

# Study on optimization method for airfoil in supersonic case based on Particle Swarm optimization(PSO)

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**Abstract.** In order to adapt to the society demand of requiring faster aircraft, this thesis aims at combining Particle Swarm optimization (PSO) with Computational Fluid Dynamics (CFD) to get a optimized airfoil which possesses a better areodynamics performance in supersonic case. This paper adopts the method of using small disturbance equation to simulate the flow field, which is more effective and stable and easier to implement. Besides, this paper adopts traditional PSO. The results show that the magnitude of ratio of lift-drag has increased by 16.29% and the magnitude of lift has risen by 13.69%, while the magnitude of drag has decreased by 2.2% after the optimization, compared with the initial airfoil. Besides, the results also reveal that the front of the optimized airfoil is wider than that of the initial airfoil and the rear of the optimized airfoil is narrower than that of the initial airfoil, which can represent that the optimized airfoil effectively enlarge the magnitude of ratio of lift-drag and lift and suppresses the drag and influence generated by shock.

**Keywords:** PSO, CFD, airfoil, supersonic, areodynamics performance.

## 1. Introduction

According to globalization, the business trades and cultural exchanges between different countries are unprecedentedly exuberant. In order to cater to these increasingly demands, the aircrafts are required to become faster and more fuel-efficient. In a way, wings are the most important part of the aircrafts. Because the wing plays a vital role in controlling the velocity of the aircraft, take-off and landing performance and Maneuverability. Besides, the wing can sustain aircrafts balanced and guarantee their operational performance when aircrafts are executing tasks. Overall, the wing is the foundation to make sure aircraft can complete all instructions given by human being. Therefore, how to improve the Pneumatic performance of the wing is kind of research area that is worth for us to explore.

Reviewing the former researches, Wang [1], their team combined SAGA with Flexible Tolerance Method to establish a new type of Genetic Algorithm, named HGA and combined it with aerodynamic analysis of the airfoil to optimize the Pneumatic performance of airfoil in transonic case; Li [2], their team combined Multi Objective Genetic Algorithm (MOGA) with Class-Shape Transformation (CST) to study on Multi-objective optimization Design and Blunt Trailing Edge of Wind Turbine Airfoil. They found that blunt trailing edge of airfoil owned a larger lift-drag ratio than that of the original airfoil; Wang [3], their team adopted surrogate model with the method of CST to simulate the shape of airfoil, and use Genetic Algorithm (GA) to optimize the Pneumatic performance of airfoil; Wang [4], their team focused on Multi Particle Swarm Optimization (MPSO) to optimize the performance of airfoil in subsonic case.

There are many kinds of intelligent optimization algorithm, such as Genetic Algorithm (GA), Simulated annealing algorithm (SAA), Particle Swarm optimization (PSO), Differential Evolution Algorithm (DE) and so on. In my study, I concentrate on optimization for airfoil in supersonic case via the method of PSO. The reason why I choose PSO as my optimization algorithm is that PSO is efficient and the rate of seeking is rapid. And PSO is easy to be implemented. I parameterize the shape of airfoil by giving it a exact function and discrete it. And then I set several particles to optimize the performance of airfoil, that is enlarging lift-drag ratio and lift coefficient, reducing drag coefficient.

## 2. Math conditions

### 2.1. Theoretic formula

The hypothesis in my study is that we consider in steady and inviscid case. And the fluid flow satisfies the Ideal Gas Law [5].

