Multi-beam Line Measurement Problem Based on Geometric Modeling and Genetic Algorithm

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Abstract. Multibeam bathymetry is a key technology to measure the depth of seawater by acoustic wave propagation. In this paper, several computational methods, such as geometric modeling, triangulation, multi-objective optimization, simulated annealing algorithm and genetic algorithm, are used to solve the actual multibeam bathymetry problem. First, the coverage width and the overlap rate between neighboring strips of multibeam bathymetry are calculated. Then, the scene is expanded from two-dimensional to three-dimensional, and the projection length is solved by using the sine theorem and the angular relationship. Again, a set of survey lines are designed so that they are as short as possible and can completely cover the entire sea area to be surveyed while meeting the overlap rate requirement between adjacent strips. Finally, the problem is abstracted into a multi-objective optimization problem by introducing relaxation variables, and a multi-objective genetic algorithm is used to find the optimal design solution. By establishing the mathematical model and applying the optimization algorithm, the researchers are able to better design the measurement wiring of the multibeam bathymetry system so that it can efficiently and accurately acquire the seabed topographic data, thus enhancing the level of the marine mapping technology.

Keywords: Multibeam detection, geometric model, simulated annealing algorithm, multi-objective genetic algorithm, multi-objective optimization.

1. Introduction

Single-beam bathymetry is a technique for measuring the depth of a body of water by utilizing the propagation characteristics of sound waves in water. Acoustic waves propagate in a uniform medium at a uniform speed in a straight line and reflect at different interfaces. Using this principle, the acoustic signal is transmitted vertically from the transducer of the measuring vessel to the seabed, and the propagation time from the acoustic wave emission to the signal reception is recorded, and the depth of the seawater is calculated by the propagation speed and propagation time of the acoustic wave in the seawater [1].

Multibeam bathymetry system is developed on the basis of single-beam bathymetry, which is capable of transmitting dozens or even hundreds of beams at a time in the plane perpendicular to the trajectory, and then the acoustic waves returned from the seafloor are received by the receiving transducer [2]. The real seabed topography varies greatly, if the average water depth of the sea area is used to design the measurement line interval, although the average overlap rate between the strips can meet the requirements, there will be a leakage in the shallow water depth, affecting the quality of the measurement; if the shallowest water depth of the sea area is used to design the measurement line interval, although the overlap rate of the shallowest water depth can meet the requirements, there will be too much overlap in the deeper water, and the redundancy of data will be large, affecting the efficiency of the measurement. Measurement efficiency is affected by the large amount of data redundancy.

For this reason, this paper applies several computational methods, such as geometric modeling, triangulation, multi-objective optimization, simulated annealing algorithm, and genetic algorithm, to solve the actual multibeam bathymetry problem. First, the coverage width and the overlap rate between neighboring strips of multibeam bathymetry are calculated. Then, the scene is expanded from two-dimensional to three-dimensional, and the projection length is solved by using the sine theorem and the angular relationship. Again, a set of survey lines are designed so that they are as short as possible and can completely cover the entire sea area to be surveyed while meeting the overlap rate requirement between adjacent strips. Finally, the problem is abstracted into a multi-objective optimization problem by introducing relaxation variables, and a multi-objective genetic algorithm is used to find the optimal design solution.

2. Model formulation and solving

2.1. Modeling and solving when the survey line is parallel to the seafloor slope

The abstract geometric model is constructed as shown in Fig. 1. In Fig. 1, the ship is traveling vertically in the direction of the paper, and the position of the ship is pointing D. DF is the direction of gravity vertically downward, and AB is the horizontal plane. two wave beams DA and DE are emitted from point D and intersect the sea bottom plane at two points AE. Since DF is vertically downward, we can see that DF is the angle bisector of ∠ADE. Cross A as AC, which is the plane of the seafloor, and ∠CAB = 1.5º.

