

Research on Bicycle Road Time Trial Based on Differential Equation

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Abstract. Bicycle road time trial is a kind of bicycle road race, including individual time trial and team time trial. The purpose of this paper is to build a power prediction model to provide the rider with an optimal power distribution strategy according to the characteristics of the track and the rider's abilities. We first analyze the energy conversion problem in the speed direction of the system composed of bicycles and people. In order to extend our model, we get the distance as a function of time. Calculate the performance of each type of rider on the two tracks, and find that the relative error of the results is about 10%, which proves that the establishment of our model is effective. Afterwards the model was applied to the track we designed, and the results were reasonable by comparing the data. Further, we consider the influence of wind speed in combination with the actual situation, to optimize our model so that the results are more accurate, and the relative error is within 5%.

Keywords: Individual Time Trial, Power Curve, Differential Equation.

1. Introduction

There are many types of road cycling, including standard, team time trials and individual time trials. The individual time trial is a race against time. Team time trial rider performance is related to route, ability and strategy.

Due to the long track of the bicycle road race, a single rider can produce different levels of power at different times, and the power and time given by a rider can vary widely from rider to rider. A rider's power curve represents the amount of time a rider can produce a given amount of power. Therefore, how to maintain physical strength and make ourselves consume power reasonably throughout the game is a problem that we need to study. Riders can choose to briefly exceed the limits of their power curve, but riders will need additional time at lower power levels to recover.

2. Model of Power and Position

2.1. The Establishment of Power and Position Model

Step 1: In the process of riding a bicycle, the rider is relatively fixed on the bicycle and assumes the role of the engine to generate power to propel the bicycle. External factors that impede the movement of the bicycle/rider system can be modeled according to basic engineering and physical principles. These factors include aerodynamic drag, rolling resistance, friction in bearings and chain drive systems, and changes in kinetic and potential energy. However, the forward movement of the bicycle is different from the ordinary rigid body motion. The structural hollow formed between the person and the vehicle will change the maximum head-on area and the wind resistance coefficient. The rotation of the wheel and the pedaling of the lower limbs will produce the Magnus effect [1]. In addition, compared with track cycling, wind changes and road undulations in the outdoor environment will make the force structure and process more complicated. In the two-dimensional coordinate system constructed in the direction of motion perpendicular to the ground, the overall force on the person-vehicle riding in the rider's hands is mainly composed of gravity F_m , ground support force F_N , tire-ground friction force f , wheel the air resistance F_{roll} during rotation, the friction F_{bear} in the bearing and chain drive system, the forward power is F_{ped} , the force diagram is shown in FIG 2.

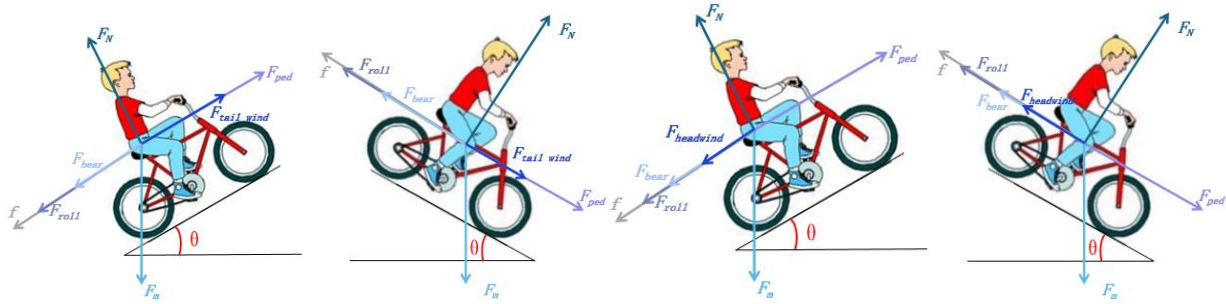


Figure 1. Force analysis diagram about four situations

Step 2: According to the discovery of A.V.Hill [2], winner of the Nobel Prize in Medicine and Physiology, it can be concluded that the relationship between time and speed during exercise is not an ordinary curve, but a rectangular hyperbola. Based on typical power profiles of determining critical power and W' (the work capacity above CP) [2]. Of which CP is the rate associated with maximal aerobic steady-state, and W' is the fixed available value above CP. Tolerate power without any recovery. The slope and intercept of the linear relationship can be derived using the data of CP and W' . According to the long-distance cycling competition, the athlete's pace can be divided into three stages: front, middle, and back [3]. Finally, the three stages are combined to form a model of the power and position relationship of the timer expert.

