Optimal Placement Algorithm for Beehives with Pollination Probability Model

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Abstract. The beehive placement problem is an important optimization problem widely applied in industrial and logistics domains. This paper proposes pollination probability model and beehive covering model, converting beehive placement problem into spatial partition problem. We perform Voronoi diagram on the farmland based on triangulated beehive points set and define an energy function on each block, decreasing the value of the energy function by continuously moving the hive to the geometric center of each block, iterating until the maximum number of iterations is reached or the maximum moving distance is less than ε. After the iteration, the minimum pollination probability in each block will be calculated. If the probability of a block is less than ξ, the number of beehives will be increased and the algorithm will be re-run until all blocks satisfy the condition. Simply enter the shape of the farmland into the model to get the optimal hive placement, the model works with all common polygonal shapes and can be applied to other similar problems with little modification.

Keywords: Pollination Probability Model, Beehive Covering Model, Voronoi Diagram.

1. Introduction

Beehive placement problem is a discrete location optimization problem, which often arise in optimizing warehouse layouts, equipment configurations, and logistics distribution center location. The significance of this problem lies in its wide practical applications and effective costs reduction. Beehive placement problem aims to maximize the efficiency or minimize the cost of honeybee collection, promoting sustainable development of beekeeping industry.

A number of scholars have also done a lot of research in this field over the past few years. V Milićević presents a functional platform for matching beekeepers, transporters, and landowners, and develops two algorithms for calculating optimal beekeeping routes to increase honey yield and minimize transportation costs [1]. Pan J S proposes a new intelligent evolutionary algorithm called Rafflesia Optimization Algorithm, based on the life habits of Rafflesia, which performs local and global searches to find optimal solutions [2]. S Dou proposes an immune wolf colony hybrid algorithm to solve the cold chain logistics distribution center location problem, which introduces constraints such as freshness and time window, improves the diversity of the wolf colony algorithm by using immune operators and memory cells, and expands the search space of the solution, showing good feasibility and robustness in simulation results [3]. M Samadi focuses on optimal localization at shopping centers using genetic algorithm as a meta-heuristic algorithm, and emphasizes the importance of accuracy and validity in selection and localization to avoid negative results such as high costs, with findings showing the effectiveness of meta-heuristic methods in achieving accurate results [4]. H Li proposes an uncertainty theory-based optimization model for equipment supporting depot location and transportation volume, using genetic algorithm to optimize the solution, to improve the support level of complex systems by addressing problems of chaotic warehouse layout and low efficiency of spare parts [5].

In this paper, we provide a novel idea of spatial segmentation combined with probabilistic models to solve site selection problem within a fixed range. We reduce the energy function by continuously adjusting the position of the points in the space segmentation iteration, and guarantee the quality of the final result through the probability model. Our model has strong robustness and high generalization ability, not only can solve the beehive placement problem, but also can be easily transferred to other similar problems with little modifications. Our work benefits in ensuring efficient pollination and improving the efficiency honeybee harvesting. Furthermore, it provides a new idea for the solution of the discrete location problem.

2. Optimal Placement Algorithm

2.1. Simplify to Set Covering Problem

First, we can reduce the problem to a Set Covering Problem [6], i.e., to cover all the crops as much as possible with a minimum amount of hives. If we assume that there are m crops and n hives, the distance matrix $D_{m\times n}$ represents the distance between the $i(1 \le i \le m)$ crop and the $j(1 \le j \le n)$ hive is d_{ij}, and in addition we use the matrix X_{ij} to denote the crop i within the coverage of the hive j, $x_{ij} = 1$ means $d_{ij} \le R$, where R represents the radius of the honey bee's high probability of collecting honey, then we can write a simple initial model:

min n

$$s.t.\begin{cases}\sum_{j=1}^{n} x_{ij} \ge 1 \quad \forall i \in \{1,2,\dots,n\} \\ x_{ij} = \begin{cases}1 & d_{ij} \le R \\ 0 & d_{ij} > R\end{cases}\end{cases} \tag{1}$$

But this model is very crude and unrealistic. In fact, according to the information, bees do not want to fly too far from the hive, and if there is pollen near the hive, they will only collect nectar nearby [8]. Actually, the likelihood of bees flying as far as possible is inversely proportional to the distance. In addition, each bee has a limited amount of time and energy to help pollinate only about 2,000 flowers per day, so if a hive covers too large an area of farmland, pollination efficiency will be too low.