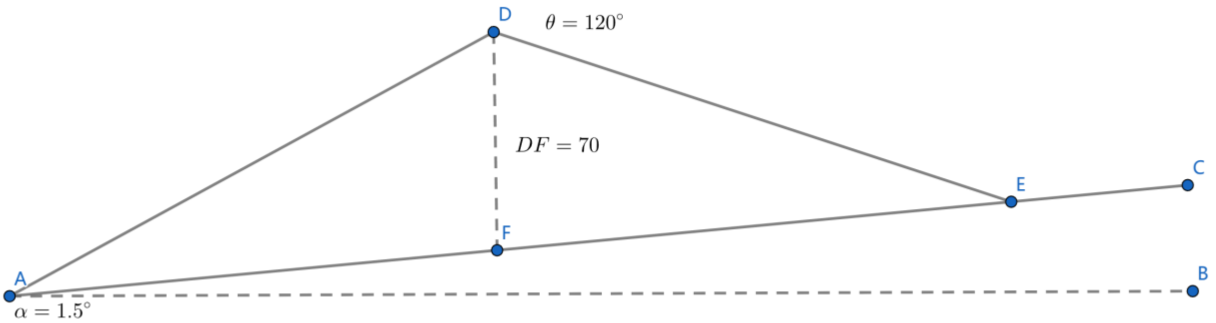

Figure 1. Schematic diagram of a single probe vessel

Obviously, since DF is vertically downward and AB is horizontal, there is <DAB + <ADF = 90º, <CAB = 1.5º, and <ADF is half of the open angle which is 60º:

$$\angle DAF = 90° - \angle CAB - \angle ADF \tag{1}$$

It can be deduced that <DAF is 28.5º and from the sum of the interior angles of the triangle, <DEA is 31.5º. Knowing that the length of DF is 70m and that the diagonal angles are known, the sine theorem can be used in triangles DAF and DFE, respectively:

$$\frac{DF}{\sin 28.5°} = \frac{AF}{\sin 60°}, \frac{DF}{\sin 31.5°} = \frac{EF}{\sin 60°} \tag{2}$$

So the length of the bandwidth to be sought is also AF + EF. After substitution, it can be solved that the coverage width is calculated as given the known depth of the seabed, D:

$$W = D\left(\frac{\sin 60°}{\sin 28.5°} + \frac{\sin 60°}{\sin 31.5°}\right) \tag{3}$$

In order to solve for coverage of neighboring survey lines, consider the case of two neighboring survey lines. And since the boat travels on the water, GD//AB. then similarly, since GJ is also vertically downward, GH//DF, GI//DF, triangles GHI and DFE are similar. For points G and D, the overlapping coverage width λ = EH for the overlapping portion of the beam. if a parallel line AB is made through J and extended DK intersects P, then the two form a triangle JKP in which <KJP = 1.5º and JP = d. Then the length of EH is clearly, relevant:

$$EH = JH + EK - JK \tag{4}$$

By solving the above expression, the width of the overlap can be obtained as:

$$\begin{aligned}\lambda &= (DK + d\tan 1.5°)\frac{\sin 60°}{\sin 31.5°} + DK\frac{\sin 60°}{\sin 28.5°} - \frac{d}{\cos 1.5°} \\ &= DK(\frac{\sin 60°}{\sin 31.5°} + \frac{\sin 60°}{\sin 28.5°}) + d(\frac{\tan 1.5° \sin 60°}{\sin 31.5°} - \frac{1}{\cos 1.5°})\end{aligned} \tag{5}$$

The width of coverage corresponding to d meters to the left or d meters to the right of the center if the line of measurement can be solved by using the formula. The depth of the seafloor corresponding to the survey line is GJ, and the same method of calculating the corresponding seafloor depth is given. If the corresponding seafloor depth is known, since triangle GHI is similar to triangle DFE, the similarity can be utilized to find the coverage width of the corresponding position:

$$\frac{W'}{D + d\tan\alpha} = \frac{W}{D} \tag{6}$$

2.2. Modeling and solving multibeam bathymetric coverage width at different points

Extending the scene from two to three dimensions turns it into a three-dimensional geometry problem, as shown in Fig. 2.

