Research on the problem of multi-beam line measurement based on the cattle plowing traversal algorithm

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Abstract. In order to study the depth and coverage width of sea water measured by the multi-beam detection system and the control of the overlap ratio range of the detection strip, the full coverage path optimization of the survey ship is carried out. In this paper, firstly, a model is established for the depth of the sea and the coverage width of the multi-beam detection system under the premise that the overlap rate is controlled within the range of $10\% \sim 20\%$ by using the property theorem of trigonometric function and geometric relation. Secondly, the Angle between the direction of the survey line and the projection of the normal direction of the submarine slope on the horizontal plane is analyzed, and the model for the depth of the sea area and the coverage width of the multi-beam detection system is further established under the constraint of the rolling attitude of the survey ship. Finally, the measured sea area is divided into molecular areas according to the overlap rate range and coverage width as standards, and then the coverage width and the number of survey lines are measured based on the cattle plowing traversal algorithm. The following results are obtained: When the survey ship starts and ends in the east-west direction and meets the requirement of $10\% \sim 20\%$ overlap rate, the number of survey lines is stable at 23, the total coverage width of the survey lines is 8.7852×10^{8} m, and the spacing of each survey line is 340.50 m.

Keywords: Cattle plowing traversal algorithm, Full Coverage Path Planning, Trigonometric Function, Multi-beam Detection System, Simple Environment.

1. Introduction

Underwater topographic survey is a basic task of Marine investigation and research, Marine resources and environmental science and deep-sea engineering construction [1] Multi-beam sounding system can simultaneously collect sounding information and backscatter intensity information for obtaining underwater topography and sonar images, and its efficient measurement method has been widely used in underwater topography and geomorphology investigation [2]. Compared with the traditional single-beam sounding technology, multi-beam sounding technology has the advantages of large measuring range, fast speed and high precision [3]. The mapping of the seabed itself can be regarded as a full-coverage path planning problem, in which model selection and path planning of the working environment are difficult to achieve the goal of full coverage of the working area [4]. At present, there are many related algorithms for full coverage path planning of mobile robots, including stochastic method [5], ISC (Internal Spiral Coverage Algorithm) [6], improved element decomposition method [7], BSO-GA algorithm [8] and so on. This paper adopts the cattle plowing traversal algorithm for path planning. Finally, the results obtained from different angles are compared.

2. Data source

The data in this paper come from the website (https://cumcm.cnki.net/) and the data will be further studied and processed.

3. Establishment and solution of the model

3.1. Multi-beam detection system coverage width and overlap rate model

First, the measuring line of the two measuring ships, the distance d between the two adjacent measuring lines and the relative position ΔD_i of the sea water depth at the time of the two adjacent measuring lines are drawn in turn. The sea water depth model at any measuring line is established according to the relationship between the center point and the measuring line and the relative position between the measuring ship and the slope surface. The sine theorem can be used to explain the relationship between the sea water depth change caused by slope α and the distance between the adjacent measuring lines.

3.1.1 Establishment of model

According to the characteristics of multi-beam transmission, the multi-beam detection of adjacent two times is simulated, and the multi-beam detection image of adjacent two times is converted into a side view.Since the change of slope α causes the change of seawater depth D, it can be assumed that the depth of seawater detected by the first and second multi-beams is D_0 and D_1 respectively, and the difference between D_0 and D_1 is ΔD, and the opening Angle of the transducer detected by the first and second multi-beams is θ_1 and θ_2 respectively. Perpendicular to A_1F_1 through point G_1 so that $G_1E_1 \perp A_1F_1$ is at point E_1, so that $A_1H_1E_1G_1$ is a rectangle (Figure 1).

