Research on optimal cutting scheme based on three-stage guillotine cut

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Abstract. Smart manufacturing has become a major part of the core competitiveness of enterprises, and better scheduling optimization methods are obtained in order to reduce the material loss in the manufacturing process. In this paper, we construct a mixed integer programming model, iterate from local optimum to global optimum by genetic algorithm, and finally solve the question B of the 2022 China Graduate Student Mathematical Modeling Competition. We get the results that the utilization rate of each scheduling solution is higher than 90%, which shows the effectiveness of the model and algorithm, and can greatly improve the utilization rate of plates.

Keywords: mixture integer programming, genetic Algorithm, cutting optimization.

1. Introduction

In recent years, smart manufacturing has become the core competitiveness of manufacturing enterprises with the advantages of personalized customization, efficient service mode of interconnection and high production efficiency. Smart manufacturing is mainly used to produce assembled product parts with many styles and decentralized processing, such as aerospace parts, electronic components, etc. The characteristics of many styles of smart manufacturing products make enterprises need to cut different styles of products in fixed-size plates, compared with the same type of products of regular size, different styles of products cut in fixed-size plates have a higher probability of waste, which gives rise to the problem of cutting optimization. The significance of solving the cutting optimization problem is to explore the rational planning and design of the layout of different styles of products on the plate, so as to reduce the waste of plate materials, improve the utilization rate of the plate, and finally help enterprises to achieve sustainable operation while reducing production costs. In this regard, some scholars have conducted research on the cutting optimization problem and obtained relevant research results.

Elsa et al. based on a tree search by keeping the position of the product cutpoints floating, search for a suitable style specification from the global and cut the product from the cutpoints, then form new floating cutpoints, and so on to achieve the overall optimal cut [1]. Han et al. start from the layout problem of electronic components, and their constructed evolutionary algorithm based on restart strategy, which makes the local search quickly jump out of the local optimum and helps to converge to the global optimum quickly, brings inspiration to the research of this paper [2]. Martin et al. constructed a top-down mixed integer linear programming model based on a binary tree, where the branch represents the cutting scheme and the root node represents the cutting object. The model exhibits better cutting quality and shorter cutting time in the example operation[3]. Polyakovskiy and M'Hallah faced a similar cutting optimization problem and accelerated the search for optimal results by constructing comprehensive constraints, so constructing suitable constraints plays an important role in model solving[4]. Russo et al. conducted a study on heuristic algorithms for solving cutting optimization problems, sorted out and refined several main algorithms, and finally verified that the anti-redundancy strategies employed by the algorithms would lead to a loss of accuracy[5]. Velasco and Uchoa assigned different weights to the parameters in the cutting model and verified the existence of optimal weights, allowing the model to obtain an optimal upper bound and thus an optimal cutting solution[6]. Furini et al. developed a complex and rigorous model with a mixed integer linear program for the cutting and dicing problem, which inspired the study in this paper, although there are arithmetic limitations in the practical application[7]. Iori et al. mainly investigated the cutting and packaging problem, and finally came up with the optimal cutting and packaging solution by summarizing and arguing the countermeasures for the four related problems in the literature[8]. In applying a mixed-integer programming model to solve the community evacuation problem, Fiete and Anne found that the introduction of a heuristic algorithm could improve the solution efficiency and enhance the conclusion certainty[9]. Baldacci and Boschetti proposed a two-level scheme for solving the cutting problem, in which the cutting styles are constituted as an ensemble at the first level, and only at the second level is the existence of a feasible cutting style for the ensemble considered. The article subsequently proposes an optimization model for the cutting surface method, which effectively reduces the number of sets in the first level, but the cutting constraints in the second level still need to be optimized[10].

In summary, the solution of the cutting optimization problem gradually tend to a mixed integer planning scheme based on heuristic algorithms, which can not only improve the accuracy of the conclusion but also shorten the computing time compared with tree search. Therefore, this paper will construct a mixed integer programming model based on genetic algorithm to carry out research on the cutting optimization problem.

2. Modeling and problem solving

2.1. Modeling ideas

Before solving the cutting optimization problem, this paper specifies that the cutting solution needs to meet the "guillotine cut" according to the cutting process, which means that each linear cut should be able to make the plate split into two pieces. As shown in Figure 1 below, the guillotine cut can cut square 1 with one cut, while square A requires two cuts, which is a non-guillotine cut and does not meet the requirements of this paper.