2.2. The Solution of Power and Position Model

Step 1: The state-of-the-art mathematical model for road cycling power, established by Martin is a nonlinear differential equation based on an equilibrium of resistance power and pedaling power P_{ped} provided by a cyclist to propel his bicycle. [4] The resistance power is composed of power due to gain in potential energy P_m , aerodynamic drag Pair, frictional losses in wheel bearings P_{bear} , rolling friction P_{roll} , and the gain in kinetic energy P_v :

$$P_m + P_{wind} + P_{bear} + P_{roll} + P_v = \eta P_{ped} \quad (1)$$

Where the efficiency factor η accounts for frictional loss in the drive chain.
Thus

$$\gamma = \frac{n_{Fron}}{n_{rear}} \quad (2)$$

Divide this equation by the angular velocity of the wheel to get the corresponding torque balance, in which the principle of leverage must be considered, and the transmission ratio γ is used, that is, the ratio of the number of teeth of the front sprocket to the number of teeth of the rear sprocket.

Thus:

$$W_m + W_{wind} + W_{bear} + W_{roll} + W_v = \frac{\eta}{\gamma} W_{ped} \quad (3)$$

And because the pedal torque is equal to the product of the pedal force and the crank length:

$$W_{ped} = F_{ped} \cdot l_c \quad (4)$$

Thus:

$$F_m + F_{wind} + F_{bear} + F_{roll} + F_v = \frac{\eta}{\gamma} \frac{l_c}{r_w} F_{ped} \quad (5)$$

Since the chain loses part of the energy when driving the rear wheel to rotate [10], the research shows that the transfer efficiency of the chain is positively correlated with the riding force. In general, the loss of the chain transfer efficiency of highway bicycles is about 98.5%, so we take $\eta = 0.985$:

$$F_{ped} = \frac{P}{v} \quad (6)$$

We consider the rotational inertia of the front and rear wheels, and we can see from the force analysis diagram:

$$F_v = (m + \frac{I_w}{r_w^2})a \quad (7)$$

And, because $a = v'(t)$

$$F_v = (m + \frac{I_w}{r_w^2})v' \quad (8)$$

$$F_m = mg \sin \theta \quad (9)$$

$$F_{roll} = \mu mg \quad (10)$$

Since the chain will lose some energy when driving the rear wheel to rotate, foreign studies have shown that the transmission efficiency of the chain is positively correlated with the riding force [5]. In general, the transmission efficiency loss of the chain of a road bicycle is about, so:

$$F_{wind} = \frac{1}{2} \rho C_D A v^2 \quad (11)$$

Where air density is ρ , resistance coefficient is C_D , frontal area is A , v is the speed relative to the rider's speed and the relative speed of airflow (depending on the wind speed and direction and the speed of the bicycle).

The rotating object will produce the dynamic effect of asymmetric fluid in the viscous fluid, and the air resistance of the wheel motion is generated by the rotation of the wheel in the air [6]. The size of the rotational resistance mainly depends on the size of the wheel and the shape of the hub, and will not change significantly with the speed of the wheel. At the same time, due to the action of the human body and the vertical beam of the bicycle, the air resistance of the rear wheel will be reduced by 25%. In general, the radii of the front and rear wheels of a road bicycle are the same as the wheel spokes, and the formula for calculating the total air resistance when the front and rear wheels rotate is [7]:

$$\begin{aligned} F_{bear} &= F_{fb} + F_{rb} \\ &= \frac{1}{2} C_w \rho v^2 \pi r_w^2 + \frac{3}{4} \times \frac{1}{2} C_w \rho v^2 \pi r_w^2 \\ &= \frac{7}{8} C_w \rho v^2 \pi r_w^2 \end{aligned} \quad (12)$$

Where, C_w is the coefficient of air resistance of the bicycle wheel.

