Maxwell's Equation and Its Applications in Electromagnetism

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Abstract. As people always admitted that James Clerk Maxwell is one of the greatest scientists in the world, but major people may do not know the source of this glory. This paper is dedicated to give a rough impression of Maxwell and his experiences. Especially, the author then focuses on the well-known Maxwell' equations and attempt to illustrate them in simplest tune so that those people with weak knowledge bases but interested in Maxwell can recognize his imperishable contribution easily. This article reviews the history of electromagnetism and then focus on Maxwell himself, especially Maxwell's equations. The author will introduce those equations separately so that readers can get a rough profile of Maxwell's equations and applying them in different fields like antenna design, spectrum and wireless communication. These applications not only improve people's living standard to a large extent, but also raise the productivity of the society. The work should be a beneficial for people in the field of electromagnetism.

Keywords: Electromagnetism; Maxwell's equations; Faraday's law; Ampere's law.

1. Introduction

James Clerk Maxwell is one of the greatest physicists in the world. Maxwell birthed in Edinburgh, the capital of Scotland. His father, an engineering with wide prospect and relatively rich, was fairly concentrated in Maxwell's mindset development which can be recognized as the base of his following success. Maxwell was hugely influenced by his father and began to be interested in math and physics at a very early age. Maxwell did his first dissertation to the royal society at the age of fifteen, which then published in Proceedings of the Royal Society [1]. This is a fantastic achievement for a young child. In sixteen, Maxwell got the offer from University of Edinburgh where he studied physics and math. Three years later, when Maxwell was nineteen years old, he studied math as his majority in University of Cambridge, following William Hopkins, who was a famous mathematician in 19th century. In Cambridge, Maxwell was so delighted by the thick academic environment that he was employed as a physical professor in University of Cambridge and Royal College London. In 1874, he took the role of the director of the Cavendish physics laboratory. Unfortunately, this great physicist died because of illness in Cambridge on November 11st, 1879 [2].

In Maxwell's century, the classic electromagnetism had just a few gains. In 1785, a famous French physicist named Coulomb proved Coulomb's law by twist scale experiment, indicating the electromagnetism has stepped into a measuring subject. After that, another great physicist named Ørsted from Denmark found the induction of a current in1820. And in 1831, British physicist Michael Faraday discovered the electromagnetic induction, like how a magnetic field induces a current, by doing myriad of experiments over almost ten years [3]. Faraday opposed the force acting at distant and proposed a new matter called "field" for the first time, which allows objects with specific property experienced a force in an area. In this way, forces can be acted by the medium---field rather than be irrelative to the distance. What's more, Faraday proposed the concept of magnetic field lines in 1831 and electric field lines from 1837 to 1838 [4]. He did such many jobs, allowing physicists use algebra to model an object. However, all his work done should be owe to his continuous experiments and fantastic hypothesis of the property of fields. It must be admitted that Faraday lacks the mathematic ability to finish the deeper illustrate by calculus in detail. After reading Faraday's statement to the electric and magnetic field, Maxwell was fairly attracted by electromagnetism. He was determined to do further study on those puzzles and build a skyscraper of physics.

2. Maxwell's Equations

2.1. A Review to Maxwell's Equations

Maxwell's equations consist of four different equations, but they are all corresponding to each other: Gauss's law, Faraday's electromagnetic induction law, Ampere-Maxwell's law and Gauss's law for magnetism. They are partial differential equations include electromagnetic terminologies like current density, electric field, magnetic field and further mathematic knowledge like double integration. The Maxwell's equation can be expressed into two different forms. In the form of integration, it reads as [5]

$$\oint_{\boldsymbol{C}} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \int_{\boldsymbol{S}} \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial \mathrm{t}}\right) \cdot \mathrm{d}\boldsymbol{a}\,, \tag{1}$$

$$\oint_{\partial} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{d}{dt}\int_{s} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{a}\,, \tag{2}$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{a} = 0\,, \tag{3}$$

$$\oint_{\theta V} \boldsymbol{E} \cdot d\boldsymbol{a} = \frac{Q_V}{\epsilon_0}. \tag{4}$$