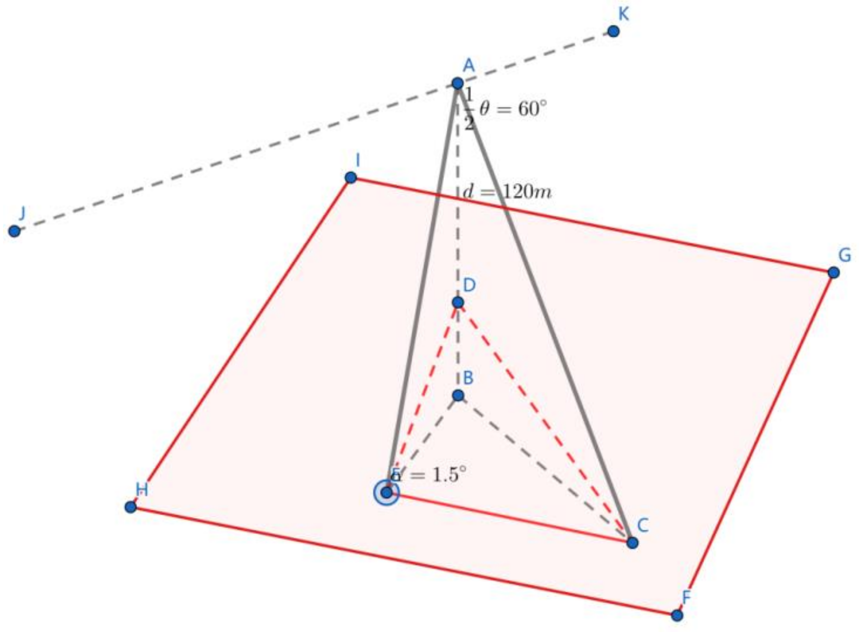

Figure 2. Schematic diagram of the three-dimensional condition

KJ is the survey line. The angle of inclination β is the angle between the line of measurement and the normal projection of the slope, the plane ABC is perpendicular to the line KJ (the line is perpendicular to the surface), and CD is the intersection line formed by the plumb line and the seafloor, which is the length to be solved for. The plane BEC is parallel to the horizontal plane IGHF. At the same time CE is perpendicular to the plane ABE and CE//FH. then it is clear that the line KJ forms an anisotropic perpendicular to the line BC at an angle of β. And, AE is perpendicular to CE, AB is perpendicular to BE, and AB is perpendicular to BC, constituting a typical tetrahedral geometric model.

In the case shown, β is an obtuse angle. If BK', the projection of KJ in the plane BEC, is made across B, the angle EBK' is β and the supplementary angle is (180-β). Then it is clear that since BK' is perpendicular to BC and BE is perpendicular to CE, then ∠BCE is reciprocal to ∠CBE, and ∠CBE is reciprocal to (180-β), which means that the angle of ∠BCE is also (180-β).

Notice that cos∠ACB = CB/AC, cos∠BCE = CE/BC, and cos∠ACE = CE/AC, which in effect forms the three-sided angle formula:

$$\cos\angle ACB \times \cos\angle BCE = \cos\angle ACE \tag{7}$$

Since <ACB is 30º and <BCE is 45º, <ACE is solvable, as is <EAC (they are mutually exclusive).

Then additionally use the tridiagonal angle formula once more, cos<EAC=EA/AC, cos<EAB=AB/EA, cos<BAC=AB/AC, so you can get:

$$\cos\angle EAC \times \cos\angle EAB = \cos\angle BAC \tag{8}$$

Therefore <EAB can also be found. And <DEB is a slope angle of 1.5 degrees, so the same can be found for <ABD. if the sine theorem is used in triangle ABD, since AD and the opposite angle are known, the length of AE satisfies the equation:

$$\frac{AE}{\sin\angle ADE} = \frac{d}{\sin\angle AED} \tag{9}$$

Also, since <EAC was solved earlier, the length of AC can be found. Since EC is perpendicular to the plane ABE, the triangle EAC forms a right triangle. the length of AC is satisfied:

$$EA = EC\cos\angle CAE \tag{10}$$

Also, knowing the lengths of AC and AD and the angle of 60 degrees in triangle CAD, you can use the cosine theorem to solve for the value of CD:

$$CD^2 = AC^2 + AD^2 - 2AC \times AD\cos 60° \tag{11}$$

Similarly, for the other beam ray, the analysis is the same as above. It is only necessary to change the angle of <BCE from 180-β to β to solve it. So the bandwidth DC + DC' can be solved.