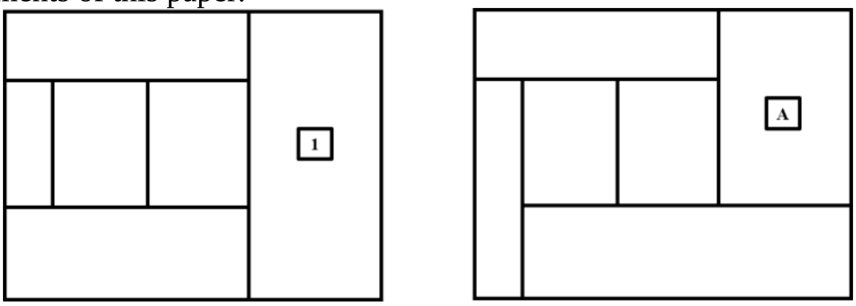

Figure 1. Example of guillotine cutting and non-guillotine cutting

Secondly, it is also necessary to define the form of different cutting stages, taking the three-stage cutting method as an example. As shown in Figure 2 below, the first stage cuts the plate longitudinally to generate modules called stripe, such as stripe1 and stripe2; the second stage cuts transversely to generate modules called stack, such as stripe1 is cut into stack1, stack2 and stack3; the third stage cuts longitudinally to generate modules called item, such as stack1 is cut into item1 and item2. On this basis, this paper specifies that the number of cutting stages does not exceed three stages, and the same stage cuts in the same direction.

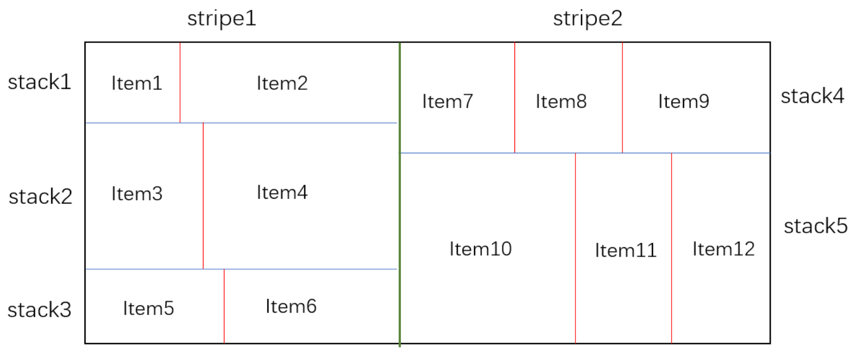

Figure 2. Example of three-stage cutting

Finally, this paper assumes that the number of plates is sufficient and only 2440 mm*1220 mm (length*width) is available based on the data of the 2022 China Graduate Student Mathematical Modeling Competition question B, and the width of the sawing slit is not considered. Thus, based on the above definitions and assumptions, this paper solves the cutting optimization problem by improving the plate utilization as much as possible while satisfying the production order demand and relevant constraints.

2.2. Modeling process

The cutting optimization problem requires that the amount of plate used should be as small as possible while fulfilling the order. According to the requirements of the competition, there are several important restrictions on the plate cutting solution in this paper, firstly, the restriction of guillotine cutting, secondly, the cutting solution is allowed to have only three stages, and finally, the items cannot be made from scrap splices. To facilitate understanding of the problem, the following Formula (1) can be introduced.

$$\frac{S_{order}}{1220*2440*N_{Plate}} = Global\ plate\ utilization\ rate \tag{1}$$

Where S_{order} indicates the total area of the project order; N_{Plate} indicates the plate usage rate. Obviously, the higher the global plate usage rate, the higher the plate utilization rate and the more effective the cutting optimization scheme. Due to the limitations of the plate cutting scheme and the large amount of item data, it is difficult to solve the optimal scheme from a global perspective, and a large number of iterations are required to obtain the optimal result. In order to improve the practicality of the mathematical model and algorithm, this paper chooses to decompose the global optimal problem into several local optimal problems, and converge to the global optimal by local optimal. In summary, the objective function in this paper is shown in Formula (2).

$$TARGET: MAX\ (Single\ plate\ utilization\ rate) \tag{2}$$

Since the objective function of maximizing the utilization of a single plate is chosen, compared to global optimization, there is necessarily a plate planning sequence. The plate planning sequence requires planning for the nth plate first and then for the n+1st original plate, and the cut items need to be eliminated in the interval of planning one by one. Regarding how to plan for the nth original plate, the three-stage cutting problem needs to be considered according to the constraints, for which a mathematical model can be built to determine the three-stage cutting point locations to solve the cutting optimization problem.

