

The Comparison of RS Codes, LDPC Codes, And Turbo Codes

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Abstract. Error correction codes have been widely used in various fields in recent years, including the communication field, storage field, digital media field, computer network field, and so on. In this paper, the development history of error correction codes is briefly summarized; and the principles and performance of three kinds of error correction control codes are systematically summarized starting from RS codes, LDPC codes, and Turbo codes. Three types of error correction codes are compared and analyzed in the field of wireless optical communication, and the error correction characteristics of different codes in three different weather conditions are compared and summarized for the applicable environments. The error correction codes are also compared in the field of aviation flight telemetry, and which codes are suitable for uplink and downlink telemetry in flight telemetry are summarized. At the same time, the paper provides an outlook on the future development trend of error correction codes.

Keywords: Aeronautical flight telemetry; Wireless optical communication system; Error correction codes.

1. Introduction

Error correction code is a technique used for error detection and correction in data transmission. It can be achieved by adding redundant information while sending data. This redundant information helps the receiver to detect and correct the errors in it. Common error correction codes include Hamming codes and RS codes. When the receiver receives data with errors, the error correction code can determine which bits are in error by examining the redundancy information and correcting it if possible.

It is used in a wide variety of applications, including communications, storage, and computer networks. In wireless communications, error-correcting codes can improve the reliability of signal transmission and reduce the number of errors that occur during data transmission. In wired communications, such as fiber optic communications and Ethernet, error correction codes ensure the quality of data transmission and increase the transmission rate. In storage systems, the reliability and durability of data can be greatly improved by employing error-correcting codes on the storage media. Even if there is some degree of damage or error in the storage medium, error correction codes are able to recover or repair the data. In computer networks, error correction codes are widely used in various protocols and standards. In general, with the continuous progress of technology and the expansion of application areas, the application of error correction code is also constantly developing and improving.

Deletion correcting code can not only correct errors but also recover lost data from the original data, which is a coding technique used for error detection and correction. Deletion correction code is commonly used in storage media that require high reliability, such as hard disk drives and flash memory. It has gone through several stages of development, gradually refining and improving the reliability and efficiency of coding.

2. History of Error Correction Codes

First, in the early 20th century, people began to pay attention to the problem of errors in data transmission and proposed the earliest concept of error correction codes. In 1910, Richard Hamming proposed the first error correction code, Hamming code [1]. Hamming code realizes the error

correction by adding redundant bits in data to realize the error correction. This simple coding scheme provided the basis for later error correction codes. This simple coding scheme laid the foundation for the later development of error correction codes.

With the rapid development of computer technology, the Code Correction Code has also been studied more deeply. In the late 1940s and early 1950s, Reed-Solomon codes and BCH codes were proposed, and Reed-Solomon codes were widely used in digital communication, especially in audio, video, and data storage, etc [2]. BCH codes were used in storage devices such as CD-ROMs and flash memories, with the advantage of being able to detect and correct multiple errors simultaneously [3].

In the 1960s, LDPC codes were proposed [4]. In the 1970s, Hadamard codes and Turbo codes became important breakthroughs in the field of corrective coding [5,6]. Hadamard codes are a kind of fast decoding codes, which have very fast decoding speed. Turbo codes are an iterative decoding technique, which can be close to Shannon's limit through multiple iterations. Turbo codes are widely used in 3G, 4G, and 5G wireless communications and other fields. In the 21st century, LDPC codes are widely used in 3G, 4G, and 5G wireless communications.

In the 21st century, LDPC codes and convolutional codes have become popular research directions for corrective codes. LDPC code is a low-density parity-check code with good performance and low coding complexity. It has been widely used in the fields of Blu-ray disk, wired communication, and wireless communication. Convolutional code is a continuous input-output code characterized by simple hardware implementation and good anti-interference performance. It has been widely used in the fields of digital television, satellite communication, and deep space exploration [7].