First introduce the Navier-Stokes Equations:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \text{div}(\rho \cdot U) = 0 \\ \frac{\partial(\rho \cdot U)}{\partial t} + \text{div}(\rho U \times U + p \cdot \vec{I}) = \text{div}(\tau) \\ \frac{\partial(\rho E)}{\partial t} + \text{div}(\rho U E) = \text{div}(k \cdot \nabla T) + \text{div}(\sigma U) \end{cases} \quad (1)$$

According to we consider in steady case, all time derivatives can be omitted. And introduce potential function  $\phi$ . Velocity field can be expressed as  $v = \nabla \phi$ . Then the equation (1) can be transform into,

$$\text{div}(\rho \cdot \nabla \phi) = 0 \quad (2)$$

which is known as the full potential equation. And under the consideration of constant entropy, it is possible to derive the relation of density,

$$\frac{\rho}{\rho_0} = \left(1 - \frac{|\nabla \phi|^2}{2H_0} - \frac{\partial_t \phi}{H_0}\right)^{1/\gamma-1} \quad (3)$$

Then we can expand the derivatives of the density, we can show that the equation (IV) can be converted into,

$$(1 - M_x^2)\phi_{xx} + (1 - M_y^2)\phi_{yy} - 2M_x M_y \phi_{xy} = 0 \quad (4)$$

In order to solve this equation, we can consider in the case of thin obstacles. And then we can get our representation of potential field,

$$\phi = U_\infty(x + \phi) \quad (5)$$

where  $U_\infty$  is uniform flow in the x-direction. And eventually the full potential equation is expressed as,

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} = 0 \quad (6)$$

### 2.2. CFD method

In my study, I use finite differences approximation for solving the former equation [6]. In order to make sure the approximation satisfies the physics condition, we need to establish two differences scheme for  $M < 1$  and  $M > 1$  respectively,

$$k_{i,j}[\phi_{i+1,j}^n - 2\phi_{i,j}^n + \phi_{i-1,j}^n] + [\phi_{i,j+1}^n - 2\phi_{i,j}^n + \phi_{i,j-1}^n] = 0 \quad (7)$$

$$k_{i-1,j}[\phi_{i,j}^n - 2\phi_{i-1,j}^n + \phi_{i-2,j}^n] + [\phi_{i,j+1}^n - 2\phi_{i,j}^n + \phi_{i,j-1}^n] = 0 \quad (8)$$

where the relation of k is,

$$k_{i,j} = 1 - M_\infty^2 - (\gamma + 1)M_\infty^2 \frac{\phi_{i+1,j} - \phi_{i,j-1}}{2h} \quad (9)$$

And next combine the equation (IX) and equation (X), we can derive our approximation for full potential equation,

$$\begin{aligned} (1 - \mu)k_{i,j}[\phi_{i+1,j}^n - 2\phi_{i,j}^n + \phi_{i-1,j}^n] + [\phi_{i,j+1}^n - 2\phi_{i,j}^n + \phi_{i,j-1}^n] \\ + k_{i-1,j}[\phi_{i,j}^n - 2\phi_{i-1,j}^n + \phi_{i-2,j}^n] = 0 \end{aligned} \quad (10)$$

Notes that  $\mu = 0$  for  $M < 1$ ,  $\mu = 1$  for  $M > 1$ . Finally, we use SOR relaxation to get our simulation.

### 2.3. Algorithm

In this study, I focus on Particle Swarm optimization (PSO). As far as we concern, PSO is a kind of bionic optimization algorithm [7]. The PSO is simulated by the collaborative process of birds seeking for food. In the primary place, PSO randomly initializes several particles with their typical position and velocity in a specific scope [8]. And due to social sharing and individual cognition, the fitnesses of particle swarm constantly increase. The core concept of the PSO is the social sharing of society. In other words, PSO is the process of obtaining the optimal solution through continuously updating the speed and position of the particles. To be more specify, in each iteration, every particle possesses its own individual extremum, named  $p_i^t best$ . And the particle swarm gain the group extremum, named  $g^t best$ , by comparing all the  $p_i^t best$  in the particle swarm. Then all the particles update their own position and velocity via tracing  $p_i^t best$  and  $g^t best$ . The theoretic formula of PSO,

$$v_i^{t+1} = \omega \cdot v_i^t + c_1 r_1 (p_i^t - x_i^t) + c_2 r_2 (p_g^t - x_i^t) \quad (11)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (12)$$

Notes that,  $\omega$  is the inertia weight, whose value control the searching ability;  $c_1$  and  $c_2$  are acceleration constants;  $r_1$  and  $r_2$  are the random numbers between 0 and 1;  $p_i^t$  and  $p_g^t$  are individual optimum solution and global optimum solution in each loop respectively.