First, we consider the "first stage" cutting scheme. Since there is no restriction on the cutting method of the first stage, there are two forms of longitudinal and vertical cutting in the first stage, which need to be discussed separately. Looking at the data, we can see that the minimum variation unit of the project is 0.5mm. In view of this, the first stage cutting can be selected by applying the enumeration method with 0.5mm as the span to find the optimal first stage cutting solution. Since the area of the plate remains the same, probing the maximization of individual plate utilization can be transformed into probing the maximization of individual plate utilization area, so the objective function (3) is as follows.

$$TARGET: MAX\{S_1、S_2\} \tag{3}$$

where S_1 denotes the maximum utilized area obtained in longitudinal cut; S_2 is the maximum utilized area obtained in transverse cut.

In order to derive the maximum utilization area in longitudinal and transverse cuts, it is necessary to first solve the problem of calculating the maximum applied area in the first stage of random cutting state, and the longitudinal and transverse cutting conditions are discussed separately in this paper. Taking the longitudinal cut as an example, the cutting line is cut at a distance of a(mm) from the leftmost end of the original plate, at which time the original plate is cut into two parts, the length and width of both left and right parts are known values. According to the definition of the three-stage cut, the left and right parts can be understood as a combination of several stacks. As shown in the Figure 3 below, the left part consists of stack1, stack3 and scrap. In the same stack, the stack is made up of items of the same width or length, so items of the same width in the case of longitudinal cutting can be stitched together to form a stack, for example, stack1 in the Figure 3 consists of item1 and scrap; stack2 consists of item2, item3 and scrap. Thus the maximum utilized area of a single plate is the sum of the area of the items in it, and the larger the area sum, the higher the utilization of the plate.

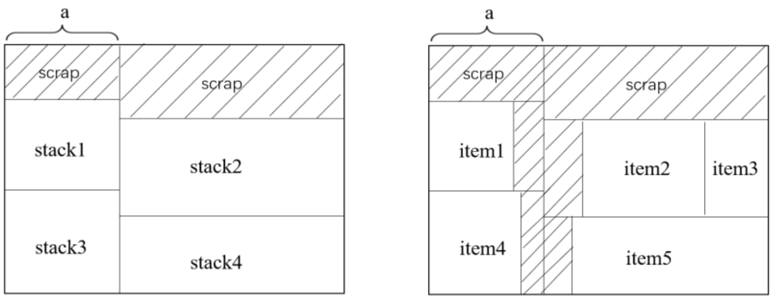

Figure 3. Composition of the **plate** in the case of longitudinal cutting

If the overall optimal cutting scheme for both left and right parts are considered at the same time, the same stack will be repeatedly applied, so the optimal cutting scheme for both left and right parts need to be considered separately. In this paper, the default cutting order in the longitudinal cut case is left then right, and the cutting order in the transverse cut case is top then bottom.

Based on the above understanding, when calculating the maximum application area in the random longitudinal cut state, the application area of each stack should also be maximized, which means that the waste area of it should be reduced as much as possible. To further maximize the stack application area, all the order items should be divided into several "stack collections" based on the principle of equal width. As shown in Figure 4, stack collection 1 and stack collection 2 consist of items with the same width but different lengths, and in the transverse cut case, the stack collection consists of items with the same length but different widths accordingly.

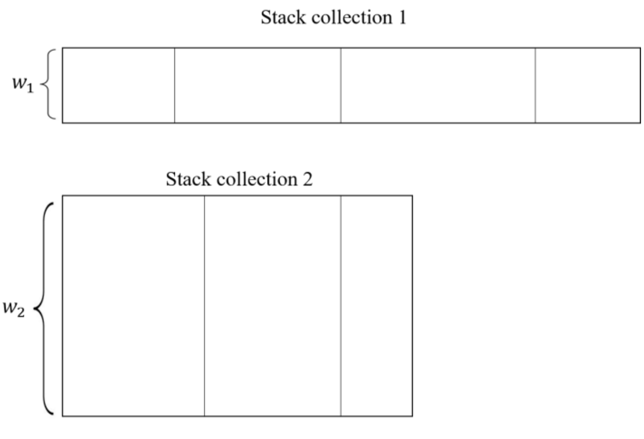

Figure 4. Examples of stack collection

Once the classification of items is completed, the stack collection can be used to solve for the current case of maximizing the stack application area. The maximum application area corresponding to all stack collections can be found according to the following Formula (4), and which items are specifically applied according to the formula subscript.