By contrast, in the form of differentiation, it is found that [6]

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{5}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \mathrm{t}} \tag{6}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{7}$$

$$\nabla \cdot \boldsymbol{D} = \rho \tag{8}$$

2.2. Gauss's law

Maxwell's equation Gauss's law refers to the theorem that the flux of electric field to a closed surface is related to the charge within the surface. This law points at the regulation of how an electronic field behaves and distributes in a space. The Gauss's law is expressed as [7]

$$\iiint_{\Omega} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) dV = \oint_{\partial\Omega} (P\, cos\, \alpha + Q\, cos\, \beta + R\, cos\, \gamma) dS \tag{9}$$

After a series of simplification, the Gauss's law can also be rewritten as

$$\oint_{\theta V} \boldsymbol{E} \cdot d\boldsymbol{a} = \frac{Q_V}{\epsilon_0} \tag{10}$$

and

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{11}$$

On the left side of the equation in integral form is the electrical flux at the surface of the curve. This is equal to the charge surrounded by the curve surface divided by Epsilon zero, which is the permittivity of the vacuum. E is the electric field. This formula applies Gauss's law quite in mathematics. By applying this formula, the total charge can be obtained simply by calculating the electrical flux.

2.3. Faraday's electromagnetic induction law

Faraday's law of electromagnetic induction refers to the law that the induced electromotive force in a closed loop is inversely proportional to the rate of change of the magnetic flux. This law demonstrates the relationship between how a changing magnetic field will induce a new electromotive force. Faraday's law is listed as [8]

$$\oint_{\partial} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{d}{dt} \int_{s} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{a} \tag{12}$$

and

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \mathrm{t}}. \tag{13}$$

The upper formula is in the form of integration. This is equivalent to electromagnetic force. Also, the side on the right of the equation is derived from the definition of magnetic flux. Faraday's law shows the connection between varying magnetic field and the inducted current. The side on the left of the equation is the integration along a loop to the electric field and the right side of the equation is about the derivative of magnetic field to the time.

2.4. Ampère–Maxwell law

Maxwell-Ampere's law refers to the law that both electric currents and time-varying electric fields can produce magnetic fields. It is firstly obtained by Ampere but then improved by Maxwell later after which he put this reviewed equation into his set of theories. This formula is listed as [9]

$$\oint_{\boldsymbol{C}} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \int_{\boldsymbol{S}} \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial \mathrm{t}}\right) \cdot \mathrm{d}\boldsymbol{a} \tag{14}$$

Maxwell-Ampère's law is derived from Ampere's law, which looks like this.

$$\oint_{\boldsymbol{C}} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \boldsymbol{I}_{\mathrm{enc}}\,. \tag{15}$$

The original formula seems much simpler than the later one. It shows the relationship between a stable current in vacuum and the integration along a closed loop to the magnetic field. However, the original formula does have some defects. What if the current is no longer stable but changing with time? Obviously, this formula won't match the real situation anymore. Them, Maxwell improved Ampere's low and published in the paper named On Physical Lines of Force in 1861. The modern one gives a J, which is defined as [10]

$$\boldsymbol{J} = \boldsymbol{J}_f + \boldsymbol{J}_M + \boldsymbol{J}_P, \tag{16}$$

with J is the total current density, J_f is the free current density, J_M is the magnetization current, and J_P is the polarization current.

2.5. Gauss's law for magnetism

Gauss's law for magnetism introduces how a magnetic field behave and distributes in a space, indicating that all the magnetic lines are closed. This equation points out that the flux of magnetic fields going through a closed surface is always none. Gauss's law for magnetism is shown by this:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{a} = 0 \tag{17}$$

and

$$\nabla \cdot \boldsymbol{B} = 0. \tag{18}$$

What can the reader gain from the formulae is that the divergence of magnetic field is equal to zero. It is sufficient to prove that there can't be any single magnetic 'particle' like an electron. The line of magnetic field is always in the shape of a loop. How about the first formula in the form of integration? What is the formula talking about? The answer is very simple. If somebody do the integration along the boundary of a closed surface to a magnetic field, he will eventually find out that the result is zero. This is due to all the magnetic field with opposite direction are cancelled.

