Design of Spontaneous Emission Spectra Amplified by S-Band Fiber and Optimization of Output Spectral Peak Power

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Abstract. The thulium-doped fiber amplifier stands out as the most promising amplifier within the S-band spectrum. This paper delves into the exploration of a 1.47μm Tm^(3+) doped fluorinated fiber amplifier as the primary subject of research. This paper approach begins by establishing the rate equation and power propagation equation for the thulium ion level structure, followed by rigorous mathematical analysis and solutions. Subsequently, we employ MATLAB programming to perform comprehensive calculations, scrutinizing the amplified spontaneous emission spectra's variation concerning Thulium-Doped Fiber Amplifier (TDFA) length and pump power. In a quest for optimal performance, this paper turn to the simulated annealing algorithm to fine-tune the doping concentration and fiber length, ultimately achieving the most favorable output spectral gain. Through systematic experimentation and parameter manipulation, we've unveiled a substantial breakthrough, with our research yielding an impressive optimal gain of 31.71 dB. The ramifications of this work extend far beyond the confines of this study. It sets a solid foundation for advancements in fiber fabrication and paves the way for the realization of fiber lasers, promising significant contributions to the expansion of S-band fiber communication systems. Our findings hold great promise and are poised to drive innovation in this critical area of telecommunications.

Keywords: MATLAB, TDFA, simulated annealing algorithm.

1. Introduction

Fiber lasers offer several advantages over solid-state and semiconductor lasers. They boast a straightforward design, efficient heat dissipation, high conversion efficiency, superior beam quality, and hassle-free maintenance. Consequently, fiber communication technology has emerged as the dominant choice in contemporary communication systems [1]. Notably, mature technologies include erbium-doped fiber amplifiers operating at 1550nm and praseodymium-doped fiber amplifiers operating at 1310nm. At present, thulium-doped fiber lasers have attracted more and more attention due to their advantages of compact structure, good beam quality, and high quantum efficiency, and have been widely used in remote sensing, biomedicine, and national defense [2]. However, the market of thulium-doped fiber amplifiers is not mature, and the research on S-band gain characteristics needs to be perfected.

Thulium-doped fiber amplifiers are the most potential amplifiers in S-band. TDFA uses thulium-doped ion fiber as a gain medium, produces particle population inversion under 793nm pump light, and achieves stimulated radiation amplification under signal light induction. In this paper, Tm31-doped fluoride fiber amplifiers at 1.47 μm are used as the research object. A mathematical model is established based on the rate equation and power propagation equation of the thulium ion level structure, and the corresponding mathematical equations are solved. TDFA length and pump power are important parameters to determine TDFA gain. In this paper, MATLAB programming is used to calculate the variation of amplified spontaneous emission spectra with the two, and the influence of different parameters on the results is discussed according to the solution results. Then, the doping concentration and fiber length are optimized by simulated annealing algorithm to obtain the best gain of the output spectrum. This paper lays a foundation for the realization of fiber fabrication and fiber laser and is of great significance for the S-band expansion of fiber communication systems.
2. Modeling

2.1. Rate propagation method based on thulium ion energy level structure

Thulium is the 69th element in the periodic table, and there is a transition in the energy level structure of \( \text{Tm}^{3+} \) that satisfies S-band optical amplification, so in this paper, thulium (\( \text{Tm}^{3+} \)) ions are used as doping materials [3]. Because the energy level structure of \( \text{Tm}^{3+} \) is very rich and has its characteristics, there is a large choice of pump wavelengths for TDFA. The absorption spectrum of thulium ion shows that the light absorption is strongest near 790nm [4]. Therefore, in this experiment, the LD with a relatively mature laser wavelength of 793nm was used as the pump source. The emission spectrum of thulium ions is shown in Figure 1.