$$S_j = w_j \max \sum l_{ji} x_i \quad s.t. \begin{cases} \sum l_{ji} x_i \leq a \\ x_i = 0/1 \end{cases} \tag{4}$$

where S_j denotes the area of the jth stack collection, w_j is the width of the jth stack collection, l_{ji} is the length of the ith item in the jth stack collection, and x_i is a judgment function that takes the value of 0 or 1. If $l_{ji}x_i$ satisfies the restriction condition, then the stack collection can continue to add items, thus increasing the utilized area; but if $l_{ji}x_i$ does not satisfy the restriction condition, then the stack collection cannot add additional items, and its maximum utilized area is the sum of the areas of the items in the current stack collection.

Once the maximum application area corresponding to the stack collection is obtained, the maximum application area of the left half can be obtained by finding which specific stack collections are applied to splice into stripes according to the following Formula (5).

$$SL_a = \max \sum S_j x_j \quad s.t. \begin{cases} \sum w_j x_j \leq 1220 \\ x_j = 0/1 \end{cases} \tag{5}$$

Similar to (4), SL_a indicates that when the longitudinal cut is made, the left half of the maximum application area when the leftmost distance of the cut axis is a(mm). S_j corresponds to the maximum area of the stack collection above, x_j is also a judgment function that takes the value of 0 or 1. If w_jx_j satisfies the restriction condition, it indicates that there is room to increase the stack, x_j takes the value of 1; but if w_jx_j breaks the restriction condition, it means that the new stack exceeds the width of the stripe and does not meet the cutting condition, x_j takes the value of 0. At this time, the maximum application area of the left half can be obtained.

After completing the solution of the maximum cutting scheme for the left half, the items contained in the cutting scheme need to be removed from the stack collection, and then the optimal cutting scheme for the right half is solved in the same way, calculated as Formula (6-8).

$$S_j = w_j \max \sum l_{ji} x_i \quad s.t. \begin{cases} \sum l_i x_i \leq 2440 - a \\ x_i = 0/1 \end{cases} \tag{6}$$

$$SR_a = \max \sum S_j x_j \quad s.t. \begin{cases} \sum w_j x_j \leq 1220 \\ x_j = 0/1 \end{cases} \tag{7}$$

$$S_1 = MAX\{SL_a + SR_a\} \tag{8}$$

Note that some of the symbols have been explained above. SR_a indicates the maximum applied area of the right half when the distance of the cutting axis from the leftmost end is a(mm) in longitudinal cutting, and S_{max} indicates the maximum applied area of the whole plate.

This completes the calculation of the maximum application area of the original slice under the random longitudinal tangent condition (the distance of the tangent point from the leftmost end is a), and the calculation principle of the maximum application area under the random transverse tangent condition (the distance of the tangent point from the uppermost end is b) is essentially the same as that of the longitudinal tangent condition, only two adjustments are needed, one is to calculate some of the values in the model, and the other is to change the classification method of the stack collection from the same width to the same length. The Formula (9-13) for calculating the maximum application area in the random transverse cutting condition is given directly below.

$$TARGET: \quad S_2 = MAX\{SU_b + SD_b\} \tag{9}$$

$$S_j = l_j \max \sum w_{ji} x_i \quad s.t. \begin{cases} \sum w_{ji} x_i \leq b \\ x_i = 0/1 \end{cases} \tag{10}$$

$$SU_b = \max \sum S_j x_j \quad s.t. \begin{cases} \sum l_j x_j \leq 2440 \\ x_j = 0/1 \end{cases} \tag{11}$$

$$S_j = l_j \max \sum w_{ji} x_i \quad s.t. \begin{cases} \sum w_{ji} x_i \leq 1220 - b \\ x_i = 0/1 \end{cases} \tag{12}$$

$$SR_b = \max \sum S_j x_j \quad s.t. \begin{cases} \sum l_j x_j \leq 2440 \\ x_j = 0/1 \end{cases} \tag{13}$$

Where SU_b and SD_b denote the maximum applied areas of the upper and lower halves when the distance of the cutting axis from the uppermost end is b(mm) in the case of transverse cut, respectively. l_j denotes the length of the jth stack collection. Judgment function x_j is applied in the same way as above.

