

The Study of Visualization and Modeling of Full Potential Equation with the Influence of Small Disturbance Based on Python

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Abstract. This paper design a program based on the full potential equation aiming to visualize the fluid field around the disturbances and see how the disturbances influence the fluid flow. In the process, this article use central difference scheme for subsonic cases, while for supersonic cases, upwind scheme is applied to rewrite the equation to get the exact models. And also calculate the error to show that the models work well. The error graph shows that the results of the modeling. Fit the full potential equation theory which means the method is feasible.

Keywords: Subsonic flow, supersonic flow, full potential equation, python modelling.

1. Introduction

With the developing of aerospace, racing and shipbuilding industry, the demand for consideration of fluid dynamics during the design processes of these artificiality is getting higher and higher. Meanwhile, many traditional design of airfoils racing cars and ships may not be applied very well anymore, which create many difficulties for some traditional design methods since they are not so efficient anymore. So a new method is needed to be applied to simulate the design and make mathematical models to represent the traditional ways of examining the feasibility of the designs.

In 1999, Chao Yan and his colleague Lei Xie had mentioned optimization of aerodynamic design and anti design based on full potential equation [1]. These theories laid a solid foundation for the application of full potential equation on the designs in computational fluid dynamics field. And then, Yihua Cao and his colleagues found these methods had a bright future on research in aerodynamics, with the development of the use of full potential equation combined with finite volume method people could explain the shark waves [2]. Guoqing Hu and his colleagues used H-O grid based on full potential equation to calculate the fluid flow around the side slipping airfoil but found some error with their method [3]. Full potential equations play a more and more important role in CFD methods. Tao Zhou and his colleagues started to use full speed potential equation and Fluent programming software to design supercritical airfoil [4]. In which they mentioned modelling based on full potential equations instead of traditional ways such as N-S equation based, they also compared the results with which they got from N-S equation based models and showed the feasibility and accuracy of modelling based on full potential equation.

From the previous researches, this article try to simulate the fluid field around the airfoil or other disturbance in a way that choose 2-D full potential equation as the basement, and also choose python programming software for modelling, trying to find a more accurate model for the design of airfoils and the researches for the features of fluid flow around a disturbance, helping the researchers who research the problems based on full potential equations to make their results more visual. Besides, finite volume method, central difference scheme and upwind scheme are used to calculate the correct models for both supersonic and subsonic cases as well as vertex scheme and cell-centered scheme. Also, 3 types of models are mentioned and they will be compared their simulation of real situations. And at last, influence factors such as mach number and size of small disturbance will be changed to see whether the models fix well.

2. Math Condition

To do the simulation of full potential equation of computational fluid dynamics the software Python is used, and the following assumptions are mentioned:

(1) The fluid flow is steady, so the vorticity is zero [5].

(2) The obstacle is small when the small disturbance approximation is applied, and we restrict the discussion to 2D.

(3) Full potential equation is solved in 2D therefore z direction is out of consideration [6].

Based on previous assumption, we started with the conservation of mass:

$$\frac{d}{dt} \int_{\Omega(t)} \rho(v, t) dv = 0 \quad (1)$$

Using differentiation of integrals:

$$\frac{d}{dt} \int_{\Omega(t)} f(v, t) dv = \int_{\Omega(t)} \frac{\partial f}{\partial t}(v, t) dv + \int_{\partial\Omega(t)} f(s, t) U(s, t) \cdot nds \quad (2)$$

Then derive the continuity equation:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \nabla \phi) = 0 \quad (3)$$

For steady flow with wall boundary condition:

$$\mathbf{V}_n = \frac{\partial \phi}{\partial n} = 0 \quad (4)$$

And under the condition of constant entropy, it is easy to derive the relation:

$$\frac{\rho}{\rho_0} = \left(1 - \frac{|\nabla \phi|^2}{2H_0} - \frac{\partial_t \phi}{H_0}\right)^{\frac{1}{\gamma-1}} \quad (5)$$

Considering expand the derivatives of the density and with assumption that it is in the steady flow case cooperate with the small disturbance approximation [7]:

$$\phi = U_\infty (x + \Phi) \quad (6)$$

The equation of the full potential equation needed as the control function of the programme shows up:

$$(1 - M^2) \Phi_{xx} + (1 - M^2) \Phi_{yy} = 0 \quad (7)$$

Where M is the mach number.