3. Applications of Maxwell's Equations

3.1. Electromagnetic waves

After a series of transformations applying to Maxwell's equations, it is possible to find out the velocity of varying magnetic field and electronic field can be approximated to 3×10^8 in vacuum, which is very close to the velocity of light. In another word, Maxwell's equations indicate that light should belong to electromagnetic waves. That is a milestone in the history of science, means that light has the same property in some ways. The spectrum of the wavelength is shown in Fig. 1.

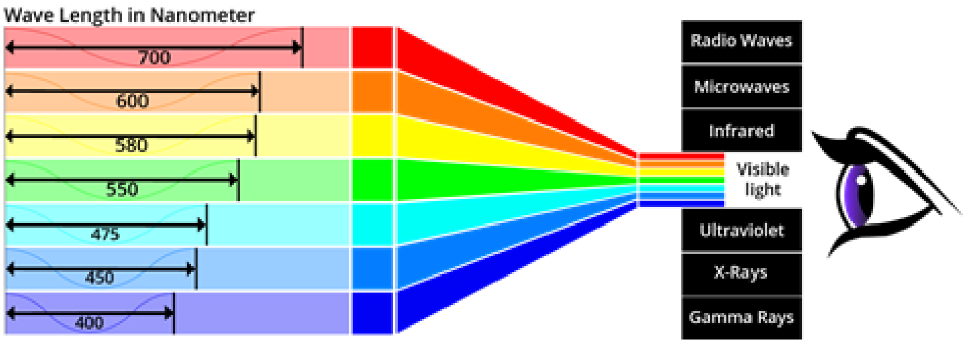

Fig. 1 Spectrum of the wavelength in nanometer.

3.2. Telecommunications

Maxwell's equations in telecommunications are applied in the field of telecommunications, and Maxwell's equations play a key role because they provide the theoretical basis for electromagnetic waves, which are the key to information transmission. Here are some specific application aspects.

Maxwell's equations are used for the design and analysis of antennas. These equations allow engineers to accurately calculate the radiation characteristics of the antenna to ensure efficient radiation and reception of the signal. By optimizing the antenna design, the performance and coverage of the communication system can be improved. Wireless communication systems, such as mobile phones, Wi-Fi, and cellular networks, rely on the propagation of electromagnetic waves. Maxwell's equations help engineers design and optimize base stations and antennas to ensure broad and stable signal coverage. They are also used to predict the propagation of signals under different environmental conditions, such as urban, rural or indoors.

4. Conclusion

This paper reviewed the history of electromagnetism and thus introduced the importance of Maxwell and Maxwell's equations. The author gives a brief introduction to Maxwell' equations and some of their applications nowadays. As people all know, Gauss's law can be used to detect charges within a surface by simply calculating the electrical flux going through the surface. By using Faraday's law, it is easier to understand how an electric field behave under a varying magnetic field. Ampère–Maxwell law enables scientists to obtain the current through a surface if they work out the magnetic field through any closed loop. Gauss's law for magnetism provides a substantial proof for the statement that a magnetic line is always closed since the divergence of a magnetic field always turns to be none. In a word, Maxwell's equations fairly simplify the calculation of electromagnetism and illustrate that magnetic fields and electric fields can be classify into the same subject. His equations potentially lead to the appearance of numerous applications like Internet and antenna design, which can benefit all the human society and people's life. The discovery of spectrum allows people to apply different types of electromagnetic waves into various industries. For example, doctors make use of X-ray to detect the body of patients for further therapy. It is widely recognized that James Clerk Maxwell is one of the greatest physicists over the whole history since recorded. His contribution has been engraved on the monument of modern society.

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