## 3. Physical model

### 3.1. Simulated conditions

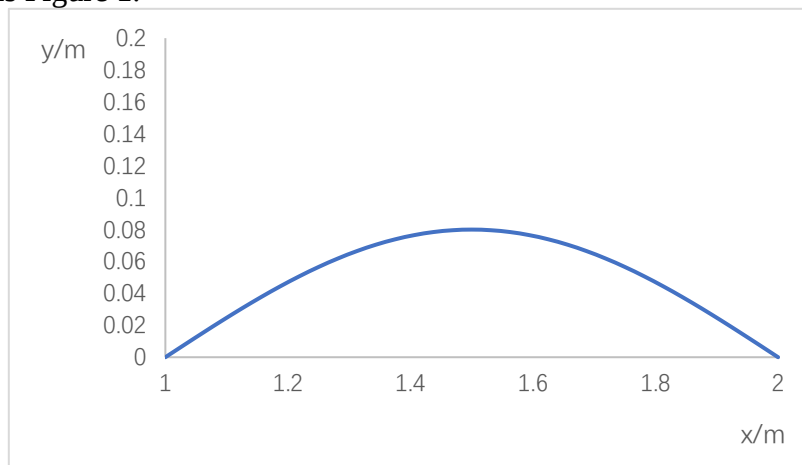
The simulation of my investigation is the potential models for fluid flow above a airfoil [9]. And I focus on supersonic case without viscous force, where the shock would be occur. the fuild flow is steady and statisfies the Ideal Gas Law. In my study, I choose scale models, that is, the model in the reality narrows down to what I require in my research.

### 3.2. Simulated model

In my study, I establish custom airfoil. The shape of the airfoil is,

$$f(x) = \beta \sin(\pi(x - 1)) \quad (13)$$

It can be seen as Figure 1.



**Figure 1.** The shape of original airfoil.

## 4. Designing process

In the design of CFD simulation, as a result of using the method of small disturbance approximation, the boundary conditions becomes

$$v = (U_{\infty} + u)f'(x) \approx U_{\infty}f'(x) \dots \dots (XVI) \quad (14)$$

where  $f(x)$  is the shape of the airfoil. And then I use finite differences approximation which is mentioned before to simulate. And I set  $M_{\infty}$ ,  $U_{\infty}$ ,  $T_0$ ,  $\rho_0$ ,  $\beta$  as initial condition. Then I use finite differences approximation mentioned in part 2 to obtain the field of potential function [10].

Besides, for the thermodynamics equation,

$$c_p T_0 = c_p T + \frac{v^2}{2} \quad (15)$$

we can derive that,

$$T = T_0 - \frac{v^2}{2c_p} \quad (16)$$

we can get the field of temper. For the equation (V), we deduce the field of density,

$$\rho = \rho_0 \left(1 - \frac{|\nabla \phi|^2}{2H_0}\right)^{1/\gamma-1} \quad (17)$$

because of our assumption of that the fluid flow satisfies the Ideal Gas Law,

$$p = \rho RT \quad (18)$$

we can get our the field of pressure by combining the equation (XVII), equation (XVIII) and equation (XIX). For the aerodynamics equations, we know that,

$$L = - \int_{LE}^{TE} p_u \cos \theta \, ds_u + \int_{LE}^{TE} p_l \cos \theta \, ds_l \quad (19)$$

$$D = \int_{LE}^{TE} -p_u \sin \theta \, ds_u + \int_{LE}^{TE} p_l \sin \theta \, ds_l \quad (20)$$

where  $p_u$  and  $p_l$  are the pressure stress on the upper and lower surface of airfoil respectively,  $\theta$  is the tangent direction of airfoil. Then we can get the ratio of lift-drag,  $K$ , by using the equation of  $L/D$ .