According to the formula described above, the maximum application area for the first stage random cut condition can be obtained by programming in python, and then the optimal first stage cut can be selected by the enumeration method with a span of 0.5mm. At the same time, since the above calculation model already includes the calculation results of the second stage cut (to determine which stacks are spliced into stripes) and the third stage cut (to determine which items are spliced into stacks), only the optimal first stage cut data is needed to obtain the complete cutting scheme in practical applications. At this point, this paper has completed the planning of the cutting scheme for maximizing the application area of a single plate, and it is only necessary to continue running the above algorithm until all the stack collections are applied to complete the global planning work.

2.3. Model Improvement

In the previous section, we modeled the optimal cutting solution for a single plate, in which the core idea is to cut only once in the first stage and apply the enumeration method to select the optimal first stage cutting method to obtain the corresponding cutting solution. If the enumeration method is applied, each plate will need to be solved 7320 times for the maximum application area under different cutting states, which is a large amount of computation, so it is necessary to optimize the model.

The first consideration is to choose between longitudinal and transverse cuts for the first stage, and determine which cut is more suitable for each plate, instead of enumerating them all as before. In the above we proposed the understanding of the stripe as a splice of the stack, according to which it can be assumed that under the restriction that the overall application area is the largest, the application area of the stack must also be the largest. The maximum application area of the stack is proportional to the number of products in the stack collection. The more products, the more options there are for splicing the stack, and the larger the area of the stack, so it can be considered that the more products in a single stack collection, the larger the overall application area of the plate. In summary, the number of products in each stack collection can be used as a criterion to determine whether the first stage is longitudinal or transverse. If the number of products in the stacks classified according to the same width principle is higher, there are more options for splicing the stacks and the longitudinal cut should be chosen; on the contrary, if the number of items in the stacks classified according to the same length principle is higher, the transverse cut should be chosen.

At present, after determining the first stage cutting method, we still need to apply the enumeration method to select the optimal first stage cutting scheme, in order to further speed up the solution, we consider replacing the enumeration method with the inspiration algorithm, which is the genetic algorithm. With the help of genetic algorithm, this paper can finally select the optimal first-stage cutting scheme through continuous iterative comparison, as shown in the following Figure 5.

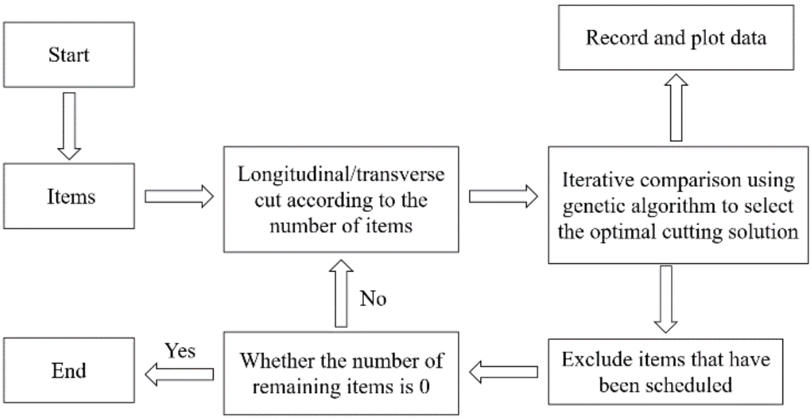

Figure 5. Flow of the **optimization** algorithm

Further using the improved model and algorithm for calculation, we found that the calculated results are not satisfactory, and by observing the algorithm plotting results, we believe that the assumption of only one cut in the first stage severely limits the efficiency of plate usage. This is because in many cases, a large area of a single item (item1) or a small number of items in the stack collection (item2 and item3) will result in more scrap and reduce the overall utilization of the plate, as shown in the following Figure 6.

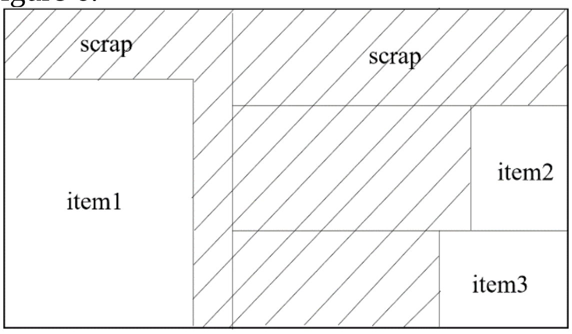

Figure 6. Problem statement

The above problem is common in the resulting optimal cutting solution, so the model must be further optimized by removing the assumption that only one cut is made in the first stage and considering the case of multiple cuts in the first stage. As can be seen from the above Figure 6, there is a large amount of scrap area in the middle region of the plate, and if the assumption of one cut can be removed, the scrap area can be utilized and thus the utilization rate of the plate can be improved.