During the development of the corrective censoring code, new coding schemes and decoding algorithms have emerged, which make the efficiency and reliability of the coding continuously improved. In today's information society, corrective coding has become one of the most important technologies to ensure the security of data transmission and storage.

3. Fundamentals of Several Error-correcting Codes

3.1. RS Code

RS code is a non-binary error correction code first proposed by Irving S. Reed and Gus G. Solomon. It is capable of detecting and correcting errors in data transmission. RS codes are suitable for different types of communication environments, especially in the presence of noise interference, such as wireless and fiber optic communications. The key idea of RS codes is to achieve error correction by adding redundant data. The sender encodes the original data and adds a certain amount of checking data to the encoded result, and the receiver uses an error correction algorithm to recover and correct the data after it is received. RS codes have an efficient error correction capability, which allows them to restore the original data efficiently even in the presence of a large number of errors.

The RS code is generally denoted by (n,k,d) , where n is the code length, k is the number of information codewords, and d is the codeword distance. Its checksum matrix is:

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1k} & 1 & 0 & 0 & \dots & 0 \\ h_{21} & h_{22} & \dots & h_{2k} & 0 & 1 & 0 & \dots & 0 \\ & & & \dots & & & & & \\ h_{r1} & h_{r2} & \dots & h_{rk} & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad (1)$$

The RS code generating polynomial is:

$$g(a) = (a - \alpha^1)(a - \alpha^2) \dots (a - \alpha^{2t}) \quad (2)$$

The RS code checksum polynomial is:

$$h(a) = a^n - 1 / g(a) \quad (3)$$

The system code is:

$$c(a) = q(a)g(a) = m(a)a^{n-k} + r(a) \quad (4)$$

The remainder equation is:

$$r(a) = m(a)a^{n-k} \pmod{g(a)} \quad (5)$$

The checksum bits are:

$$g(a) = g_{2t}a^{2t} + g_{2t-1}a^{2t-1} + \dots + g_1a + g_0 \quad (6)$$

$$m(a)a^{n-k} = m_k a^{n-1} + m_{k-1} a^{n-2} + \dots + m_1 a^{n-k} + 0x^{n-k-1} + \dots + 0x + 0 \quad (7)$$

3.2. LDPC Code

LDPC code is a binary error correction code, named after the lower "density" of its checksum matrix. LDPC code was first proposed by Robert G. Gallager in 1962. But in recent years, LDPC codes have attracted renewed attention due to the development of related technologies. LDPC codes have good error correction capability and low decoding complexity. The main idea of LDPC codes is to achieve efficient error correction by introducing a large number of sparse check matrices in the coding process. The sender encodes the data and adds the checksum bits to the encoded result. The receiver uses an iterative decoding algorithm to back-calculate and infer the received data to recover the original information.

The checksum matrix H has a majority of "0s" and a very small number of "1s". The distribution of "1" elements is very sparse, so it is low density. The checksum matrix of a regular LDPC code is H, which needs to satisfy the following three conditions:

- (1) Each row has the same number of "1" elements, all k.
- (2) Each column has the same number of "1" elements, all j.
- (3) Any two columns in the matrix have at most one position with a value of "1" in the same position.

The length of H is M×N, and the codeword after encoding is x, then HxT=0T. Coding is done by Gaussian elimination. x can be divided into two parts, which are the body bit s and the checksum bit p, and x is denoted by x=(s,p), then the system encoding can be realized in two steps:

- (1) The N × M information bits to be transmitted are assigned to the system bit s.
- (2) For the i-checksum bit pi can be obtained by backward recursion.

$$P_i = \sum_{j=1}^{N-M} H_{i,j} S_j + \sum_{j=1}^{i-1} H_{i,j+N-M} P_j \quad (8)$$

3.3. Turbo Code

Turbo code is an error correction coding technique commonly used in digital communication systems, especially in wireless communications. It improves the coding performance by introducing two parallel convolutional encoders and an interleaver.