In the design of PSO algorithm, in order to simplify the calculation, I set a weighting function,

$$g(x) = c_1 \cdot K + c_2 \cdot L + c_3 \cdot D \quad (21)$$

where  $c_1, c_2, c_3$  are the weighting coefficients.

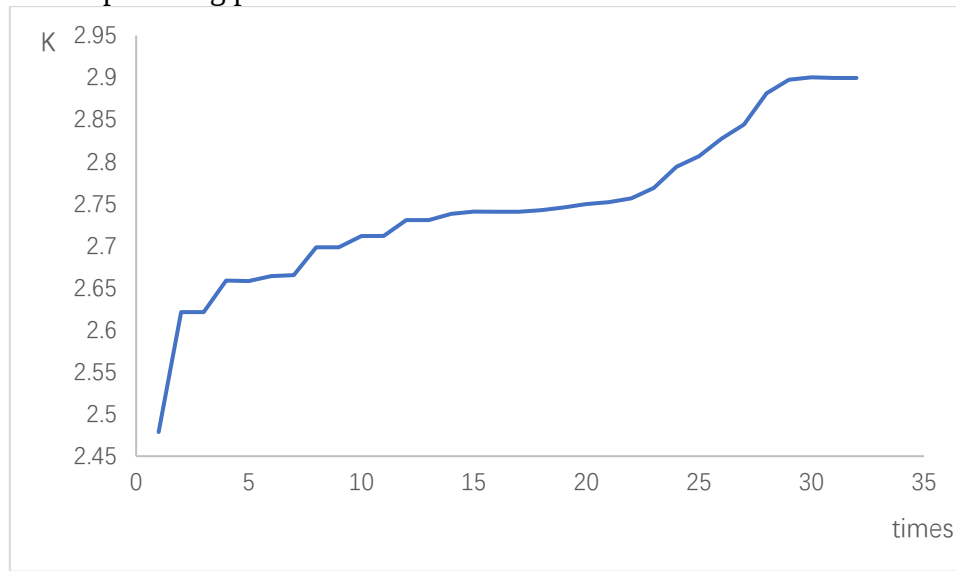
The crux in my study is how to integrate CFD simulation and PSO algorithm. In order to deal with it, the shape of the airfoil is represented by the discrete points that I divided. And these discrete points play the role of variables of the PSO algorithm. In more detail, the position of discrete points correspond to the  $x_i^t$  in the equation (XIV). By altering the position of the discrete points, the PSO can figure out a certain position of the discrete point where the airfoil possess a better aerodynamics performance.

## 5. Result and Discussion

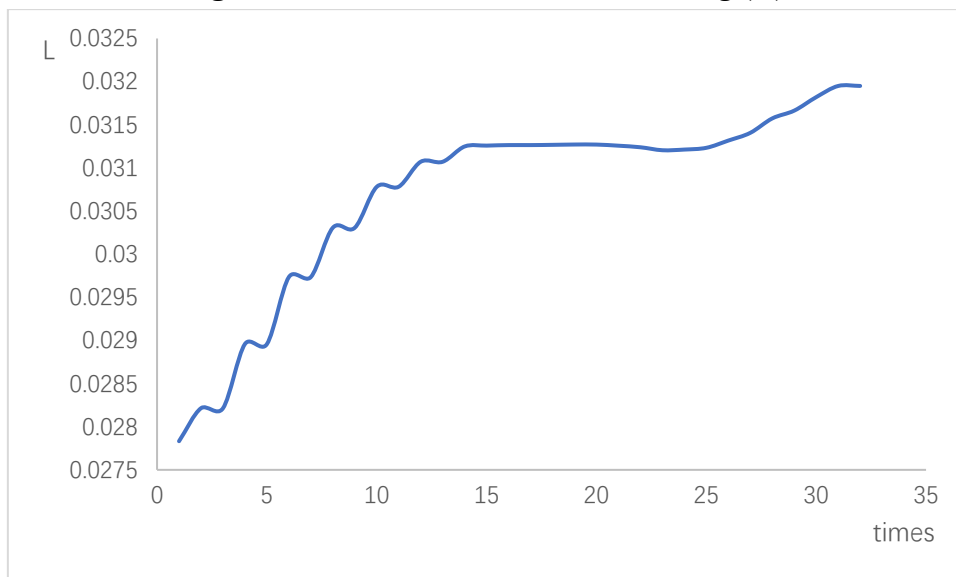
### 5.1. The graph of aerodynamic performance