At the bottom we specify the boundary condition:

$$\frac{\partial \phi}{\partial n} = -\frac{\partial \phi}{\partial y} = U_\infty f'(x) \quad (8)$$

3. Physical modeling

3.1. Boundary conditions

Base on the previous functions, as a popular programme langue, python is used to do the model work. Figure 1 shows the 2dimensional structure of the full potential equation model [8].

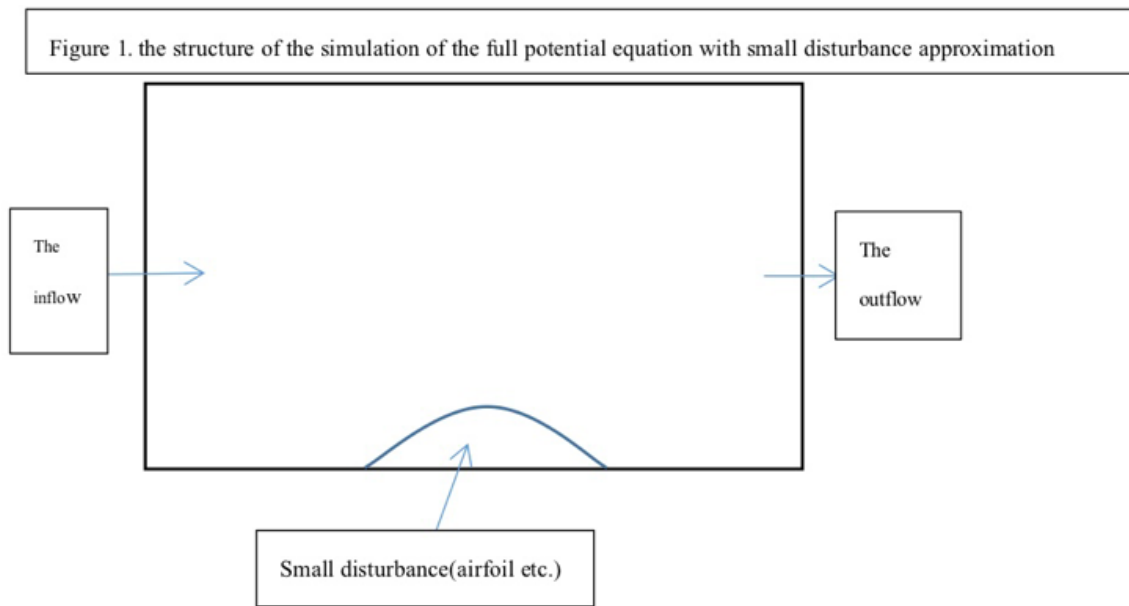


Figure 1. The 2dimensional structure of the full potential equation model.

Finite volume method as well as finite differences approximation were used to discretize the previous equations. Based on the discrete forms of the equations, the modeling was done in three steps [9]:

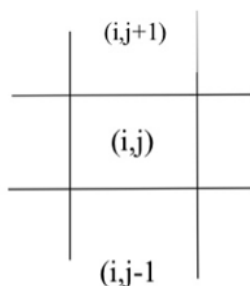
- (1) $(1-M^2)\phi_{xx} + \phi_{yy}=0$, where M is a fixed number.
- (2) $(1-M^2)\phi_{xx} + \phi_{yy}=0$, where M is a function of ϕ , and $M<1$ $M>1$ are both relevant.
- (3) $(\rho)\phi_{xx} + (\rho)\phi_{yy}=0$, where ρ is a function of ϕ .

Also, for subsonic case (Mach number <1), the central difference scheme is applied, and for supersonic case, the upwind scheme is used. Thus the control function of the model is given by the discrete form of the full potential equation with central difference scheme and upwind scheme respectively for two situations [10].