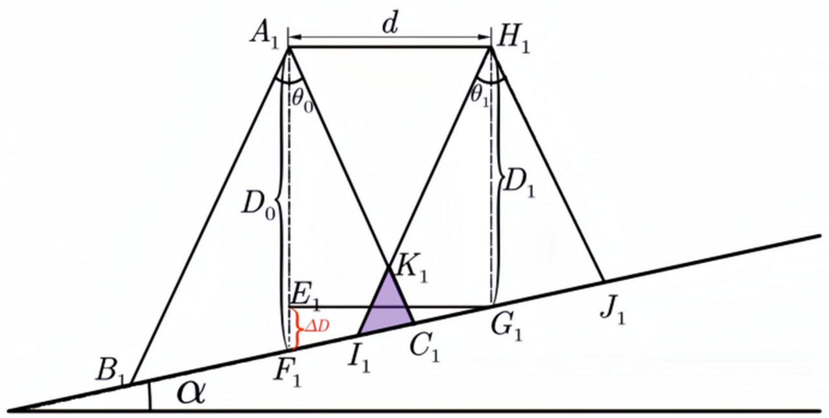

Figure 1. Two-dimensional diagram of seawater depth at any measurement line

According to the parallel line theorem, we can get:

$$\angle E_1G_1F_1 = \alpha, \angle E_1F_1G_1 = \frac{\pi}{2} - \alpha, \tag{1}$$

In $\Delta E_1F_1G_1$, according to the sine theorem:

$$\frac{\Delta D}{\sin\alpha} = \frac{d}{\sin(\frac{\pi}{2} - \alpha)}, \tag{2}$$

The relationship between the spacing d and ΔD of two adjacent measurement lines can be found out by equation transformation:

$$\tan\alpha = \frac{\Delta D}{d}, \tag{3}$$

For the image of single multi-beam detection, make the angular branch line AE of $\angle BAC$ and cross the edge of BC at point E, then:

$$\angle BAE = \angle EAC = \frac{\theta}{2}, \tag{4}$$

Let the length of side BE be α_1, the length of side FC be α_2, extend AE to point G , perpendicular to AG through point B and point C, and cross line AG at point E and point G respectively (Figure. 2).

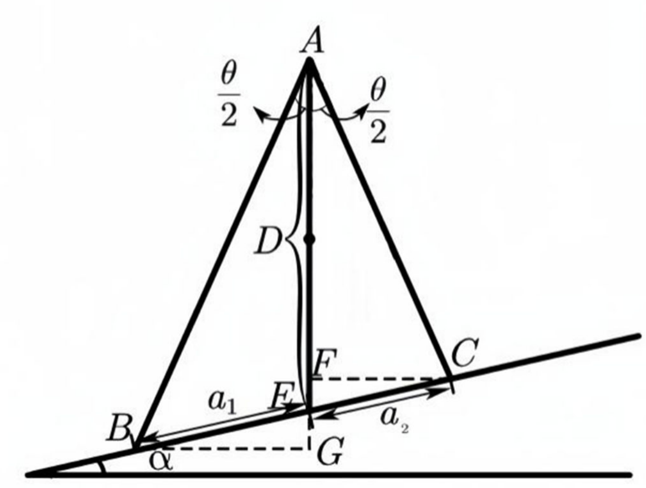

Figure 2. Calculate the coverage width diagram at any measurement line

According to the parallel line theorem:

$$\angle EBG = \angle FCE = \alpha, \tag{5}$$

The coverage width W of multi-beam detection obtained can be expressed as:

$$BE + EC = BC = \alpha_1 + \alpha_2 = W, \tag{6}$$

Using the law of cosine to get the formula:

$$\begin{cases} BE^2 = AB^2 + AE^2 - 2AB\cdot AE\cos\dfrac{\pi}{2} \\ BE^2 = AB^2 + AE^2 - 2AB\cdot AC\cos\dfrac{\pi}{2} \end{cases}, \tag{7}$$

The equations are combined to obtain the coverage width W, which is divided into two parts α_1 and α_2:

$$\begin{cases} (\sin^2\alpha - \cos^2(\dfrac{\theta}{2}))\alpha_1^2 + 2D\sin\alpha\sin^2(\dfrac{\theta}{2})\alpha_1 + D^2\sin^2(\dfrac{\theta}{2}) = 0 \\ (\sin^2\alpha - \cos^2(\dfrac{\theta}{2}))\alpha_1^2 - 2D\sin\alpha\sin^2(\dfrac{\theta}{2})\alpha_1 + D^2\sin^2(\dfrac{\theta}{2}) = 0 \end{cases}, \tag{8}$$