2.2. Pollination Probability Model

We then introduce a probabilistic pollination model to evaluate the odds that the crop will be pollinated in a given time. We assume that there are a total of n hives and that the probability function of at least one bee in the hive flying away from the hive d km is $p(d)$. Based on the objective situation that the probability decreases with distance [7], we assume that

$$p(d) = e^{-\frac{d^2}{\kappa}} \tag{2}$$

It is clear that $\boldsymbol{p(0) = 1, p(\infty) = 0}$. According to a research of pollination range [7], honeybees can travel up to 20 km, but typically stay within 6 km of their hive. Therefore here we take κ = 60, then we have $\boldsymbol{p(d) > 0.5(d \le 6)}$.

Then for a single crop, we set the distance from the hive $\boldsymbol{i(1 \le i \le n)}$ to that crop to be $\boldsymbol{d_i}$, then the probability of a bee coming to collect honey in a day is

$$P = 1 - \prod_{i=1}^{n}\left[1 - p\left(d_i\right)\right] \tag{3}$$

Then we want each crop to have more than ξ probability of being able to be pollinated in time t, i.e.

$$1-(1-P)^t = 1-\left[1-\prod_{i=1}^{n}\left(1-p\left(d_i\right)\right)\right]^t \ge \xi \tag{4}$$

This probabilistic model helps us to evaluate how good a hive placement scheme is in terms of pollination. It is important to note that for beekeepers, the goal of keeping bees is to harvest as much honey as possible, while for growers, the goal of keeping bees is to pollinate each crop as much as possible and thus harvest the fruit. The two goals are very different, and the question emphasizes "a parcel of land containing crops that benefit from pollination", so we only consider pollination as the goal, that is, to spread out the hives to cover as many crops as possible.

Next we started to consider the minimum number of hives needed to pollinate as many crops as possible on the field. To minimize the cost of hives, we need to have each hive cover as many crops as possible and as close to the hive as possible. Ostensibly we are solving for the number of hives, but in fact we are looking for an efficient hive arrangement, and the two are closely related.

2.3. Voronoi diagram

Laying out a beehive is essentially dividing the farmland spatially, and we want the crops within each plot to be the closest to the beehive on that plot. Therefore, we can use Voronoi diagram shown in Figure 1 to spatially partition farmland.

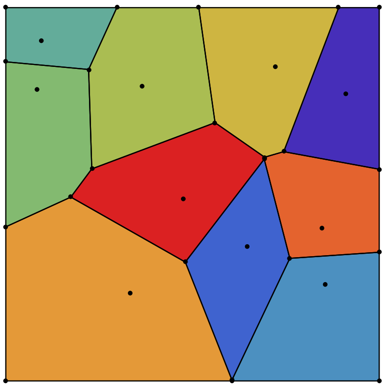

Figure 1 Voronoi diagram

After determining all the centers, the Voronoi diagram can divide all the points in the surface into the blocks whose nearest centers are located, in this problem the centers are the beehives. Suppose the Surface is S and there are k centers $\{c_i\}(1 \leq i \leq k)$, and each partitioned block $\{S_i\}(1 \leq i \leq k)$ corresponds to the center c_i, then S_i can be expressed in the form of a point set as

$$S_i = \left\{ p \in S | \ d\left(c_i, p\right) = \min\left\{ d\left(c_j, p\right) | \ j \in \{1, 2, \ldots, k\} \right\} \right\} \tag{5}$$