Specifically, Turbo code coding is performed in the following steps:

- (1) The input data is divided into several consecutive blocks.
- (2) Each block is fed into two mutually independent convolutional encoders, called Encoder 1 and Encoder 2.
- (3) Encoder 1 converts the input data into a series of redundant bits and outputs them.

(4) Encoder 2 receives the output of Encoder 1 and generates another set of redundant bits.

(5) An interleaver interleaves the outputs of the two encoders to eliminate the effects of sequential errors.

(6) The final output contains the original data and the redundant bits to form the encoded data block.

On the decoding side, iterative decoding algorithms (e.g., iterative decoding, e.g., using the Dimitrov decoding algorithm) are generally employed to iteratively decode the received coded data to provide better error correction performance.

Overall, Turbo codes greatly improve the performance of channel coding by introducing parallel encoders and interleavers, which fully utilize the redundant information between encoders. It is widely used in various wireless communication standards such as 3G, 4G, and 5G.

4. Code Comparison in Wireless Optical Communication

4.1. The Usage of Error Correction Codes in Wireless Optical Communication

Wireless optical communication is a technology that uses optical wavelengths for wireless data transmission, which uses the atmosphere as a transmission coal for optical signal transmission. However, the communication is susceptible to atmospheric effects, which can degrade the quality of the communication, thus channel coding becomes very important. It can be used to improve the immunity to interference during channel transmission and to improve the communication performance of the system.

The key features of error correction code in wireless optical communication include error detection and correction, by adding the error correction code, the receiver can detect whether the errors occurred during the data transmission process, and to a certain extent, can correct these errors. In wireless optical communication, because the optical signal is susceptible to atmospheric diffusion, multipath effect, noise, and other factors, error correction code can improve the reliability of the receiving end of the data; improve system capacity, error correction code can increase the redundancy of data transmission, thus reducing the probability of errors in data transmission, making the system able to carry more users and data traffic; enhance the ability of anti-jamming, wireless optical communication is often subject to interference from other light sources and electromagnetic interference, error correction code can improve the system's resistance to these interferences, reducing the occurrence of data transmission errors; extend the communication distance, in optical signal transmission, due to signal attenuation and other factors resulting in a decrease in the signal-to-noise ratio will increase the probability of transmission errors. Using the error correction code can effectively improve the channel capacity and extend the transmission distance of wireless optical communication.

4.2. Simulation

Simulation study on the performance of RS code, LDPC code, and Turbo code in wireless optical communication system. Firstly, the simulation is carried out to compare the BER of all types of codes under different weather conditions, which are sunny, cloudy, rainy, and snowy. The information is grouped using RS codes, 9 symbols are divided into one group, and 6 micro-supervised code elements are added to the end of each group. RS (30, 10) is simulated using a pipelined decoding algorithm. Fig 1 shows the result.