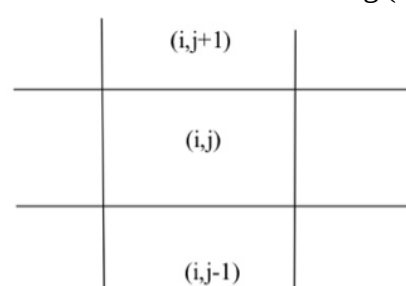
In order to get a numerical solution, we also applied boundary conditions. There are two different boundary conditions. Dirichlet boundary condition and Neumann boundary condition [11]:

(1) Dirichlet boundary condition specifies the value of the solution of the differential equation at the boundary (Figure 2(a)).

(2) Neumann boundary gives the directional derivative of the normal line outside the boundary of the unknown function. It is also the boundary condition we choose to use in our modeling (Figure2(b)).



(a) Solution value at the boundary



(b) Model Boundary Conditions

Figure 2. Two different boundary conditions.

To make the simulation more real and have a more universal usage, a small disturbance whose shape is controlled by the function:

$$\beta \times \sin(\pi \times (x-1))$$

Besides, Gauss-Seidel method is applied to solve the discrete forms of the full potential equation.

3.2. Design process

From the above preparation, the control functions can be expressed as follow [12]:

(1) Subsonic case

$$eq = -C_0 \times \varphi_{i,j} + (B_{yp} \times \varphi_{i,j+1} + B_{ym} \times \varphi_{i,j-1} + A_{xp} \times \varphi_{i+1,j} + A_{xm} \times \varphi_{i-1,j}) \quad (9)$$

Where

$$C_0 = A_{xp} + A_{xm} + B_{xp} + B_{xm} \quad (10)$$

(2) Supersonic

$$eq = -C_0 \times \varphi_{i,j} + (B_{yp} \times \varphi_{i,j+1} + B_{ym} \times \varphi_{i,j-1} - A_{xp} \times \varphi_{i+1,j} - A_{xm} \times \varphi_{i-1,j} + A_{xm} \times \varphi_{i-2,j}) \quad (11)$$

Where the central point is

$$C_0 = -A_{xp} + B_{xp} + B_{xm} \quad (12)$$

For model 1:

$$A_{xp} = 1 - M^2, A_{xm} = 1 - M^2, B_{yp} = 1, B_{xm} = 1 \quad (13)$$

Where M represents the mach number.

For model 2:

$$A_{xp} = (1 - \mu) \times k_{ij}, A_{xm} = \mu \cdot k_{i,j} + (1 - m) \times k_{i,j}, A_{xmm} = \mu \cdot k_{i,j}, B_{yp} = 1, B_{xm} = 1 \quad (14)$$

In this case, M represents the mach number:

$$1 - M^2 = k_{i,j} = 1 + M_\infty^2 - (\gamma + 1) \cdot M_\infty \cdot \frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta h} \quad (15)$$

Where

$$\Delta h = \frac{3}{\varphi.shape[0]} \quad (16)$$

And the central point is changed into:

$$C_0 = A_{xp} + (-1)^\mu \times A_{xm} + B_{ym} + B_{yp} \quad (17)$$

For model 3:

$$A_{xp} = \rho_{xp}, A_{xm} = \rho_{xm}, B_{yp} = \rho_{yp}, B_{ym} = \rho_{ym} \quad (18)$$

In this case, take Bym as an example:

$$\rho_{ym} = \rho_0 \times \left[\frac{1 - (\varphi_x^2 + \varphi_y^2)}{2H_0} \right]^{\frac{1}{\lambda-1}} \quad (19)$$

Where

$$\Delta h = \frac{3}{\varphi.shape[0]} \quad (20)$$

$$\rho_y = \frac{(\varphi_{i,j+1} - \varphi_{i,j-1})}{2h} \quad (21)$$

$$\varphi_x = \frac{\varphi_{i+1,j+1} - \varphi_{i-1,j+1} + \varphi_{i+1,j-1} - \varphi_{i-1,j-1}}{4h} \quad (22)$$

And gamma=1.4, $\rho_0=1$, with the scheme shown in Figure 3:

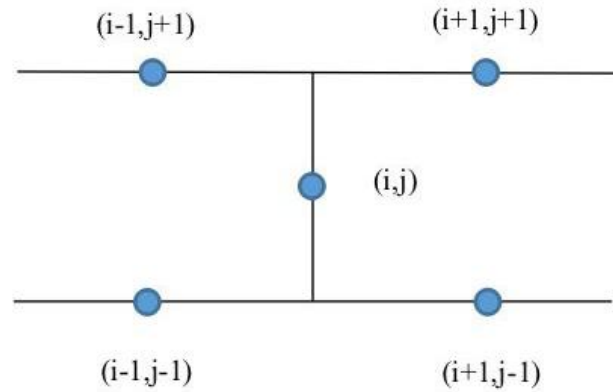


Figure 3. Model 3 schematic.

Notice in this case, A_{xp} and A_{xm} are restricted with the function:

$$\varphi_y = BC - F(i \times h) \times U_\infty + BC - F((i+1) \times h) \times U_\infty \quad (23)$$

At the boundary where:

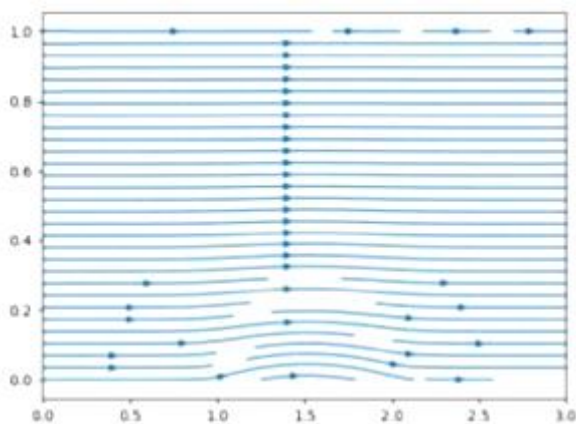
$$F(x) = \beta \times \cos(\pi \times (x-1)) \quad (24)$$

$$U_\infty = \left(\frac{|M_\infty^2 \times (\gamma-1) \times H_0|}{1 + M_\infty^2 \times \frac{\gamma-1}{2}} \right)^{0.5} \quad (25)$$

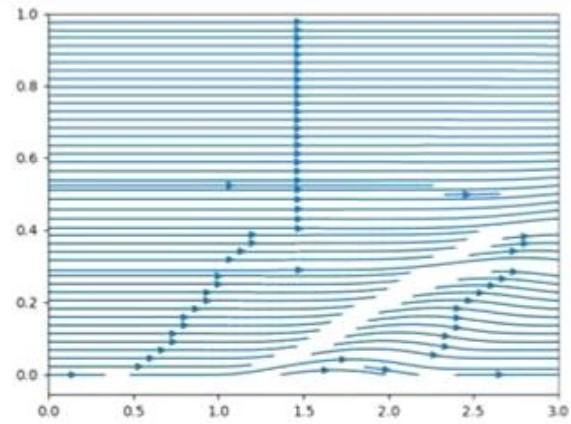
Which can be derived from the full potential equation with a constant M infinity and $H_0=1$.

4. Results and discussing

As the first step, model1 was built first. For subsonic case, beta =0.1 and mach number= 0.3, the result is shown in Figure 4(a), in it the small disturbance is laid in the middle and the fluid flow is crossing the disturbance in the way that is excepted at the beginning. And besides it is the error graph(Figure 4(a)) which shows that the error of the simulation is very small and it fits the theory very well. After it is the graph for the supersonic case in which Mach number=1.3 and beta=0.1 (Figure 4(b)) and its error graph (Figure 4(b)). It also fits well.