![Emission spectrum of thulium ion](image)

**Fig. 1** Emission spectrum of thulium ion [3]

Thulium ion has a three-level energy structure. The \( \text{Tm}^{3+} \) ion in the ground state \( ^3H_6 \) level is excited to \( ^3H_4 \) level after absorbing the pump light with the wavelength of 793nm, decays to \( ^3F_6 \) level through the photon self-quenching process and radiates photons at the same time. The \( ^3H_6 \) level particles are excited to the \( ^3F_4 \) level, and the \( \text{Tm}^{3+} \) ions at the \( ^3F_4 \) level transition to the ground state and emit laser [5]. To simplify the model, \( N_1 \) represents the ground state \( ^3H_6 \), \( N_2 \) represents the excited state \( ^3F_4 \), and \( N_3 \) represents the excited state \( ^3H_4 \), as shown in Figure 2.

![Energy-level diagram of thulium ion](image)

**Fig. 2** Energy-level diagram of thulium ion (Photo/Picture credit: Original)

The three-level rate equation is a dynamic equation used to describe the transition between the excited state and the ground state in a three-level system. The rate equation describes the effect of these transitions on the population density of excited and ground states by considering the rate of transitions between various energy levels:

\[
\frac{dN_1(z)}{dt} = -W_p(z)N_1(z) + A_{21}N_2(z) \tag{1}
\]

\[
\frac{dN_2(z)}{dt} = -W_{23}(z)N_2(z) - A_{21}N_2(z) + W_{32}(z)N_3(z) + A_{32}N_3(z) \tag{2}
\]
\[
\frac{\partial N_3(z)}{\partial t} = W_{23}(z)N_2(z) - A_{32}N_3(z) - W_{32}(z)N_3(z) + A_{43}N_4(z) \\
N = N_1(z) + N_2(z) + N_3(z)
\]

(3)

Where, \( W_p(z) = \frac{\sigma_{14}P_p(z)}{\hbar v_{14}A_{eff}} \), \( W_{23}(z) = \frac{\sigma_{23}(v_{23})P_s(z)}{\hbar v_{23}A_{eff}} \), \( W_{32}(z) = \frac{\sigma_{32}(v_{32})P_s(z)}{\hbar v_{32}A_{eff}} \), \( A_{eff} = \pi r^2 \), \( W_p \) means Pump light absorption rate, \( W_{23} \) means signal light absorption rate, \( W_{32} \) means stimulated emission rate of signal light, \( A_{32} \) means non-radiative transition probability, \( A_{21} \) means radiative transition probability, \( r \) means fiber core radius, \( N \) means total number of ion-doped elements. Using MATLAB software to solve the differential equations, the particle population density of each energy level can be obtained.

### 2.2. Power propagation equation

The power propagation equation is a mathematical equation that describes the power propagation process of optical signals in optical fiber. Based on the wave equation of light and the transmission characteristics of optical fiber, it is used to describe the attenuation of optical signal in optical fiber, the propagation loss, and the relationship between the optical signal power and the change of distance [6].

In optical fibers, the propagation of optical signals is affected by several factors, including diffraction, absorption, scattering, and dispersion. The power propagation equation takes these factors into account and provides a mathematical framework to describe the variation of the optical signal power in the fiber. Pump power, signal power, ASE power the change of \( (P_p, P_s, P_{ase}) \) with propagation length is called the power propagation equation.

\[
\frac{dP_p(z)}{dz} = -\Gamma_p (\sigma_p N_1(z) + \alpha_a) P_p(z)
\]

(5)

\[
\frac{dP_s(z)}{dz} = \Gamma_s (\Delta_v(\sigma_{32}N_3(z) - \sigma_{23}N_2(z) - \alpha_s) P_s(z)
\]

(6)

\[
\frac{dP_{ase}(z)}{dz} = \pm \Gamma_{ase}[\sigma_{32}N_3(z) - \sigma_{23}N_2(z) - \alpha_s) P_s(z) \pm 2\sigma_{32}N_3(z)\Delta_v
\]

(7)

\[
G(P_p(z), P_s(z), N_2, N_1, z) = \frac{P_s(z)}{P_s(0)} (100\%)
\]

(8)

\[
G(P_p(z), P_s(z), N_2, N_1, z) = 10 \times log10\left(\frac{P_s(z)}{P_s(0)}\right) (dB)
\]

(9)

Where \( \alpha \) means loss factor of light material, \( \Delta_v \) means frequency half high full width, \( N_i \) means the number density of particles in the I-th energy level, \( G \) means signal gain, and \( z \) means fiber length.