2.3. Optimization strategy for line-of-sight design under specified overlap rates

Take the center of the sea surface as the origin, the north-south direction as the x-axis, the east-west direction as the y-axis, and vertically upward as the z-axis positive direction to construct the three-dimensional right-angled coordinate system as shown in Fig. 3:

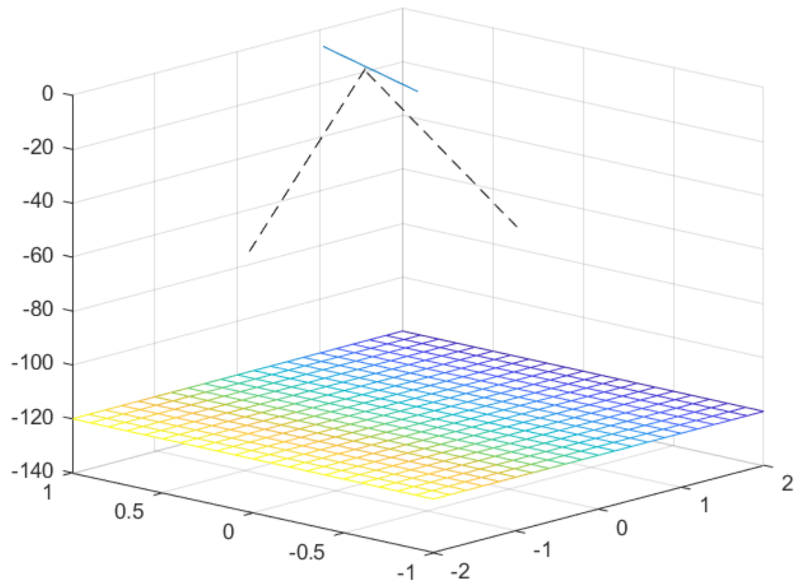

Figure 3. Three-dimensional planar schematic

For a survey line, if the angle of inclination is known, the direction vector of the line is:

$$\vec{v} = (\cos\beta, \sin\beta, 0) \tag{12}$$

As for the two beams emitted by the ship traveling along the survey line, since the maximum opening angle is known to be 120°, the direction vectors of the left and right beams can be found to be:

$$\begin{aligned} \vec{v_1} &= (-\cos\frac{\theta}{2}\sin\beta, \cos\frac{\theta}{2}\cos\beta, -\sin\frac{\theta}{2}) \\ \vec{v_2} &= (\cos\frac{\theta}{2}\sin\beta, -\cos\frac{\theta}{2}\cos\beta, -\sin\frac{\theta}{2}) \end{aligned} \tag{13}$$

The analytical equations of the seafloor slope in a three-dimensional coordinate system can be written:

$$\begin{gathered} z = -x\tan\alpha - h \\ -1852 \le x \le 1852 \\ -3704 \le y \le 3704 \end{gathered} \tag{14}$$

The equation of a line in three-dimensional space is constructed with a direction vector at its core. If the analytic equation of the line is written in the form of a parametric equation, there is an analytic equation in the case where the line passes through the point P(x0,y0,0):

$$\begin{cases} x = t\cos\beta + x_0 \\ y = t\sin\beta + y_0 \\ z = z_0 \end{cases} \tag{15}$$

The coordinates of the drop point of the ship's beam at point P on the line of measurement can be found by relating the left and right lines of measurement to the equation of the plane. For each line, it is parallel to the line passing through the origin, and the distance between the two neighboring lines is d. According to the distance condition, it can be calculated that the neighboring lines must pass through a point. For example, for a line passing through the origin, there must be a point P on the line to the right of the line such that OP = d while OP is perpendicular to the line. The coordinates of this point P can be solved for:

$$P=(-d\sin\beta, d\cos\beta, 0) \tag{16}$$

Associating the above conditions, one can solve for the set of coordinates of the drop points $\{w_{i1}\}$ and $\{w_{i2}\}$ formed when the vessel P moves on the survey line for each of the inscribed lines l_i, which in turn allows one to solve for the length L formed by all the drop points.

Since the stylus lines are all parallel, the projected straight lines corresponding to the left or right beams are also parallel, and the maximum distance projected when the tilt angle is β does not exceed the length of the diagonal line. In addition, for survey line i, the width W of the survey line can be measured by the distance between the left and right survey lines, and the coverage width is also similarly shaped. So the complete optimization form is:

$$\begin{gathered}\min_{n,d,\beta} L=\sum_{i=1}^{n}(\|w_{i1}\|+\|w_{i2}\|)\\ s.t.\begin{cases}\|w_{11}-w_{n2}\|\le 2\sqrt{5}\times 1852\\ 10\%\le \dfrac{\|w_{(i+1)1}-w_{i2}\|}{\|w_{i1}-w_{i2}\|}\le 20\%\end{cases}\end{gathered} \tag{17}$$