(a) Beta=0.1 and mach number= 0.3



(b) Mach number=1.3 and beta=0.1

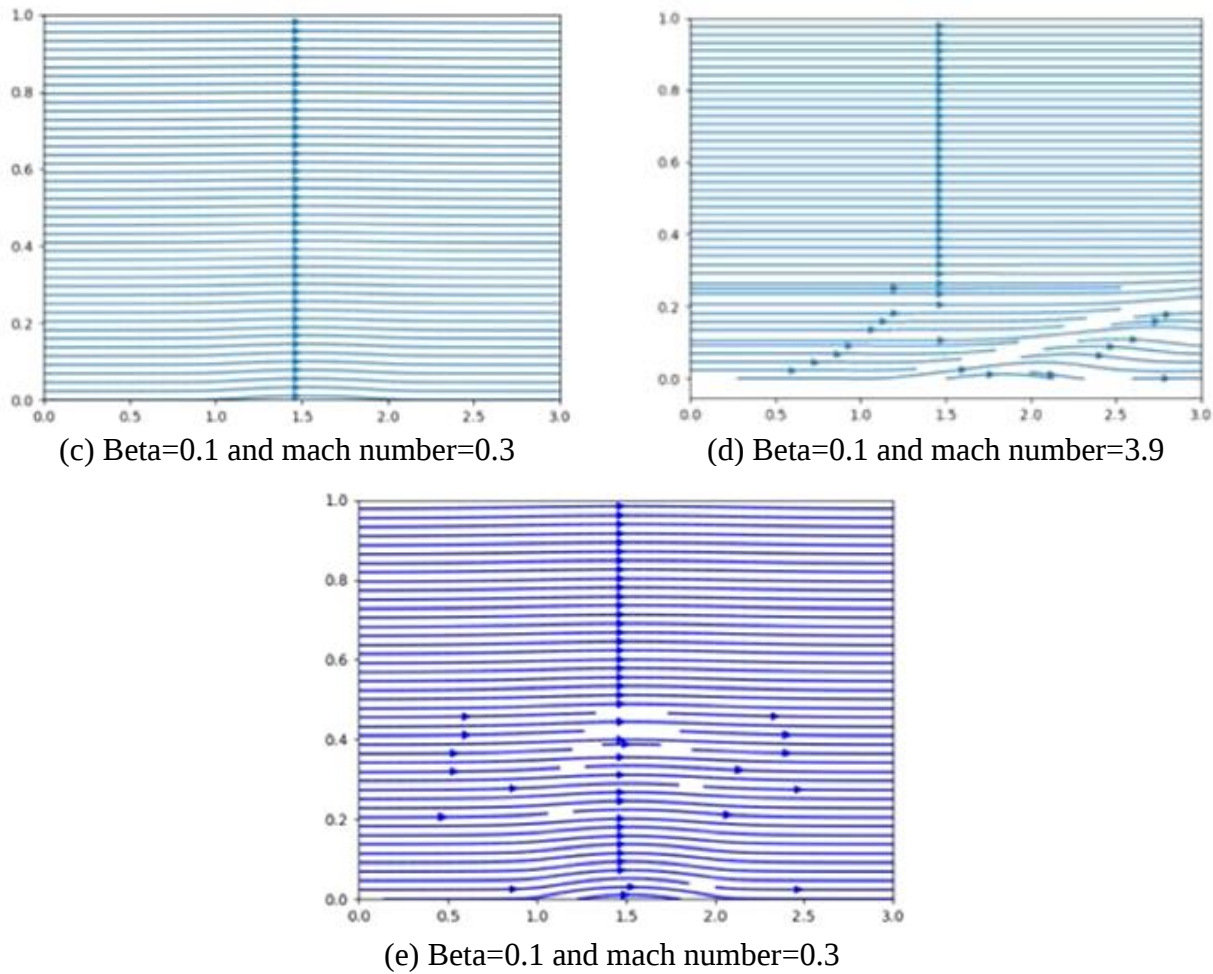


Figure 4. Model results under different beta and mach numbers.

4.1. The influence of obstacle

To be more specify, take model 2 as example, mach number is set to be 3.5 and compare the influence of the small disturbance (Figure 5), it shows that with the size of the disturbance become larger the the shark angle becomes larger, which consistent with the hypothesis that $\theta \propto \beta$ where β represents the size of the disturbance. This also implies with the different disturbance used in the simulation process, this full potential equation based method always works well.