Next, let's briefly introduce how to generate a Voronoi diagram. Notice that there is one and only one center in each division, and the center is the nearest center for any point in the division. Then we consider the boundary of each block, all boundary lines are common edges of two blocks, which means that the points on the boundary line have the characteristics of two blocks, i.e., the distance to the center of two blocks is equal, so we can conclude that any boundary line is on the perpendicular line of two of the center points. It's known that the intersection of the three perpendiculars of a triangle is the circumcenter, so we just need to divide the surface into triangles, and then connect all the circumcenters according to a certain pattern. We can adopt the following algorithm steps to build the Voronoi diagram [8]:

1. Build triangular mesh for the center points using the Delaunay triangulation algorithm.
2. Calculate the coordinates of the circumcenter of each triangle.
3. Traverse all triangles and connect the circumcenters of each adjacent triangle to the circumcenter of the current triangle.
4. If there is no adjacent triangle on a certain side, draw a perpendicular from the circumcenter to the side.
5. All Voronoi edges can be obtained after the traversal.

2.4. Beehive Covering Model

After determining all the centers, the Voronoi diagram can divide all the points in a space into the blocks whose nearest centers are located. In this problem the centers are the beehives. Suppose the surface is S and there are k centers $\{c_i\}(1 \le i \le k)$, and each partitioned block $S_i(1 \le i \le k)$ corresponds to the center c_i, then S_i can be expressed in the form of a point set as

$$E(S_i) = \sum_{p \in S_i} \| p - c_i \|^2 \tag{6}$$

In fact we do not consider here the distribution of crops on the farmland, because we want to propose a hive placement scheme oriented to the farmland, so that we can guarantee a certain probability of pollination opportunities anywhere on the farmland. If a crop-oriented consideration is needed, i.e., the distribution of crops is heterogeneous, then we can just consider the energy function of all crops on this divided block to the center, i.e., $p_{crop} \in S_i$.

Now our aim is to make the energy function of each division block as small as possible. Also the pollination probability of each crop is above ξ. Hence the final model is summarized as follows

$$\min \sum_{j=1}^{n} E(S_j)$$

$$\begin{cases} p(d_{ij}) = e^{-\frac{\|c_i - p_j\|^2}{\kappa}} & i \in (1,2,\dots,m\}, \; j \in \{1,2,\dots,n\} \\ P_i = 1 - \prod_{j=1}^{n} \left[1 - p(d_{ij})\right] & i \in \{1,2,\dots,m\} \\ 1 - (1 - P_i)^t \le \xi & i \in \{1,2,\dots,m\} \\ E(S_j) = \sum_{p \in S_j} \| p - c_j \|^2 & J \in \{1,2,\dots,n\} \end{cases} \tag{7}$$

2.5. Optimal Beehive Placement Algorithm

If the crops are uniformly distributed on the farmland, then in fact the energy function can be simply equivalent to

$$E(S_i) = \iint_{S_i} \| p - c_i \|^2 \, \mathrm{d}\sigma \tag{8}$$

Which can be regarded as the moment of inertia of a uniform mass distributed plane with its rotational axis vertically through c_i.

According to the parallel axis theorem, the moment of inertia is minimized when the axis of rotation passes through the center of mass. The intersection of rotational axis in the plane is here equivalent to the beehive. Also when the mass is distributed evenly, the center of mass is located at the geometric center.

Notice that all the regions are polygons in a Voronoi diagram. Hence we assume that a polygon consists of N vertices $\{(x_i, y_i)\}(0 \le i \le N-1)$, then the coordinates of the geometric center of the polygon (C_x, C_y) can be written as

$$C_x = \frac{1}{6A} \sum_{i=0}^{N-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \tag{9}$$

$$C_y = \frac{1}{6A} \sum_{i=0}^{N-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \tag{10}$$

$$A = \frac{1}{2}\sum_{i=0}^{N-1}\left(x_i y_{i+1} - x_{i+1} y_i\right) \tag{11}$$

Where A is the area of the polygon and (x_N, y_N) is assumed to be the same as (x_0, y_0), meaning that vertice $N-1$ must be connected to vertice 0 to form a closed polygon.