In my configuration, the  $\beta$  in the equation (XV) equates to 0.08,  $c_1, c_2, c_3$  in the equation (XXII) are tantamount to 0.6, 0.25, -0.15 respectively. Viewing the figure of the variation of ratio of lift-drag ( $K$ ), seen as Figure 2, we can know that there is a upward trend in the magnitude of  $K$  during the process of optimizing. And the magnitude of  $K$  for the optimized airfoil is 2.88263, while that for the initial airfoil is 2.47873, has increased by 16.29%. When observing Figure 3, we can understand

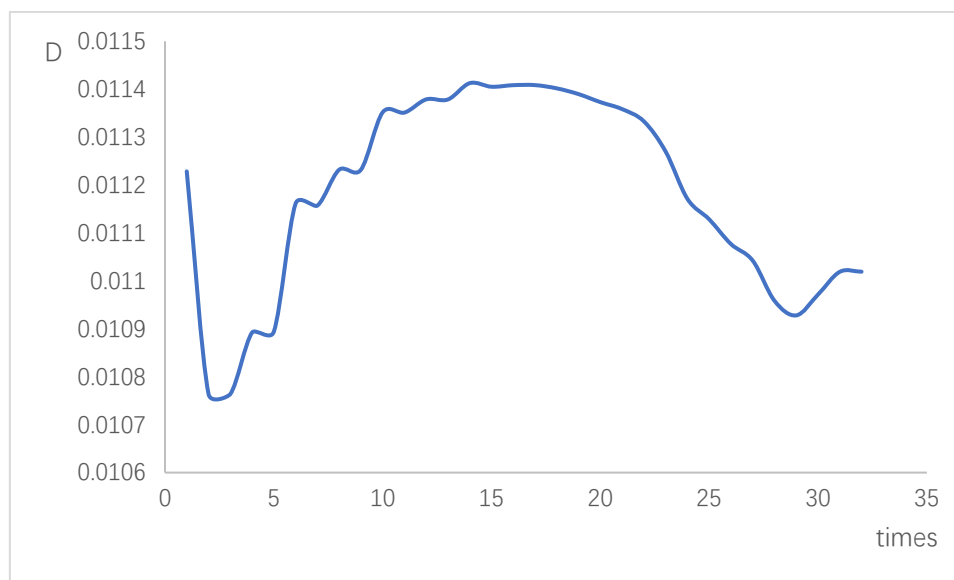
that the magnitude of lift goes up generally. And the magnitude of lift has increased by 13.69% during the process, rising from 0.02783 to 0.03164. Besides, we can realize that the magnitude of drag varies fluctuantly during the process of optimizing in the Figure 4. The magnitude of drag for the optimized airfoil is 2.2% less than that for the initial airfoil. From seeing Figure 2, Figure 3 and Figure 4 together, we can draw a conclusion that, during the optimization, the increase rates of magnitude of K and lift vary from rapid to slow. In the stage of sluggish increase, the magnitude of drag reaches extreme value. And from then on, the magnitude of drag starts to decrease after a period of increasing. This means that in the late stage of optimization, drag plays a leading role. These three figures also show that the process of optimizing performs well.