So the model can be transformed into:

$$mg \sin \theta + \frac{1}{2} \rho C_D A v^2 + \frac{7}{8} C_w \rho v^2 \pi r_w^2 + \mu mg + (m + \frac{I_w}{r_w^2})v' = \frac{\eta}{\gamma} \frac{l_c}{r_w} \frac{P}{v} \quad (13)$$

Step 2: We also took into account the physiological conditions of the players themselves and established a physiological model: using a three-parameter model.

$$t = \frac{W}{P - CP} + K \quad (14)$$

Where K is the constant, W is the anaerobic working capacity, P is the power, and CP is the critical power.

Step 3: And because the rider had the maximum power at that time p reaches the maximum, and it can be seen by substituting:

$$K = -\frac{W}{P_{\max} - CP} \quad (15)$$

That is:

$$t = \frac{W}{P - CP} - \frac{W}{P_{\max} - CP} \quad (16)$$

Thus:

$$P = \frac{W(P_{\max} - CP)}{t(P_{\max} - CP) + W} + CP \quad (17)$$

In summary, the resulting model is:

$$\left\{ \begin{array}{l} mg \sin \theta + \frac{1}{2} \rho C_D A v^2 + \frac{7}{8} C_w \rho v^2 \pi r_w^2 + \mu mg + \left(m + \frac{I_w}{r_w^2} \right) v' = \frac{\eta l_c}{\gamma r_w} \frac{P}{v} \\ P = \frac{W(P_{\max} - CP)}{t_m(P_{\max} - CP) + W} + CP \end{array} \right. \quad (18)$$

Thus:

$$v' = \frac{r_w^2}{m r_w^2 + I_w} \left[\frac{\eta l_c}{\gamma r_w} \frac{1}{v} \left(\frac{W(P_{\max} - CP)}{t(P_{\max} - CP) + W} + CP \right) - \frac{1}{2} \rho C_D A v^2 - \frac{7}{8} C_w \rho v^2 \pi r_w^2 - \mu mg - mg \sin \theta \right] \quad (19)$$

From the relationship between distance and time, the model of distance and time is obtained:

$$S = \int_0^t v dt \quad (20)$$

Thus (the Model 2 is that)

$$\left\{ \begin{array}{l} v' = \frac{r_w^2}{m r_w^2 + I_w} \left[\frac{\eta l_c}{\gamma r_w} \frac{1}{v} \left(\frac{W(P_{\max} - CP)}{t(P_{\max} - CP) + W} + CP \right) - \frac{1}{2} \rho C_D A v^2 - \frac{7}{8} C_w \rho v^2 \pi r_w^2 - \mu mg - mg \sin \theta \right] \\ S = \int_0^t v dt \end{array} \right. \quad (21)$$

Step3: In addition, we also need to consider the situation at the corner. For the force analysis of the corner [8], we only consider the force in the direction of velocity, and we find that our model is still applicable.

Since the human-vehicle system needs to be inclined when the bicycle is turning, the angle between the vehicle body and the vertical ground is set as θ . The stress analysis is shown in:

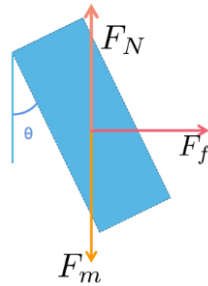


Figure 2. Force analysis diagram of Bicycle Turning

Among them: gravity is F_m , ground support force is F_N and friction force is $F_f = F_{wind} + F_{bear} + F_{roll}$, to ensure that the car does not tip over when cornering, there are:

$$\begin{cases} F_m = F_N & (1) \\ F_f = \frac{mv^2}{k} & (2) \end{cases} \quad (22)$$

Where k is the curvature radius of the turning.