After removing the assumption of only one cut, the model improvement in this paper basically follows the original model, which is to select the longitudinal/transverse cut according to the number of items in the stack collection, and then use the maximum application area as the fitness function to search for the optimal solution using genetic algorithm, in which the biggest change is reflected in the generation of the solution set. The improved model needs to consider additional cut cases, so for larger solution sets, multiple fitness functions are needed to calculate the maximum application area in different cases, but these fitness functions are essentially similar to the calculation idea when only one cut is made in the first stage, so the idea can be followed to construct the first stage model that can have multiple cuts.

2.4. Model Results

Based on the above cutting optimization model and ideas, this paper attempts to solve the cutting optimization problem of the 2022 China Graduate Student Mathematical Modeling Competition. For the first group of 752 order items data in the problem, the cutting optimization is carried out in the fixed-size plate of 2440 mm*1220 mm, and finally a total of 91 plates are used to complete the cutting of all items, and the utilization rate of the plates reaches 91.8%. This paper constructs a coordinate system with the lower left corner of the plate as the origin, and derives the item IDs and their corresponding coordinates in the plate. For the sake of illustration, only the cutting of plate 1 and plate 2 are given in the Table 1.

Table 1. Layout Results of plate 1 and 2

No	ID	X coordinate	Y coordinate	X axis length	Y axis length
1	41	956	0	198	2198
1	167	981	2251	238	118
1	453	1154	0	58	2224
1	530	0	2251	981	174
1	741	0	0	956	2251
2	36	0	1981	1198	370
2	69	0	0	1063	1981
2	118	841	2351	338	78
2	151	1063	0	98	1884
2	373	0	2351	841	88
2	425	1161	0	58	1970

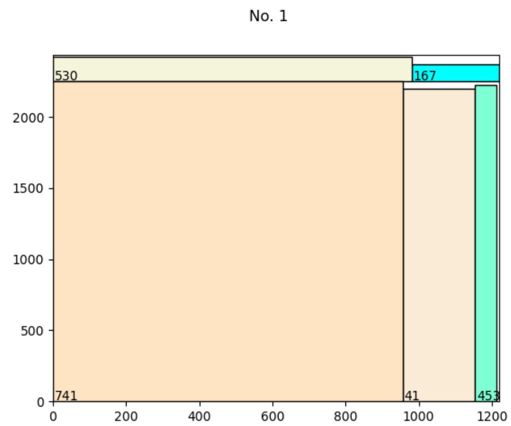

Figure 7. Layout of plate 1

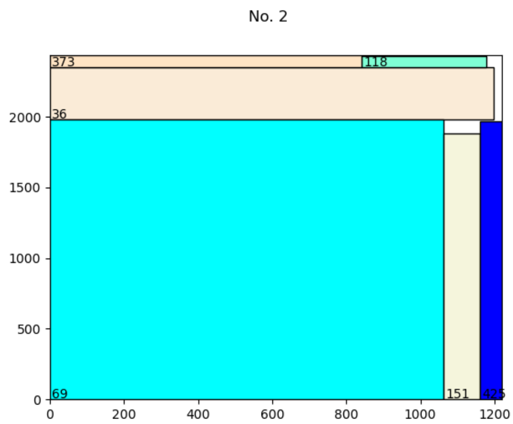

Figure 8. Layout of plate 2

Combined with the above Figure 7, Figure 8 and Table 1, plate 1 and plate 2 significantly show the situation of one cut and two cut in the first stage. Plate 1 carries out the first stage cutting from longitudinal axis 2251, dividing the plate into upper and lower parts, while plate 2 carries out the first stage cutting from longitudinal axis 1981 and 2351, dividing the plate into upper, middle and lower parts, both of which achieve high plate utilization. This reflects the necessity of considering the hypothesis of multiple cuts in the first stage. This paper continues to solve other data, and finds that the utilization rate of the second group and the third group is 91.07% and 94.08% respectively, which shows the effectiveness of the algorithm in this paper.