**Figure 2.** The variation of ratio of lift-drag (K).



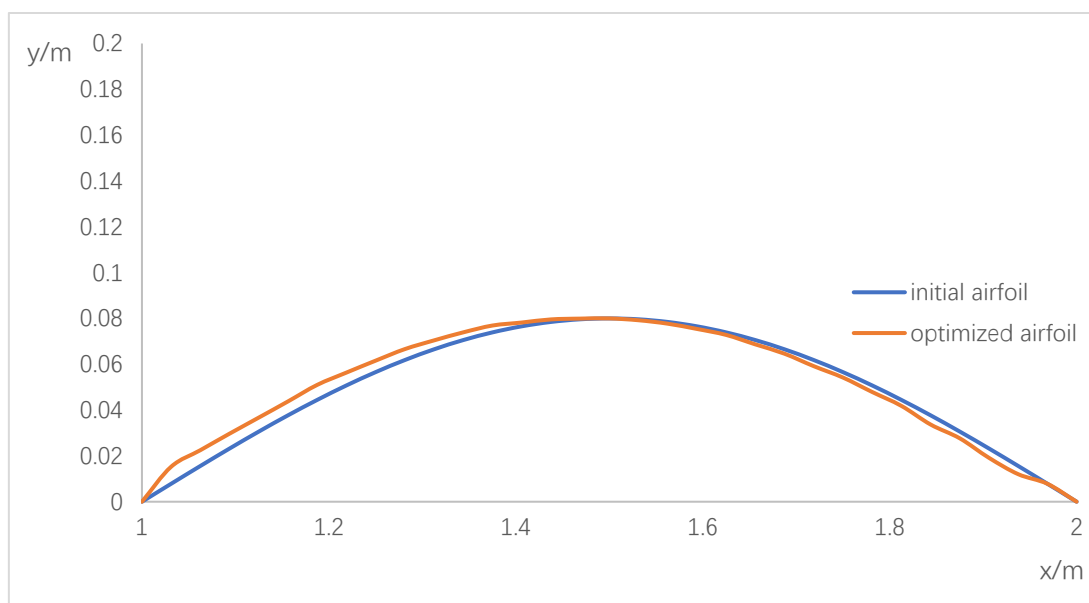
**Figure 3.** The variation of magnitude of lift (L).



**Figure 4.** The variation of magnitude of drag (D)

## 5.2. Compared with original model

Viewing the Figure 5, the orange line represents the optimized airfoil and the blue line represents the initial airfoil. We can realize that the the front of optimized airfoil is wider than that of initial airfoil and the rear of the optimized airfoil is narrower than that of the initial airfoil. The wider front of optimized airfoil can effectively enlarge the magnitude of lift. Besides, as we can see that, in the Figure 4, in the late stage of the optimization, the magnitude of drag steadily decreases while the magnitude of K and lift slowly increase, shown in the Figure 2 and 3. Hence, the narrower rear of optimized airfoil can greatly reduce the drag and influence generated by the shock. Overall, the optimized airfoil has a better areodynamics performance compared with the initial airfoil.



**Figure 5.** The comparison between initial and optimized airfoil.

## 6. Conclusion

This paper combines PSO with CFD to derive a optimized airfoil possessing a better areodynamics performance in supersonic case. The fact proves that the magnitude of ratio of lift-drag has increased by 16.29% and the magnitude of lift has risen by 13.69%, while the magnitude of drag has decreased

by 2.2% after the process of optimizing, compared with the initial airfoil. That is, it is feasible to use PSO to optimize the airfoil.

The wider front of optimized airfoil can generate more lift, compared with the initial airfoil. Besides, the narrower rear of the optimized airfoil would effectively reduce the drag and influence produced by the shock, thereby the airfoil can own a better aerodynamics performance. Overall, the optimized airfoil is not refined yet, and needs further amelioration.

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