Therefore we designed an optimization algorithm for hive placement with a specified initial number of hives n:

1. Generate n random center point coordinates in the farmland.
2. Generate a Voronoi diagram of these random points through the previously mentioned algorithm.
3. Calculate the geometric center coordinate of each block.
4. Adjust the center point to the geometric center of the block and record the adjustment distance $\{d_i\}(1 \le i \le n)$.
5. Exit the loop when $\max\{d_i\} < \varepsilon$ or the maximum number of iterations is reached, otherwise regenerate the Voronoi diagram for the new centers and return to step 3.
6. Calculate the minimum pollination probability $P_{\min}$ for each block, and if $P_{\min} < \xi$, then $n = n + 1$ and return to step 1.

Obviously, the minimum pollination probability of each block must appear on the vertices of the polygon, so it's only necessary to calculate the pollination probability of all vertices in the polygon and take the smallest one as $P_{\min}$.

The flowchart of the whole algorithm is shown in Figure 2:

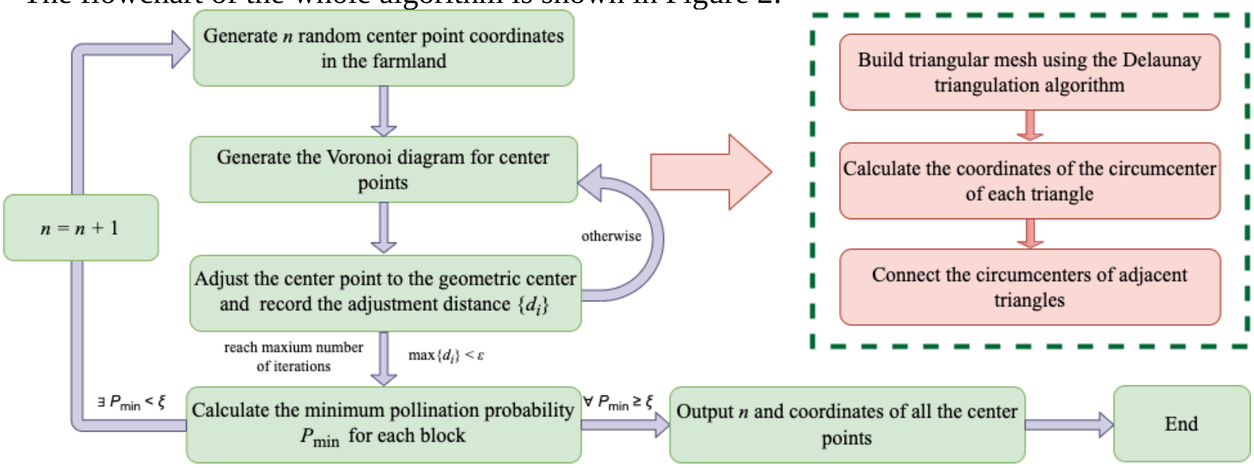

Figure 2 flowchart of the above algorithm

3. Results

3.1. Set up an application scenario

We assume that there is a farmland with a size of 81,000m^2, on which rapeseed flowers are planted. Based on modern planting methods of rapeseed [9], the interval between rapeseed flowers is about 0.1m. Furthermore, there are approximately 20 flowers on a rapeseed flower.

3.2. Estimation of the number of beehives

Based on the above data we can estimate the number of flowers in the whole farmland to be approximately

$$\text{flower} = \frac{81{,}000m^2}{0.1\text{m} \times 0.1\text{m}} \times 20 = 162{,}000{,}000 \tag{12}$$