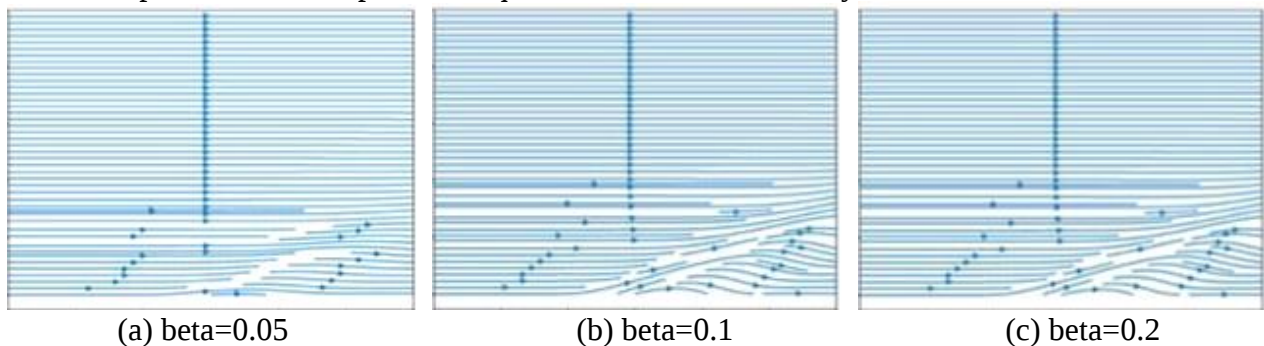


Figure 5. The influence of obstacle.

4.2. The influence of Mach number

In this case, the size of the disturbance is fixed with $\beta = 0.1$, and the Mach number increased gradually from 2.5 to 5.3 (Figure 6). With the increase of Mach the shark angle becomes smaller, so

it satisfy the hypothesis that $\theta \propto \frac{1}{Mach}$, hence the models can also fits the situation when the experiment needs different Mach speed condition.

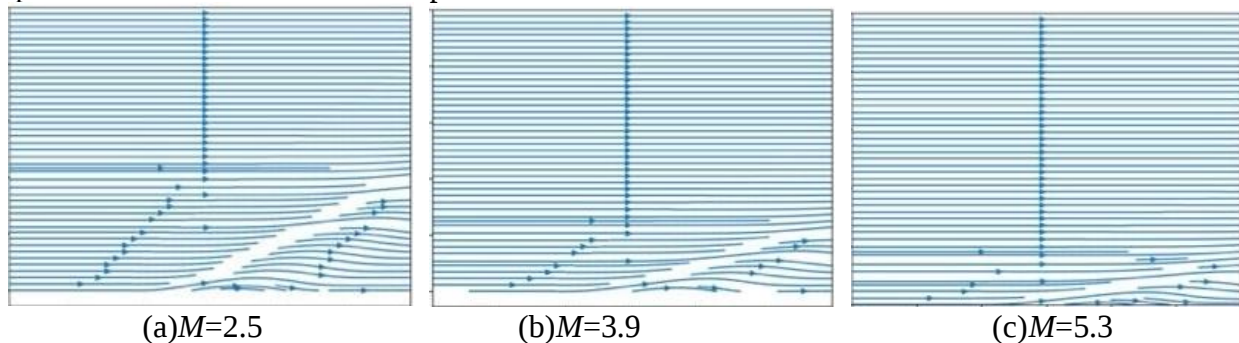


Figure 6. The influence of mach number.

From the above, the models can work well in the simulation process no matter how the influence factors are changed, so it is a reliable and feasible method.

5. Conclusions

The study on full potential equation has a long history, more and more studies on airfoil design, fluid flow simulation around obstacles start to choose full potential equation as the base instead of N-S equation, also, as the development of programming in modeling, a better way for the visualization of the full potential equation instead of physical experiments has occurred. The python programme which choose full potential equation with different parameters as the control function is a wonderful choice. Parameters include: obstacle size β , mach number M , shark angel θ . And from the results, the following conclusion are obtained:

From the error graphs, the errors of the simulations are small enough to be ignored, so the simulations are reliable.

From the relations between the shark angle θ and obstacle size β , mach number M which are $\theta \propto \frac{1}{Mach}$ $\theta \propto \beta$, the simulations are well qualified.

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