The relationship between covering width and seawater depth and the spacing of two adjacent measurement lines is obtained:

$$W = D_i \cdot \left| \frac{\sin\theta\cdot\cos\alpha}{\sin^2\alpha - \cos^2\dfrac{\theta}{2}} \right|, \tag{9}$$

$$W=(D-d\cdot\tan\alpha)\cdot\left|\frac{\sin\theta\cdot\cos\alpha}{\sin^2\alpha-\cos^2(\frac{\theta}{2})}\right|, \tag{10}$$

When the slope is α, the overlap rate between adjacent strips η is:

$$\eta=1-\frac{\mathrm{d}}{W\cdot\cos\alpha}, \tag{11}$$

Thus, a model for measuring the depth of the sea and the coverage width of the multi-beam detection system is established:

Objective function:

$$\begin{cases} D_i(t)=D-\mathrm{d(t)}\cdot\tan\alpha \\ \mathrm{W(t)}=\mathrm{D_i}(t)\cdot\left|\frac{\sin\theta\cdot\cos\alpha}{\sin^2\alpha-\cos^2(\frac{\theta}{2})}\right| \end{cases}, \tag{12}$$

Constraints:

$$\begin{cases} \eta(t)=1-\frac{\mathrm{d(t)}}{W(t)\cdot\cos\alpha}\in[0.1,0.2] \\ \mathrm{t}\in[0,+\infty) \end{cases}, \tag{13}$$

3.1.2 Simulation measurement of the model

Assuming that the opening Angle θ of the multi-beam transducer is 120°, the slope α is 1.5°, and the seawater depth at the center point of the sea area to be measured is $70m$, the model is used to calculate the following data, as shown in Table 1 below:

Table 1. The calculation result of the model after importing the data to be measured

Measure the distance of the line from the center point/m	Sea depth/m	Covering width/m	The overlap rate with the previous line/%
800	49.05	170.33	-17.42%
600	54.29	188.51	-6.09%
400	59.53	206.70	3.24%
200	64.76	224.88	11.07%
0	70.00	243.07	17.72%
-200	75.24	261.26	23.45%
-400	80.47	279.44	28.43%
-600	85.71	297.63	32.80%
-800	90.95	315.81	36.67%

3.2. Coverage width model of multi-beam detection system with Angle constraint

The coverage width of the multi-beam detection system is measured, and the Angle β formed by the projection of the survey line direction and the normal direction of the submarine slope on the horizontal plane is further considered. Since the track formed by the point with the same distance between the measuring ship and the central point of the sea area is a ring, in order to simulate the actual situation more intuitively, the spherical coordinate system of the sea area to be measured is established (Figure. 3).