According to simulation results [10], the number of bees in a hive can be up to 70,000 in summer, and the number of forager bee is about 1/4-1/3 of the total number of bees, so there are about 20,000 bees outside the hive, and the number of flowers collected by one bee per day is about 2,000, so the number of flowers collected by a hive in a day can reach 2, 000 × 20, 000 = 40, 000, 000. Considering that all bees spread outward from the same starting point, so in fact some of the flowers will be picked by more than one bee, then the actual number of flowers picked is about 40, 000, 000 × 0.6 = 24, 000, 000, thus the number of hives needed is about

$$162,000,000 / 24,000,000 \approx 7 \tag{13}$$

Since shapes of farmlands is various, we test the algorithm by considering several classical farmland shapes: triangle, square, irregular quadrilateral and pentagon. By inputting the coordinates of the boundary vertices and centers, the above algorithm is run to generate the optimal location for hive placement. Figure 3-10 shows the results of the algorithm running on different shapes of farmland.

3.3. Results of Different Shapes

Triangle

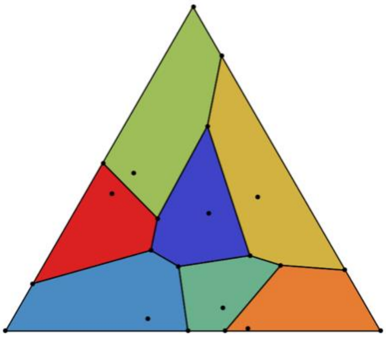

Figure 3 initial Voronoi diagram

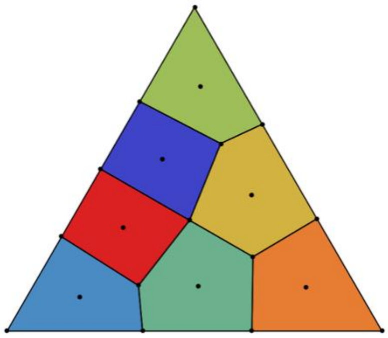

Figure 4 final Voronoi diagram

Square

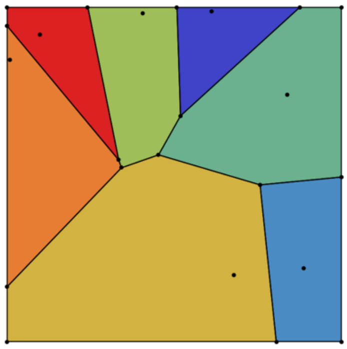

Figure 5 initial Voronoi diagram

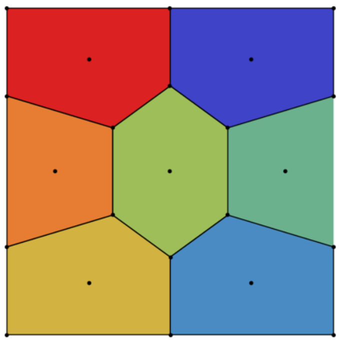

Figure 6 final Voronoi diagram

Irregular Quadrilateral

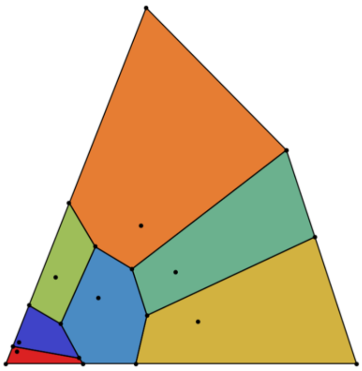

Figure 7 initial Voronoi diagram

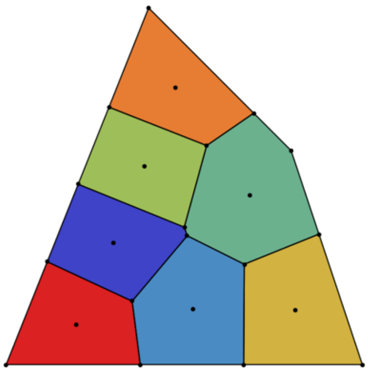

Figure 8 final Voronoi diagram

Regular Pentagon

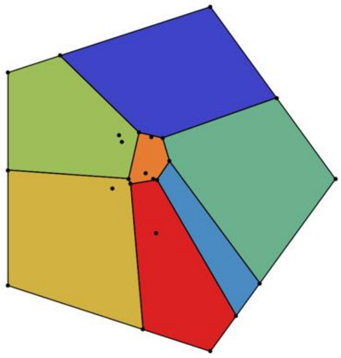

Figure 9 initial Voronoi diagram

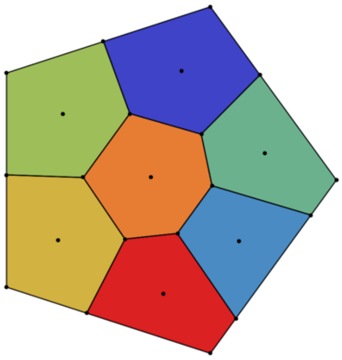

Figure 10 final Voronoi diagram

3.4. Promotion

We found that the algorithm converged quickly and achieved good results in the above samples. The main reason is that the number of hives is not very large and the distribution of points is limited. In order to maximize the performance of the algorithm, we increase n to 100 and test it in a hexagonal farmland, and obtain the following results in Figure 11-12:

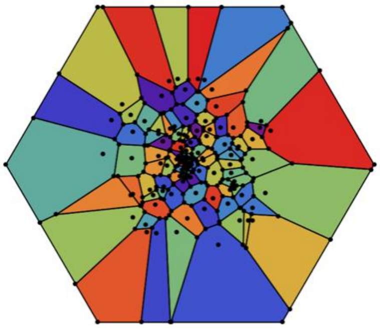

Figure 11 initial Voronoi diagram

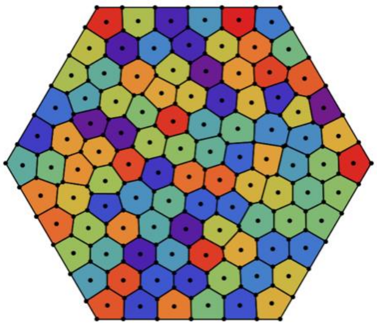

Figure 12 final Voronoi diagram

Although the run time is longer than all the above samples, the results are still very satisfactory. We can see that all the division blocks are very close to the square hexagonal shape, which is the same as the structure inside the beehive, indicating that bees are the structuralists in nature.

Also in the process of finding photos of farmland, we saw some circular farmland, however our algorithm only works for polygonal bounded farmland. So we intend to simulate it with positive polygons, and below we approximate the circle with positive decagon while taking n = 50 and get the following results in Figure 13-14:

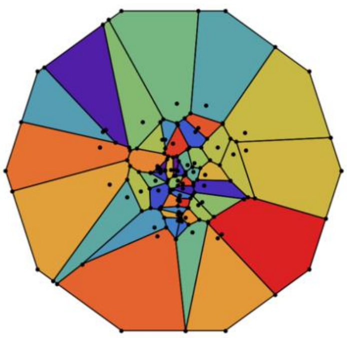

Figure 13 initial Voronoi diagram

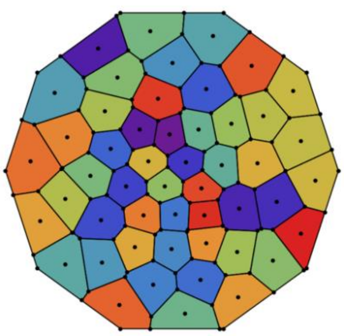

Figure 14 final Voronoi diagram

4. Conclusions

In this paper, we solved the beehive placement problem brilliantly, and introduce the pollination probability model to guarantee the quality of the results. When applying our model, it's only necessary to give the specific shape of the farmland even with no need to specify the exact number of beehives. For different shapes of farmland with different numbers of hives, the algorithm is well adapted, demonstrating good robustness.

In fact, we have abstracted beehive placement as a spatial segmentation problem to make the model more generalizable. Our model can be applied to most problems that require site selection within a fixed range, such as irrigation siting problem, radio tower siting problem and distribution center location problem. Hence our model can solve other problems with little modification and thus has great practical application value.

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