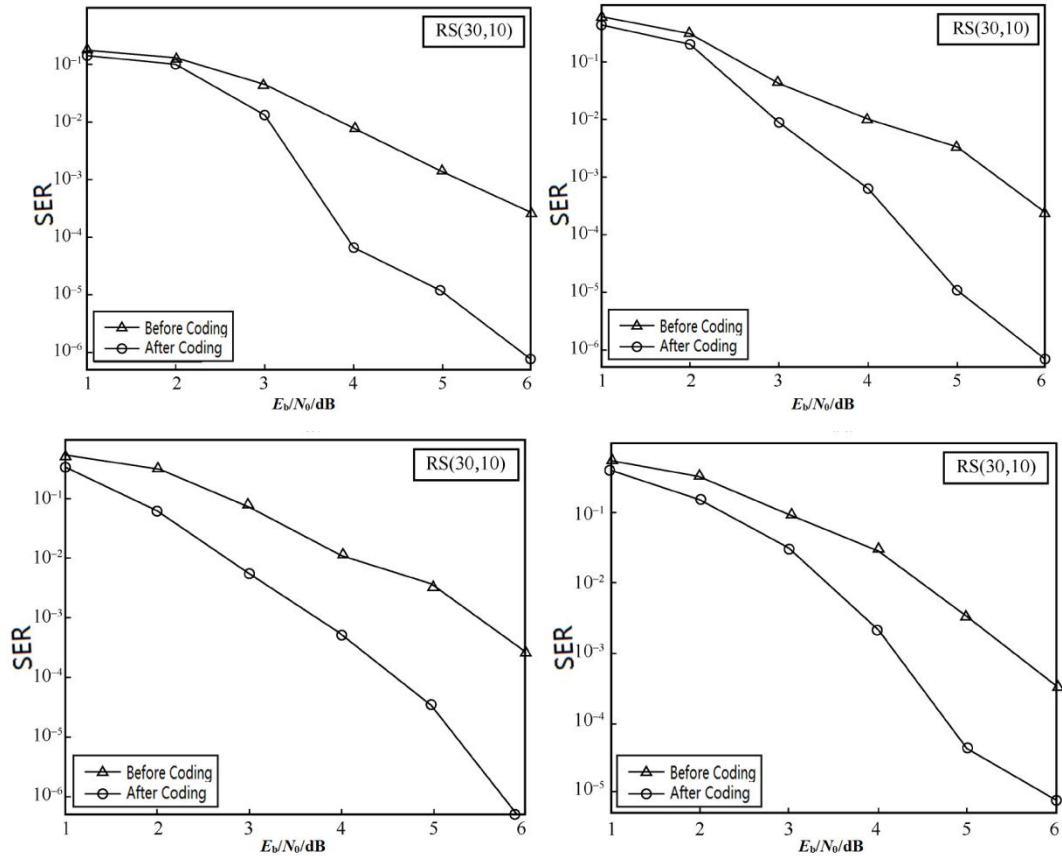


Fig. 1 SER before and after error correction of RS codes under different weather conditions [8]
 Simulation is done for LDPC codes. Fig 2 shows the result.:

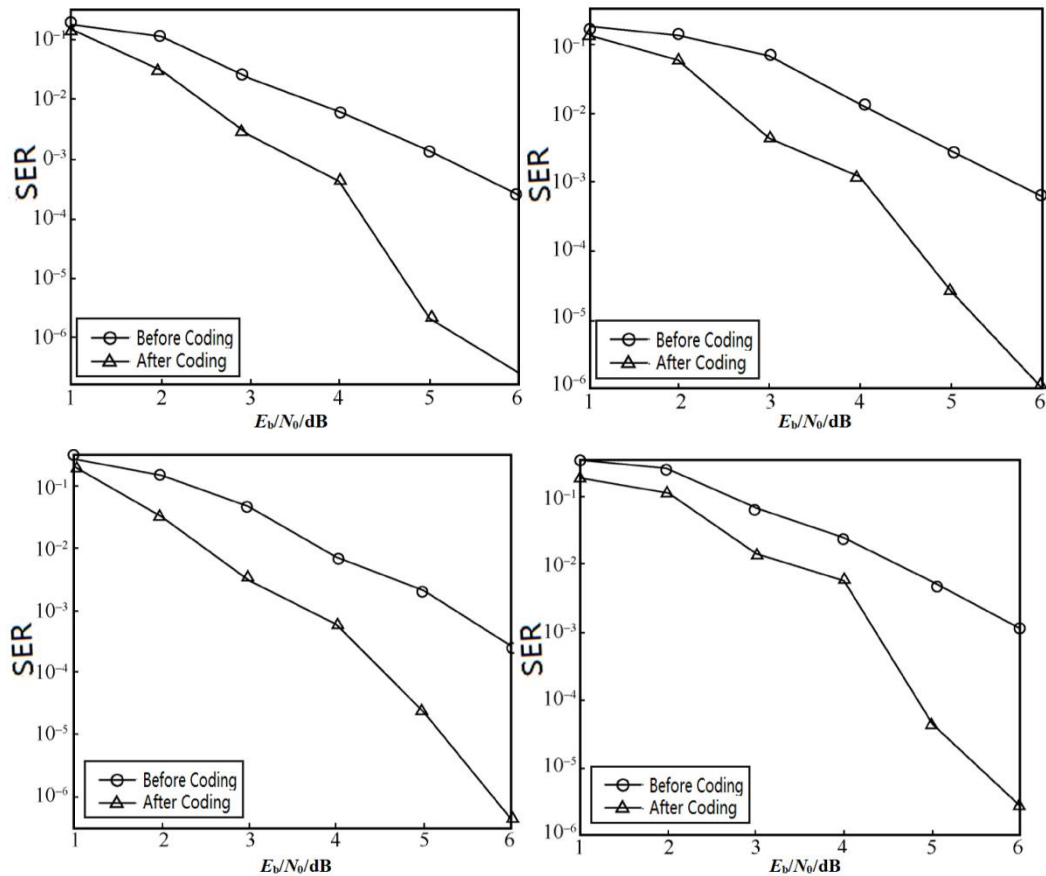


Fig. 2 SER before and after error correction of LDPC codes under different weather conditions

Simulation is also done for turbo codes. Fig 3 shows the result.:

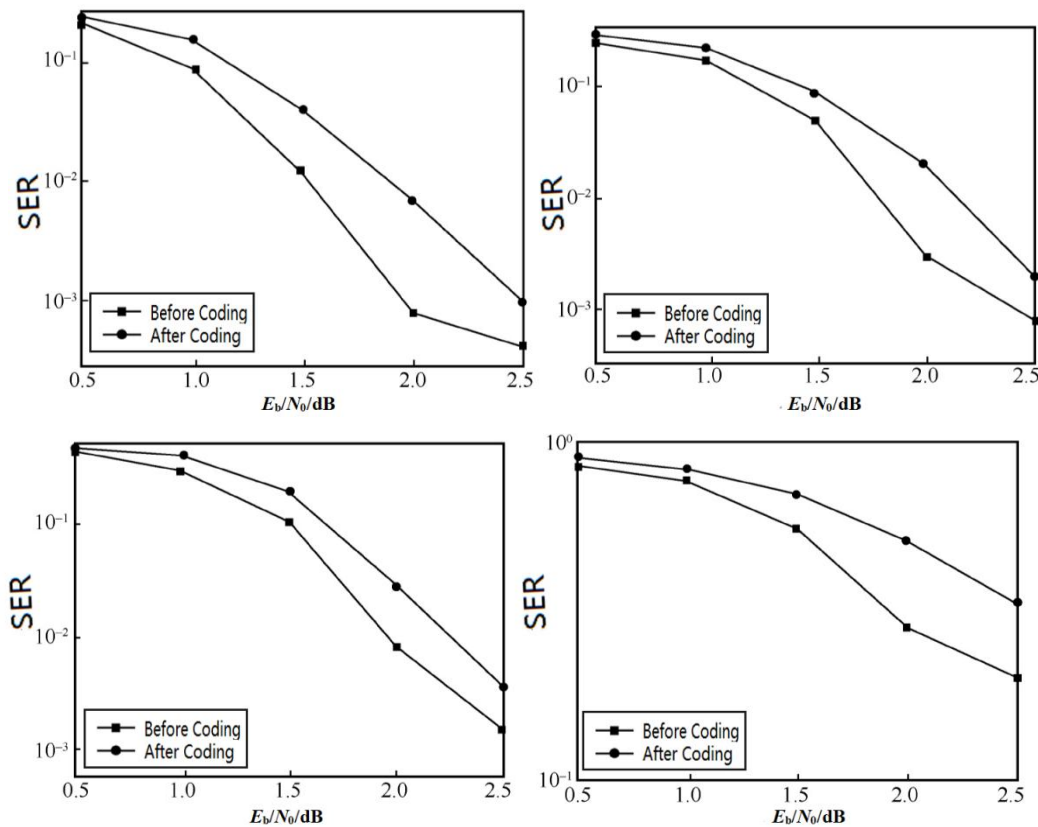


Fig. 3 SER before and after error correction of Turbo codes under different weather conditions