For this large-scale complex optimization problem, a simulated annealing algorithm is considered for solution. The simulated annealing algorithm was proposed by Kirkpatrick et al. and is effective in solving the local optimal solution problem. The simulated annealing algorithm is derived from the process of crystal cooling. If the solid is not in the lowest energy state, heating and then cooling the solid, as the temperature slowly decreases, the atoms in the solid are arranged according to a certain shape, forming a high-density, low-energy regular crystal, which corresponds to the globally optimal solution in the algorithm. Whereas, if the temperature drops too quickly, it may cause the atoms to lack enough time to arrange themselves into the structure of a crystal, resulting in an amorphous body with higher energy, which is the locally optimal solution. Therefore it is possible to give it in a little increase in energy according to the process of annealing, and then in cooling, if the increase in energy, jumped out of the local optimal solution, this annealing is successful [3].

The simulated annealing algorithm consists of two parts i.e. Metropolis criterion and annealing process.The Metropolis criterion accepts the new state with a probability rather than using fully deterministic rules and is known as Metropolis criterion which is less computationally intensive [4]. Moving from a particular solution to a new solution is essentially a measure of its energy change, if the energy jumps in a decreasing direction then this iteration is accepted, if the energy increases instead, it is not necessarily rejected but accepted with a certain sampling probability. This probability value satisfies the Metropolis definition:

$$P=e^{-\frac{E(n+1)-E(n)}{T}} \tag{18}$$

Direct use of the Metropolis algorithm may result in an optimization search so slow that it is not practical to use, in order to ensure that the convergence in a limited time, it is necessary to set the parameters that control the convergence of the algorithm, in the above formula, the parameter that can be adjusted is T. T, if it is too large, will result in annealing too quickly to reach a local optimum will end the iteration, and if it is taken to a smaller value, then the computation time will be increased,.

The annealing temperature table is used in practical applications, where a larger value of T is used at the beginning of the annealing process and gradually reduced as the annealing proceeds.

By modeling and programming the solution, the iterative curve results of the simulated annealing algorithm can be obtained as shown in Fig. 4:

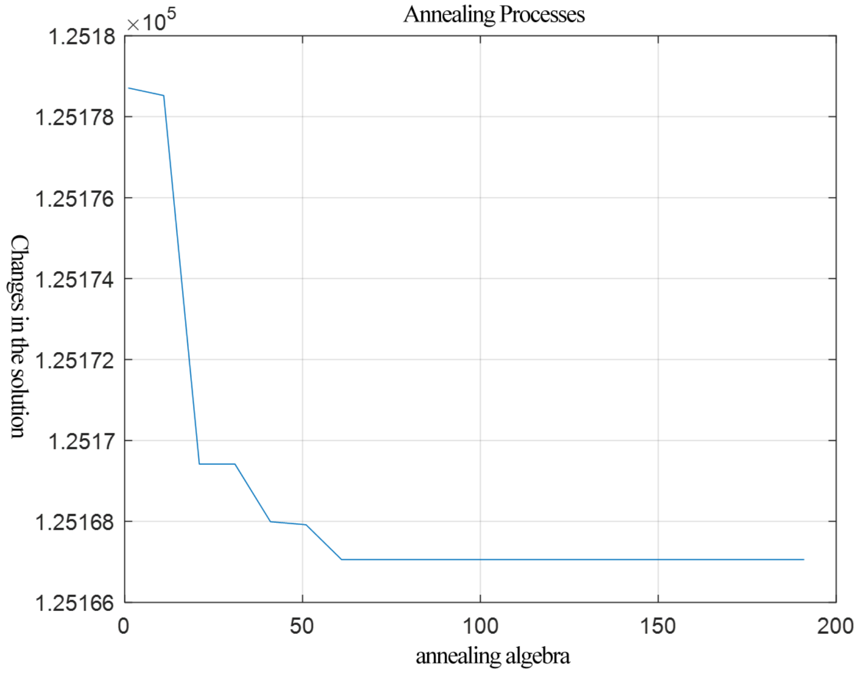

Figure 4. Annealing process

As can be seen in Figure 4, the simulated annealing algorithm is converging gradually. The final solution is that the optimal number of survey lines is 21 with a spacing of 320m and an inclination angle of 33º. The final optimal length obtained is 125963m.