3. Practical production guidance

In this paper, the first stage of cutting is divided into longitudinal cutting and transverse cutting. Obviously, for different order requirements, the final plate utilization results of transverse cutting and longitudinal cutting are different. According to the calculation results, the utilization rate of the longitudinal cutting plate obtained in this paper is higher than that of the transverse cutting plate. In combination with the fact that the number of products with the same width is more than the number of products with the same length in the data, it can be concluded that if there are more products with the same width, the longitudinal cutting position should be selected on the long side of the plate to maximize the utilization rate of the plate; If there are many products with the same length, the transverse cutting position should be selected on the wide edge of the plate to maximize the utilization of the plate. After several groups of data tests, this paper verifies the above conclusions.

The conclusion can be explained through thinking, demonstration and literature review. When there are many items with the same width, more consideration will be given to splicing items to make stacks longer. Therefore, it is necessary to control the length by slitting on the long side to achieve reasonable planning. On the contrary, if more consideration is given to items splicing to make the stacks wider, the width needs to be controlled by wide side crosscutting to achieve reasonable planning. In the production process, the enterprise shall reasonably select the cutting method according to the length and width.

4. Conclusion

This paper first analyzes the data conditions and constraints given by the 2022 Chinese Postgraduate Mathematical Modeling Contest question B, and finds that solving the product cutting scheme from the perspective of global optimization has problems such as large computation and complex algorithm. Therefore, this paper starts from the local optimization, uses the mixed integer programming model to formulate the local optimal plate cutting scheme, and proposes and proves the solution idea that determining the optimal "first stage cutting" can determine the local optimal plate utilization rate, thus transforming the problem into the "first stage" optimal cutting problem. In order to facilitate the establishment of the "first stage" optimal cutting model, this paper proposes the concept of "stack collection" based on relevant literature, which shows strong practicality. In order to solve the problem of large amount of computation and strict assumption of "the first stage", this paper introduces the longitudinal/transverse cutting selection method based on "stack collection" and uses genetic algorithm to expand the solution set to solve the problem. Finally, this paper uses Python to program the improved optimal cutting model to solve the problem, and obtains the A1 cutting scheme with a utilization rate of 91.8%; A2 cutting scheme with utilization rate of 91.1% and A3 cutting scheme with 94.1% utilization rate. The utilization ratio of the three cutting schemes is higher than 90%, which shows the excellent effectiveness of the model and algorithm, and greatly improves the utilization ratio of plates.

References

[1] Elsa S, Fernando OJ, Tiago S, Leandro M, Antónia CM. The Floating-Cuts model: a general and flexible mixed-integer programming model for non-guillotine and guillotine rectangular cutting problems[J]. Omega, 2023, 114.

[2] Han Lei, Wang Handing, Wang Shuo. A Surrogate-Assisted Evolutionary Algorithm for Space Component Thermal Layout Optimization [J]. Space: Science & Technology, 2022, 2022.

[3] Martin M, Morabito R, Munari P. A top-down cutting approach for modeling the constrained two- and three-dimensional guillotine cutting problems. Journal of the Operational Research Society 2021; 72(12): 2755–2769.

[4] Polyakovskiy S, M'Hallah R. Just-in-time two-dimensional bin packing. Omega 2021; Volume 102: 102311.

[5] Russo M, Boccia M, Sforza A, Sterle C. Constrained two-dimensional guillotine cutting problem: upper-bound review and categorization. International Transactions in Operational Research 2020; 27(2): 794–834.

[6] Velasco AS, Uchoa E. Improved state space relaxation for constrained two-dimensional guillotine cutting problems. European Journal of Operational Research 2019;272(1):106–20.

[7] Furini F, Malaguti E, Thomopulos D. Modeling two-dimensional guillotine cutting problems via integer programming. INFORMS Journal on Computing 2016; 28(4): 736–51.

[8] Iori M, de Lima VL, Martello S, Miyazawa FK, Monaci M. Exact solution techniques for two-dimensional cutting and packing. European Journal of Operational Research 2021; 289(2): 399–415.

[9] Fiete KK, Anne G. The isolated community evacuation problem with mixed integer programming[J]. Transportation Research Part E, 2022, 161

[10] Baldacci R, Boschetti MA. A cutting-plane approach for the two-dimensional orthogonal non-guillotine cutting problem. European Journal of Operational Research 2007; 183(3): 1136–49.