The experimental results show that although the error correction code can improve some of the performance of the wireless optical communication system, the effectiveness is more general in foggy and snowy weather, and the BER increases greatly in snowy and foggy weather. In bad weather, the BER can be up to 10^{-3} or even 10^{-2} . After using the error correction code, the BER can be improved to 10^{-5} . After correcting the error by Turbo code and RS(30,10) code, the BER is below 10^{-5} . The BER can be maintained at 10^{-4} in light to medium rainy weather, and after error correction by LDPC code, the BER can be maintained at 10^{-6} . In sunny and cloudy weather, the average BER is less than 10^{-7} , but after error correction, the BER can be better than 10^{-9} .

5. Code Comparison in Aeronautical Flight Telemetry

5.1. The Usage of Error Correction Codes in Aeronautical Flight Telemetry

Aeronautical flight telemetry is a technology that transmits data and information about an aircraft via radio signals. It can monitor and record various parameters of an aircraft in real-time, such as speed, altitude, attitude, and engine status. However, due to the transmission medium, the time-varying characteristics of interference, and the influence of noise, telemetry signals will occur in the transmission process of the error code error [9]. Thus, the error correction coding technique is very important and can be used to improve the anti-jamming ability of the system.

The main role of error correction coding in aviation flight telemetry is to ensure data integrity: in aviation communication and navigation systems, the integrity of the data is crucial, error correction coding can detect and recover errors in the data transmission process, to ensure the accuracy and integrity of the data; to prevent interference, aviation flight there are many factors that may cause data interference, such as electromagnetic interference, noise, etc., the error correction coding by adding redundant information to detect and correct errors caused by these interferences; improve reliability and fault tolerance, as data transmission in aviation flights often faces complex and

unforeseen environmental conditions, data is likely to be interfered with or partially corrupted when the error correction coding can still recover and rebuild the original data, thus avoiding possible malfunctions and accidents; reduce the cost of communication: error correction coding reduces the frequency of data transmission errors, and this reduces communication costs, otherwise, the presence of errors would result in the need to re-transmit data or perform other types of corrective operations, which would increase communication burdens and time delays.

5.2. Simulation

Bit error rates are compared for RS codes, LDPC codes, and Turbo codes while using convolutional codes as a reference standard. In order to evaluate the performance, a simulated direct expansion-based externally measured communication system in a simulated Gaussian channel environment is simulated. In this context, a Gaussian channel is a channel with Gaussian-distributed noise interference. It is a common stochastic channel model that can simulate the actual noise and interference in various wireless transmissions. Direct spreading is a digital modulation technique that works by dot-multiplying a low-speed data sequence with a high-speed pseudo-random code sequence to extend the bandwidth of the signal to a larger range. At the receiving end, the same spreading code is used for rebroadcasting to restore the original low-speed data sequence. In contrast, an external test communication system is a communication system that is tested and verified in a wireless environment. Such systems typically consist of one or more transmitters and one or more receivers and are used to simulate real communication scenarios and evaluate system performance.

The code rate of 1/2 and 2/3 is used for all the codewords. convolutional code is used with code rate 1/2 (2,1,4) and code rate 2/3 (3,2,4) with Viterbi decoding algorithm and LDPC code is used with code of DVB-S2 standard for deep-space communication with BP decoding algorithm and the number of simulations is 106 times. Turbo codes are generated using codes generated by a recursive system of convolutional codes with generating polynomials (13,15) and a maximum a posteriori probability decoding algorithm.