### 2.3. Parameter setting

Due to the difficulties in manufacturing fluoride fibers and splicing them onto standard silica fibers, TDFA has not been accepted by the telecommunications industry. Unlike fluoride glass, tellurate oxide glass is highly stable [7]. Therefore, our data simulation is derived from a Tm\(^{3+}\)-doped tellurite amplifier.

In the literature, Ti: sapphire laser with a wavelength of 0.79 um and scanning spectrometer with InGaS detector were used as equipment to excite Tm\(^{3+}\) fluorescence at 1.47 um and record emission spectrum. According to the experimental results in the literature, thulium-doped tellurite glass can cover the S-band, which meets our data simulation requirements. We selected the signal wavelength of 1470nm for the calculation process. Stimulated-emission cross section of \( ^3H_4 \rightarrow ^3F_4 \) at 1.47\( \mu m \) is 0.36 and the Absorption cross section for pump \( ^3H_6 \rightarrow ^3H_4 \) at 0.79\( \mu m \) is 0.89 [8].
3. Calculation Method

This paper uses MATLAB as a programming tool. Based on the rate propagation equation and power propagation equation of thulium ion, two methods of ergodic solution and simulated annealing algorithm are used to solve the model, and the amplified spontaneous emission spectra are obtained.

3.1. Traversal solution to draw a three-dimensional graph

3.1.1 Algorithm thinking

In the traversal solution, we first set fixed parameters: input signal power, input main pump power, input signal bandwidth power, stimulated-emission cross-section of \(^3H_4 \rightarrow ^3F_4\) at 1.47 \(\mu\)m, absorption cross-section for pump \(^3H_6 \rightarrow ^3H_4\) at 0.79 \(\mu\)m.

Set emission cross section \(\sigma_{em} = 0.36 \times 10^{-24} m^2\). The data are derived from Mira Naftaly et al. ’s study of Tm\(^{3+}\) doped tellurate glass as the main body of a 1.47\(\mu\)m broadband amplifier [8]. The relative absorption cross section \(\sigma_{abs}(\lambda)\) can be derived from the calculated \(\sigma_{em}\) using the McCumber equation as follows [9]:

\[
\sigma_{em} = \sigma_{abs}(\lambda) \cdot \frac{Z_L}{Z_U} \cdot \exp \left( \frac{\Delta E - \hbar \lambda}{k_B T} \right)
\]

where \(T\), \(h\), and \(k_B\) are the temperature, Planck’s constant, and the Boltzmann constant, respectively. \(\Delta E\) represents the energy difference between the lowest energy levels of the up and down state Stark. \(Z_L\) and \(Z_U\) represent partition functions of the up and down states respectively.

This paper uses the method proposed by Minis-calco and Quimby to calculate partition functions [10].

In this paper, the ergodic solution is performed by changing the variable fiber length and doping concentration. Through the power propagation equation of 2.2, the relationship between the amplified spontaneous emission spectrum and the fiber length and doping concentration can be obtained, and three-dimensional plots can be made.

3.1.2 Interpretation of result

Using MATLAB for the traversal solution, the variation of amplified spontaneous emission spectra with fiber length and doping density can be calculated. The emission spectrum of thulium ions is shown in Figure 3.

![Fig. 3 Amplification of spontaneous emission spectrum](Photo/Picture credit: Original)

According to the results, when the fiber length is 2.5m and the thulium ion doping concentration is \(2 \times 10^{25} \text{ / m}^2\), the signal gain is close to the peak, about 31.7085dB. When the signal gain and pump power are fixed, the larger the ion doping concentration, the shorter the fiber length required. Due to the particularity of thulium ion, it is not easy to form a particle population inversion, so the peak gain will not decrease with the increase of doping concentration but tends to be flat. At the same doping concentration, with the increase of fiber length, the gain increases rapidly, reaches the maximum value, and then decreases slowly.
3.2. simulated annealing algorithm