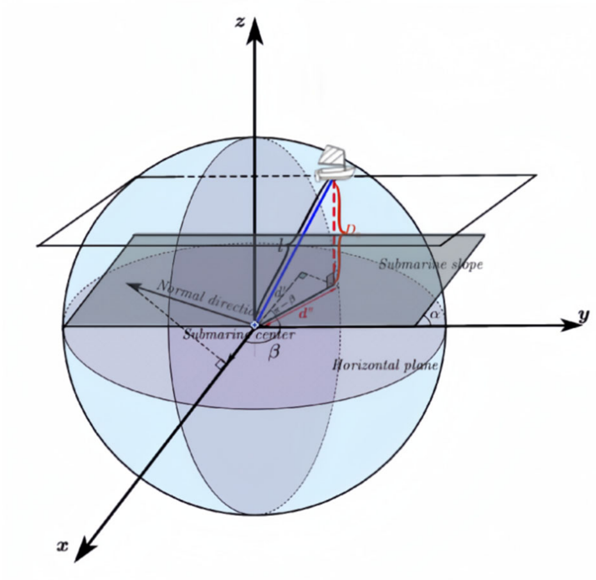

Figure 3. Diagram of 3D model of survey ship detection

3.2.1 Establishment of model

On the basis of the model established above for measuring the depth of sea water and the coverage width of the multi-beam detection system, the constraint condition of Angle β formed by the projection of the direction of the survey line and the normal direction of the submarine slope on the horizontal plane is added. In the spherical coordinate system, the xy plane represents the Angle β formed by the projection of the direction of the survey line and the normal direction of the submarine slope on the horizontal plane, and the xz plane represents the Angle α formed by the plane perpendicular to the direction of the survey line and the intersection line of the submarine slope. Abstract the side view of the multi-beam detection system at the center point of the sea area to be tested (Figure 4)

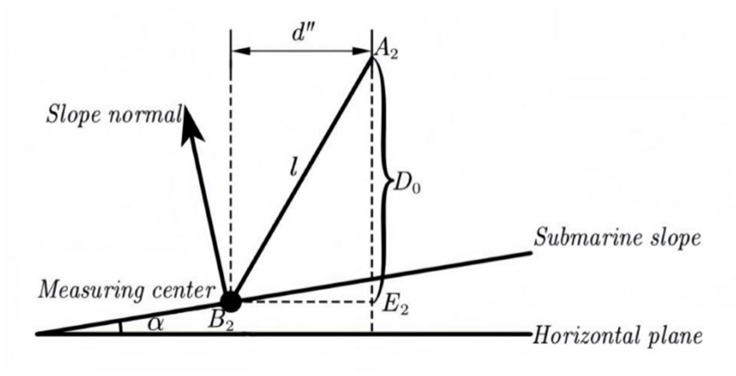

Figure 4. Distance diagram of center from measurement line

The relationship between the distance d' of the two adjacent lines and the projection d'' of the horizontal plane and the depth D_0 and the Angle β at the center of the sea area is obtained:

$$\begin{cases} d' = d'' \cdot \cos(\pi - \beta) \\ d'' = \sqrt{l^2 - D_0^2} \end{cases}, \tag{14}$$

Further, the relationship between the spacing of the two adjacent measurement lines d' and the depth of the sea water D_0 at the center point of the sea area can be obtained:

$$d' = \sqrt{l^2 - D_0^2} \cdot \cos(\pi - \beta) = -\sqrt{l^2 - D_0^2} \cdot \cos\beta, \tag{15}$$

On the basis of the model established above, the relationship between the coverage width with Angle constraints and the spacing of two adjacent measurement lines is obtained:

$$W = \left(D - \mathrm{d}'\tan\alpha\right)\times\left|\frac{\sin\theta\cos\alpha}{\sin^2\alpha - \cos^2\frac{\theta}{2}}\right|, \tag{16}$$

A multi-beam detection system with Angle constraints is established to model the measurement coverage width of the measured sea area:

Objective function:

$$W = \left(D + \sqrt{l^2 - D_0^2}\cdot\cos\beta\cdot\tan\alpha\right)\cdot\left|\frac{\sin\theta\cdot\cos\alpha}{\sin^2\alpha - \cos^2\frac{\theta}{2}}\right|, \tag{17}$$

Constraints:

$$\begin{cases} \eta(t) = 1 - \dfrac{\mathrm{d(t)}}{W(t)\cdot\cos\alpha} \in [0.1, 0.2] \\ \beta \in [0, 2\pi) \end{cases}, \tag{18}$$

3.2.2 Simulation measurement of the model

Assuming that the opening Angle θ of the multi-beam transducer is 120°, the slope α is 1.5°, and the seawater depth at the center point of the sea area to be measured is $120m$, the following data can be calculated by using the model, and Table 2 can be obtained:

Table 2. The calculation result of the model after importing the data to be measured