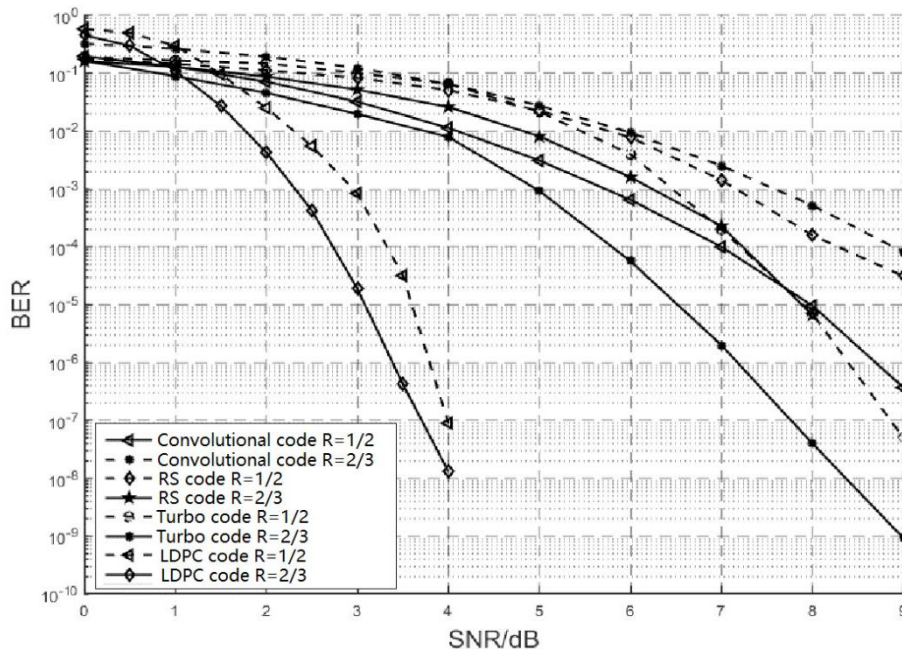


Fig. 4 Comparison of BER performance of different code words with different code rates [10]

It can be seen from Fig 4 that the smaller the code rate in the same code type, the smaller the BER. For the same code rate, the best performance among different code types is LDPC code, followed by Turbo code, convolutional code, and finally RS code. Considering the speed and performance aspects, different channel codes are used for upstream and downstream telemetry links, the convolutional code

with faster decoding speed is suitable for upstream telemetry links, and the LDPC code with good performance is suitable for downstream telemetry links.

6. Conclusion

This paper systematically summarizes the principles and performances of three kinds of error correction control codes and analyzes and summarizes the three kinds of error correction codes in the field of wireless optical communication and aviation flight telemetry. Each of these coding techniques has its own advantages and disadvantages and is suitable for different application scenarios. In general, RS codes are suitable for environments with more burst errors and poor transmission quality, LDPC codes are suitable for high-speed communication systems with high capacity and low noise, and Turbo codes are suitable for communication environments with high BER. According to the specific communication requirements and channel conditions, the appropriate error correction coding technique can be selected to improve the reliability of data transmission.

Error correction codes have several future development trends:

(1) More efficient coding schemes. To meet the growing demand for data transmission, future error correction code schemes will focus more on efficiency. Researchers are working to develop new coding schemes that provide higher error correction capability, and lower redundancy overhead. This will allow faster and more reliable data transmission.

(2) Adaptation to new media. With the emergence of new types of media, such as quantum computing, storage, and communication technologies, error correction codes need to adapt to these new environments. For example, quantum error correction codes are being actively researched to solve the problem of errors in quantum bits. These new error correction codes will play an important role in protecting and processing quantum information.

(3) Increase in code length. Code length refers to the number of bits used to detect and correct errors in an error correction code. In the future, the code length of error correction codes will increase as data capacity and transmission rates increase. Longer code lengths can provide higher error correction capability, but they also come with higher computational complexity and storage overhead. Therefore, researchers need to find a balance to provide an efficient error correction mechanism.

(4) Combining other techniques. The future development of error-correcting codes will also involve combining with other related technologies. For example, combining machine learning and artificial intelligence can provide smarter error correction capabilities. By analyzing a large amount of data and patterns, the system can learn and improve the error correction process, thereby improving error correction performance.

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