Thus,

$$\mu mg = \frac{mv^2}{k} - \frac{1}{2} \rho C_D A v^2 - \frac{7}{8} C_w \rho v^2 \pi r_w^2 \quad (23)$$

And the rider wants faster through the turning, naturally requires faster speed, k constant at each turning, the greater the ground friction $F_{roll} = \mu mg$, and the maximum ground static friction $F_{roll} = \mu mg$, so the vehicle does not fall when turning conditions are:

$$\mu mg \geq \frac{mv^2}{k} - \frac{1}{2} \rho C_D A v^2 - \frac{7}{8} C_w \rho v^2 \pi r_w^2 \quad (24)$$

Thus, When the rider at the corner is better to pass at this speed:

$$v_{\max} = \sqrt{\frac{8\mu kmg}{8m - 4\rho k C_D A - 7\rho k C_w \pi r_w^2}} \quad (25)$$

3. Applied to Windy environment

Based on the original model, we consider the effects of wind speed and direction [9], the wind direction is divided into upwind and downwind, mainly affecting:

When tailwind $v_{wind} \geq 0$, then headwind $v_{wind} < 0$.

We further analyze the problem and optimize the model, considering the effect of wind speed on the model [10]. Observing the column chart and the error analysis chart, we find that the relative error is less than 5%, indicating that the result is more accurate, as shown in Figure 3 and Figure 4.

Table 1. Analysis under wind conditions (Tokyo)

name	Gender	the initial value	Grade	residual error	relative error
Time Tria	Male	3369.93	3536.74	-166.81	4.95%
Rouleur	Male	3404.72	3568.15	-163.43	4.80%
Time Tria	Female	1815.12	1948.50	-78.54	4.20%
Rouleur	Female	1869.96	1863.95	-48.83	2.69%

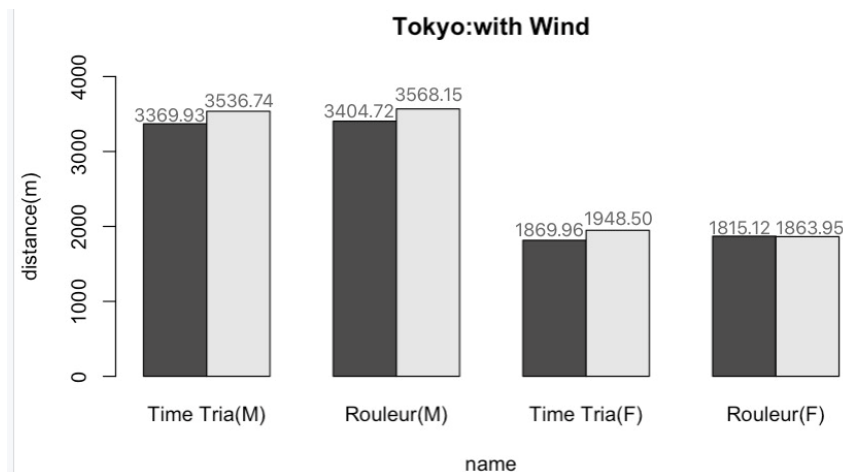


Figure 3. Analysis results comparison chart (Tokyo)

Table 2. Analysis under wind conditions (UCI)

name	Gender	the initial value	Grade	residual error	relative error
Time Tria	Male	2867.80	3006.89	-139.09	4.85%
Rouleur	Male	2873.20	3009.34	-136.14	4.74%
Time Tria	Female	2176.00	2246.72	-70.72	3.75%
Rouleur	Female	2189.30	2259.67	-70.37	3.21%

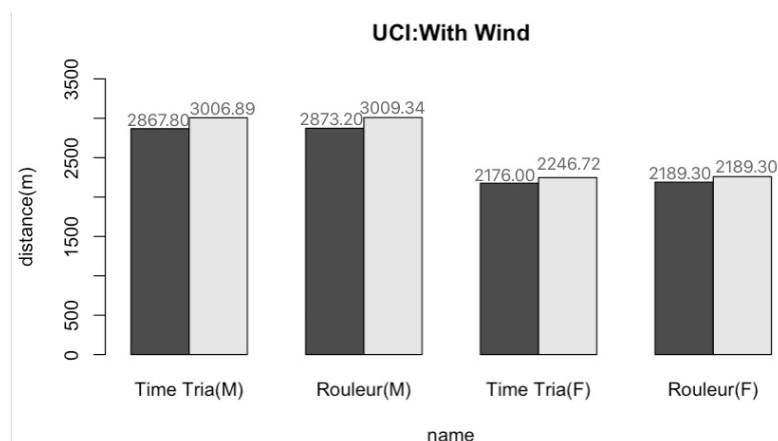


Figure 4. Analysis results comparison chart

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