2.4. Optimization strategy for line-of-sight design considering submarine surfaces

At this point, the seafloor is no longer a plane but a surface. The shape of the surface needs to be analyzed. First of all, to visualize and analyze the data, we establish the length and width as the x-axis and y-axis of the coordinate system and the seafloor depth as the z-axis, respectively, and then choose to represent the values for different depths from shallow to deep. By fitting the data, the seawater depth z can be solved to satisfy the relational equation with x and y:

$$z = 3.173 + 17.6x + 4.491y + 6.4x^2 - 9.6y^2 + 3.2xy \tag{19}$$

The R2 score of the measured line fit was tested to be 0.9648, which is a reasonably good fit, and the model was successfully fitted and can be approximated using this equation.

The design principle of the survey line is similar. However, at this point, since the seafloor is no longer planar, the beam projections form not parallel lines but curves. The set of curves $\{w_{i1}\}$ and $\{w_{i2}\}$ can be obtained from the projection. Based on the set of projected points, it is possible to determine whether the boundary of the area is included or not, which is also called complete coverage. Another way to determine complete coverage is to solve for the area of the set of points formed by the projection interval. Based on the area calculation, slack variables d_1^+ and d_1^- are introduced, where the positive slack variable represents the excess of the area formed by the beam projection over the area of the seafloor region, and the negative slack variable is the excess over the area of the seafloor region. The goal is to make the negative relaxation variable as small as possible, i.e.,:

$$\min d_1^- \qquad\qquad \min d_1^-$$
$$s.t.\begin{cases} S(w_{i1}, w_{i2}) + d_1^+ + d_1^- = S_0 \\ S_0 = 4*1852*5*1852 \end{cases} s.t.\begin{cases} S(w_{i1}, w_{i2}) + d_1^+ + d_1^- = S_0 \\ S_0 = 4*1852*5*1852 \end{cases} \tag{20}$$

And in order to calculate the coverage width and repeat width of the curves, the distance between two curves w_1 and w_2 is defined as the minimum value of the distance between the two curves by taking the point PQ, PQ respectively. That is:

$$\|w_1 - w_2\| = \min_{\substack{P\in w_1 \\ Q\in w_2}} \|P - Q\| \tag{21}$$

In addition, the goal of keeping the length of the survey line as short as possible needs to be met:

$$\min_{n,d,\beta} L = \sum_{i=1}^{n} (\|w_{i1}\| + \|w_{i2}\|) \tag{22}$$

Therefore, this is a multi-objective optimization problem. The three objectives are solved according to the weight combination method, and the final optimization problem can be obtained by combining the weights with 0.2,0.2,0.6:

$$\min_{n,d,\beta} L = 0.6\sum_{i=1}^{n} (\|w_{i1}\| + \|w_{i2}\|) + 0.2\sum_{i=1}^{n} d_{2i}^+ + 0.2 d_1^-$$
$$s.t.\begin{cases} S(w_{i1}, w_{i2}) + d_1^+ + d_1^- = S_0 \\ S_0 = 4*1852*5*1852 \\ \dfrac{\|w_{(i+1)1} - w_{i2}\|}{\|w_{i1} - w_{i2}\|} + d_{2i}^+ + d_{2i}^- = 20\% \\ \|w_1 - w_2\| = \min\limits_{\substack{P\in w_1 \\ Q\in w_2}} \|P - Q\| \end{cases} \tag{23}$$

For this multi-objective optimization, traditional algorithms such as the Hungarian method and Monte Carlo method have been difficult to solve because the dimensionality of the problem's variables is too high. With the maturity of evolutionary computation and population intelligence algorithms, more and more intelligent algorithms have been used to solve function optimization problems, among which, genetic algorithms are a class of stochastic search algorithms that draw on the natural selection and natural inheritance mechanisms in biology, searching for function extremes by simulating the hereditary, mutation and other natural phenomena of organisms [5]. Its main features are that it operates directly on structural objects without the qualification of derivation and function continuity; it has better global optimization search ability; it can automatically obtain and guide the optimized search space without the need of definite rules, and adaptively adjust the search direction.

Genetic algorithm draws on the concept of biology, the first need to encode the problem, usually after the function is encoded into binary code, randomly generate the initial population as the initial solution. Subsequently, one of the core operations of the genetic algorithm - selection, usually the selection of the first to calculate the fitness of the individual, according to the fitness of different to take different selection methods for selection, commonly used methods are fitness ratio method, expected value method, ranking method, roulette method, etc. [6].