3.2.1 Introduction to simulated annealing theory

The project employs a heuristic optimization technique known as the simulated annealing algorithm, ideal for addressing optimization challenges within the search space. Simulated annealing emulates the temperature variation observed during the annealing process in solid materials [11]. As solid materials are heated, internal particles undergo disorder, raising their internal energy levels. Upon cooling, particles tend to reorganize and reach equilibrium at various temperatures. The simulated annealing algorithm incorporates a stochastic search approach to regulate randomness within the search space through a control parameter, i.e., temperature. It commences from a higher initial temperature and probabilistically accepts new states. As the temperature parameter decreases, the search space narrows, facilitating the random discovery of the global optimal solution for the objective function. In this study, the application of a simulated annealing algorithm is harnessed for data processing to maximize gains.

3.2.2 Based on MATLAB algorithm

This paper discusses the variation trend of gain, pump power, signal power, and ASE power with the increase of fiber length through the MATLAB program. The results of simulated annealing show that:

1) The gain increases linearly with the increase of the fiber length. When the fiber length approaches 2.39m, the gain reaches a maximum of 31.68dB, and then slowly decreases. The gain curve with fiber length is shown in Figure 4.

![Fig. 4 Gain curve with fiber length](Image)

Fig. 4 Gain curve with fiber length (Photo/Picture credit: Original)

2) The signal light power increases with the increase of the fiber length. When the fiber length approaches 2.47m, the signal light reaches a maximum value of $1.48 \times 10^{-3} dB$, and then slowly decreases linearly. The pump power curve with fiber length is shown in Figure 5.

![Fig. 5 Pump power curve with fiber length](Image)

Fig. 5 Pump power curve with fiber length (Photo/Picture credit: Original)
(3) The gain decreases linearly with the increase of fiber length. When the fiber length approaches 2.4m, the signal light tends to the minimum value of zero and then remains unchanged. The ASE power curve with fiber length is shown in Figure 6.

![ASE power curve with fiber length](Photo/Picture credit: Original)

(4) The signal light power increases with the increase of the fiber length. When the fiber length approaches 2.47m, the signal light reaches a maximum value of $7.5 \times 10^{-5} \text{dB}$, and then slowly decreases. The result of the simulated annealing procedure accords with the general rule. The signal power curve with fiber length is shown in Figure 7.

![Signal power curve with fiber length](Photo/Picture credit: Original)

3.3. Results and discussion

Through the traversal solution, it can be obtained that when the fiber length is 2.5m and the thulium ion doping concentration is $2 \times 10^{25}$ / m$^2$, the signal gain is close to the peak, which is 31.7085dB. Through the simulated annealing algorithm, the maximum gain of the result is 31.7112dB. Comparing the two methods, the results are very different. The gain, pump power, signal power, and ASE power obtained by the two methods have the same variation trend with the change in fiber length. Therefore, the solution and output results of the simulated annealing optimization algorithm are correct. Although the two results are similar, the simulated annealing algorithm has obvious advantages: less programming effort, easy implementation, and a global optimal solution can be found.

4. Conclusion

In this paper, the output signal gain characteristics of thulium-doped tellurate glass fiber amplifiers are analyzed and simulated. A mathematical model is established based on the rate equation and the transmission equation, and the simulation is carried out by ergodic solution and simulated annealing algorithm. The effects of thulium ion doping concentration, fiber length, and pump power on the output signal gain characteristics of the amplifier are studied and analyzed in detail. The variation of
amplified spontaneous emission spectra with fiber length and doping concentration is obtained by simulation. When the pump’s optical power is 700mw, the output peak gain is about 31.71dB. At the same time, the variation trend of gain, pump power, signal power, and ASE power with the increase of fiber length is obtained.

When the signal gain and pump power are fixed, the larger the ion doping concentration, the shorter the fiber length required, which is conducive to the development of miniaturization and modularization of the system. How to choose the appropriate pump power and fiber length is a problem that needs to be considered when the device is applied. The pump power in this paper is a fixed value, and how to choose the pump power is still to be studied, and the problems of resource waste and fiber bearing capacity should be considered.

References