Covering width/m		Measure the distance of the ship from the center of the sea/NM							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Measurement line direction Angle/°	0	427.60	466.02	517.14	567.86	618.48	669.06	719.62	770.16
	45	424.41	451.57	487.72	523.58	559.38	595.14	630.89	666.63
	90	405.78	367.36	316.24	265.52	214.97	164.33	113.77	63.22
	135	408.98	381.81	345.66	309.80	274.01	238.24	202.49	166.75
	180	405.78	367.36	316.24	265.52	214.91	164.33	113.77	63.22
	225	408.98	381.81	345.66	309.80	274.01	238.24	202.49	166.75
	270	405.78	367.36	316.24	265.52	214.91	164.33	113.77	63.22
	315	424.41	451.57	487.72	523.58	559.38	595.14	630.89	666.63

3.3. Path optimization model of multi-beam detection system based on traversal algorithm of cattle farming

Cattle plowing traversal algorithm is a commonly used full-cover path planning algorithm, also known as reciprocating mulch method [9]. In a rectangular sea area 2 nautical miles long from north to south and 4 nautical miles wide from east to west, the water depth at the center point of the sea area is $110m$, the west is deep and the east is shallow, the slope is 1.5°, the opening Angle of the multi-beam transducer is 120°, and the overlap rate between the adjacent strips meets the requirement of $10\% \sim 20\%$. Since there are no obstacles to block the survey line path in the rectangular sea area (that is, simple environment [10]), this paper adopts the traversal method of cattle ploughing and Matlab software to solve the full coverage path planning, and finally compares the measurement results of the north-south and east-west rectangular sea area respectively.

3.3.1 Establishment of model

According to the above two models, the coverage width obtained by the multi-beam detection system at each location of the sea area to be measured is obtained. The depth of the sea water at the center point of the known sea area is $110m$, the slope is 1.5°, the opening Angl of the multi-beam transducer is 120°, and the starting side length of the measuring ship x is obtained.

$$d = W \cdot (1-\eta), \tag{19}$$

Then the number of lines A is obtained:

$$n = [\frac{x}{d}], \tag{20}$$

Finally, the depth and coverage width of each point on each line are determined, and the path planning model of the multi-beam detection system is established:

Objective function:

$$W = \left(D + \sqrt{l^2 - D_0^2} \cdot \cos\beta \cdot \tan\alpha\right) \cdot \left| \frac{\sin\theta \cdot \cos\alpha}{\sin^2\alpha - \cos^2\frac{\theta}{2}} \right|, \tag{21}$$

Constraints:

$$\begin{cases} d = W \cdot (1-\eta) \\ n = [\frac{x}{d}] \\ \eta \in [0.1, 0.2] \\ \beta \in [0, 2\pi) \end{cases}, \tag{22}$$

3.3.2 Model solving and visualization processing

The rectangular sea area was visually processed to obtain the east-west cross-cutting diagram of the sea area topography to be measured (Figure 5a) and the north-south cross-cutting diagram of the sea area topography to be measured (Figure 5b).

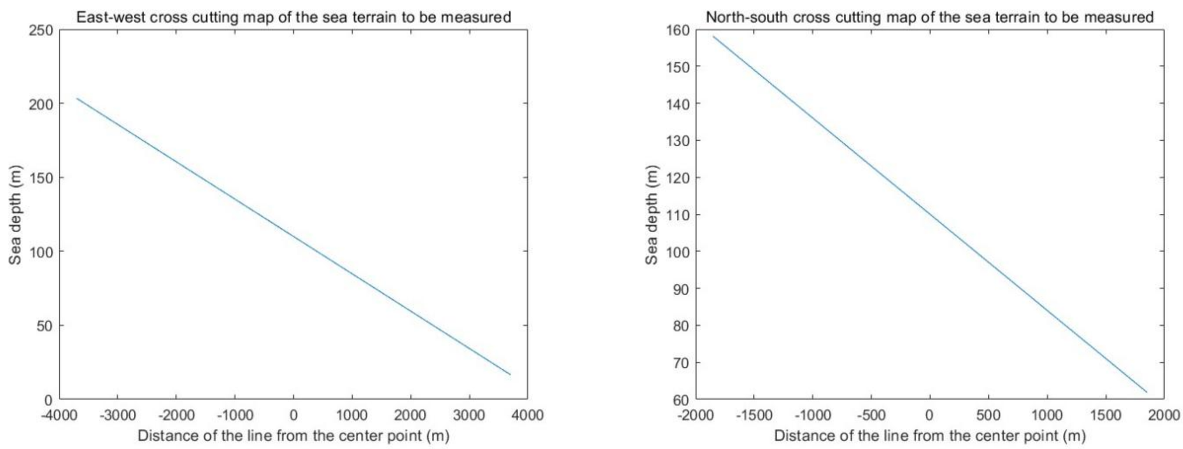

Figure 5. Topographic cross-cutting map of the sea area to be measured in east-west (a) and north-south (b) directions

