mach number to 1.5 and 1.7. The cloud charts of local Mach number are shown in Figure 14 and 15 respectively. As the Mach number increases, the shock wave angle decreases and the Mach disk disappears. The interaction area between the two front shock waves becomes smaller, and the normal shock wave part of the rear shock wave expands.

For a completely supersonic flow field, the four disturbance points each produce an oblique shock wave. Figure 16 gives the supersonic flow field for an incoming Mach number of 3.

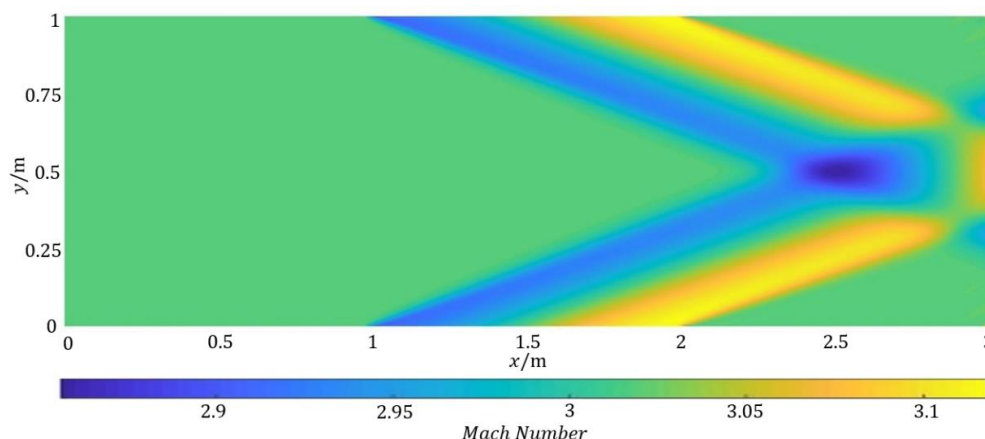


Figure 16. Cloud chart of local Mach number ($M_\infty = 3$)

4. Conclusion

The transonic and supersonic inviscid steady flow in a two-dimensional pipe with a bump on one side of the wall or symmetrical bumps on both sides is solved based on a simplified form of modified small disturbance equation (MSD).

The case of a small bump on one side of the wall. If the flow is transonic, the shock wave generated at the front and rear disturbance points of the bump are both λ -shaped. As the incoming Mach number increases, the two λ -shaped shock wave gradually merge into a single λ -shaped shock wave. In the case of supersonic flow field, the shock waves generated at the front and rear points are oblique shock wave and parallel to each other. They do not interfere with each other.

The case of symmetrical bumps on both sides of the wall. If the flow is transonic, the two front disturbance points produce two λ -shaped shock waves. They interfere with each other and form a Mach disk when $M_\infty = 1.3$. The shock waves generated by the two rear disturbance points combine into a single bow-shaped shock wave. As the incoming Mach number increases, the area of interaction between the two λ -shaped shock waves decreases and the normal shock wave part of the bow-shaped shock wave expands. If the flow field is supersonic, the four disturbance points will each generate an oblique shock wave.

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