In nature, mutation of genes and crossover combinations of chromosomes are common phenomena, and here too, mutation and combinations need to occur according to a certain probability after selection. Repeat the above operation until convergence, the solution obtained is optimal. By modeling and programming the solution, the iterative results of the genetic algorithm can be obtained as shown in Fig. 5:

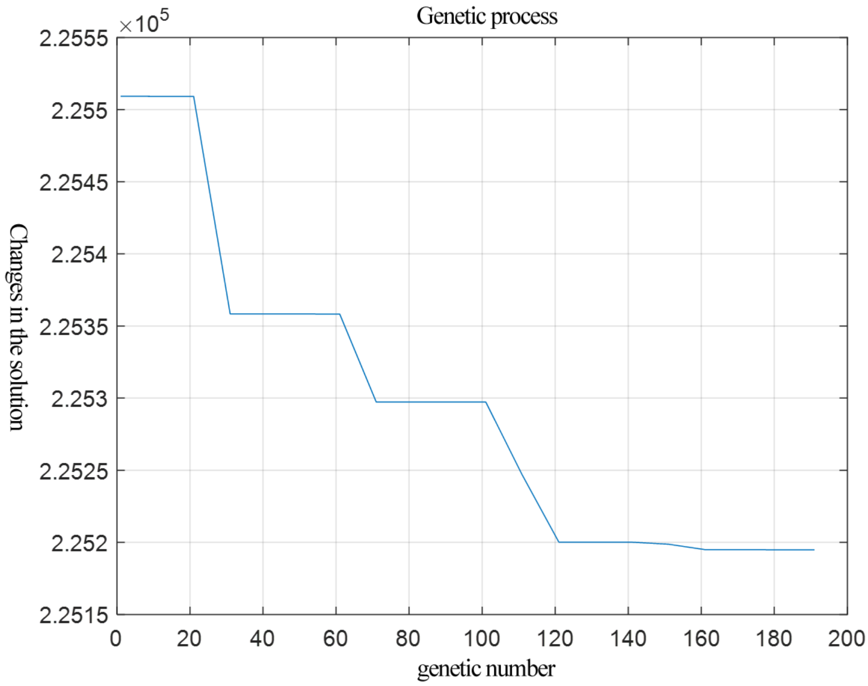

Figure 5. Genetic algorithm iteration

As can be seen from Fig. 5, the iteration curve of the genetic algorithm is constantly converging, and it is considered that the model has reached the optimization. Finally, the optimal number of survey lines is 32, the tilt angle is 15.4º, and the spacing of survey lines is 343 m. In the overlapping area, the total length of the part with overlapping rate more than 20% is 43,601.69 m. In the overlapping area, the total length of the part with overlapping rate more than 20% is 43,601.69 m.

3. Conclusion

In this paper, several computational methods, such as geometric modeling, triangulation, multi-objective optimization, simulated annealing algorithm and genetic algorithm, are used to solve the actual multibeam bathymetry problem. First, the coverage width and the overlap rate between neighboring strips of multibeam bathymetry are calculated, and the sinusoidal theorem in triangulation is repeatedly used to solve the distance between the left and right two beams' landing points, also known as the bandwidth, and subsequently the duplicated portion is solved by geometric relations. Then, the scene is extended from two to three dimensions, and the sine theorem is utilized to solve for the projected length in three dimensions, and a quadrangular cone is constructed by making an auxiliary plane, which is solved by using the three-sided angle theorem. Again, a set of measurement lines are designed so that they are as short as possible and can completely cover the whole sea area to be measured, and at the same time satisfy the requirement of overlap rate between neighboring strips, multiple parallel lines are introduced, and the number of lines, the distance of the lines and the tilt angle are taken as the decision variables, and the total length is taken as the objective function, and the constraints on the complete coverage and the overlap rate are taken into account, which can be solved using the simulated annealing algorithm. Finally, it is abstracted as a multi-objective optimization problem by introducing relaxation variables, which is solved using a multi-objective genetic algorithm. By establishing the mathematical model and applying the optimization algorithm, the researchers are able to better design the measurement wiring of the multibeam bathymetry system so that it can efficiently and accurately acquire the seabed topographic data, thus enhancing the level of the marine mapping technology.

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