The path planning model of the rectangular sea area was measured by a multi-beam detection system in two directions (east-west direction and north-south direction) in accordance with the rectangular sea area, and the measurement results in both directions were obtained respectively (Figure. 6).

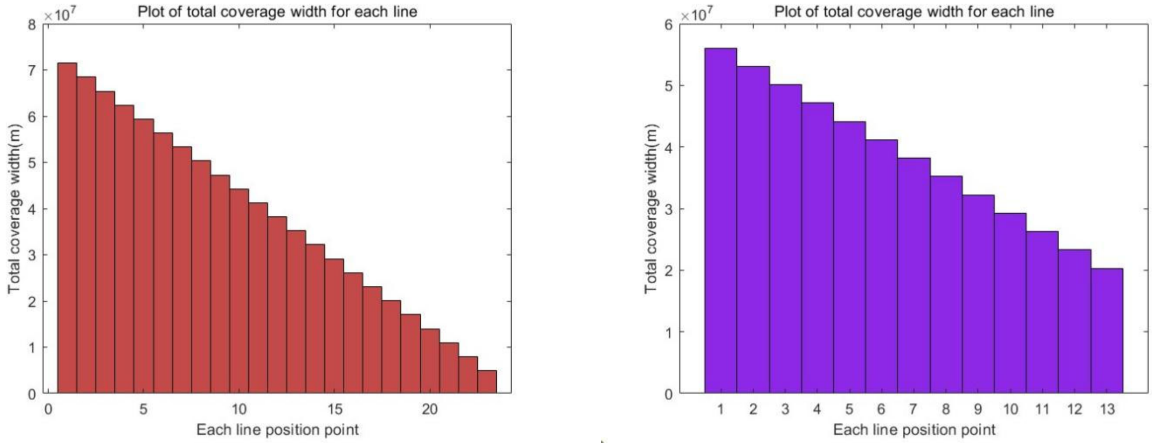

Figure 6. Total coverage width of each line in east-west and north-south directions

3.3.3 Model result analysis

The following results were obtained after the model was run several times to measure the coverage width and the number of survey lines: when the survey ship started and ended in the east-west direction and the overlap rate was $10\% \sim 20\%$, the number of survey lines was stable at 23, the total coverage width of the survey lines was 8.7852×10^{8} m, and the spacing of each survey line was 340.50 m. When the survey ship starts and ends in the north-south direction and meets the requirement of overlap ratio, the number of survey lines is stable at 13, the total coverage width of the survey lines is 4.9655×10^{8} m, and the spacing of each survey line is 333.74 m.

By comparison, it can be seen that the path length and total coverage width of the rectangular sea area are better when the measuring ship takes the east-west direction as the starting and ending point, and the path length is 85192 m.

4. Conclusions

In this paper, the relationship model between seawater depth and cover width is established. Then the model is further optimized, and the coverage width model of the multi-beam detection system with Angle constraint is established. Finally, the path planning problem of the sea area to be measured under simple environment is analyzed by the multi-beam detection system based on the cattle plowing traversal algorithm. It is found that the path length obtained by the surveying ship should be better than that obtained by surveying the sea area from the east-west direction. The measured coverage width can fully cover the sea area to be measured and ensure that the measured line belt meets the overlap rate within the range of $10\% \sim 20\%$. This model maps the seabed conditions of the measured sea area under a simple environment by the multi-beam sounding system, and at the same time, the overlap rate can be controlled to ensure full coverage of the measured sea area..

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