Research on Multibeam Bathymetry Based on Geometric Analysis Models and Brute Force Search Algorithm

Ziyu Liu[1], Yuxiang Zhao[1,*], Pu Wang[1], Jinli Ju[1], Weiquan Lu[1], Jiarong Zhang[2], Wei Zheng[3], Ni Li[3]

[1]School of Information Engineering, Xi'an Mingde Institute of Technology, Xi'an, 710124, China

[2]School of Intelligent Manufacturing and Control Technology, Xi'an Mingde Institute of Technology, Xi'an, 710124, China

[3]School of General Studies, Xi'an Mingde Institute of Technology, Xi'an, 710124, China

* Corresponding Author Email: lzy2398642388@gmail.com

Abstract. In recent years, multibeam sonar systems have gradually replaced single-beam sonar systems. They have become widely accepted and extensively used in the field of seabed depth detection because they overcome the lack of data between survey lines inherent to single-beam systems. The issue of high redundancy in seabed depth data has been a long-standing and challenging problem to solve. To minimize the redundancy rate of seabed depth data as much as possible, this paper conducts a study on the redundancy rate of multibeam sonar systems using different depth measurement methods based on geometric analysis and brute-force search algorithms. This paper establishes a geometric analysis model that utilizes a brute-force search algorithm to enumerate all possible outcomes and selects the optimal solution. The results, upon comparison, show that the redundancy rate of the equidistant uniform survey line method is one-sixth of that of the traditional grid measurement method, indicating good practicality and significantly improving the data space utilization of seabed depth measurement systems.

Keywords: Geometric Analysis Model, Brute-force Search Algorithm, Multibeam Sonar System.

1. Introduction

The multibeam echosounder system is widely used in marine exploration [1], and with the extensive use of electronics, computers, new materials, and new processes, multibeam depth measurement technology has made breakthrough progress [2]. Redundancy rate is an important indicator of the efficiency of seabed depth measurement. The redundancy rate directly affects the volume of seabed depth data. Reducing redundancy, saving data space, and simplifying positioning methods are of significant importance for seabed depth measurement. Traditional seabed depth measurement methods are based on a grid measurement model, which suffer from a high redundancy rate and difficulties in achieving precise positioning.

2. The Basic Principle of the Brute-force Search Algorithm

2.1. Brute-force Search Algorithm

The brute-force search algorithm is a fundamental computer algorithm whose primary principle is to find a problem's solution by enumerating all possible solutions within the solution space. It does not rely on any specific properties of the problem but rather simply attempts every possible solution and selects the one that satisfies certain conditions. The advantage of the brute-force search algorithm lies in its simplicity and intuitiveness; for problems of small scale or with a smaller solution space, it can quickly find the optimal solution.

2.2. Determining the Subject of Discussion for the Brute-force Search Algorithm

Given the parameters of the coverage area, the depth at the center point, and the opening angle of the sonar on the small boat, when using the equidistant uniform survey line method for depth measurement of the surveyed sea area, the objects of discussion are only the angle between the horizontal projection of the small boat's navigation direction vector and the seabed slope's normal vector, and the interval between survey lines during the boat's depth measurement. When using the traditional grid method for depth measurement of the surveyed area, there is no need to use the brute-force search algorithm. The angle between the horizontal projection of the boat's navigation direction vector and the seabed slope's normal vector is the same as that in the equidistant uniform survey line method, and the interval between the boat's survey lines is determined by the actual parameters of the surveyed sea area.

3. Results

3.1. Establishment of the Geometric Analysis Model

To establish the geometric analysis model, an absolute spatial Cartesian coordinate system is set up with the central point of the sea surface above the surveyed area as the origin and the vertical upward direction as the positive direction of the z-axis. For any seabed slope, three non-collinear points on the slope are selected, and their depths are measured to obtain their Cartesian coordinates. Based on these three points, any two points are connected to form two non-collinear spatial vectors, which define the entire seabed slope and allow for the calculation of the slope's normal vector.

Let the two vectors be $\vec{a}=(x_1,y_1,z_1)$ and $\vec{b}=(x_2,y_2,z_2)$.

The normal vector $\vec{p}$ is calculated as follows:

$$\vec{p}=\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=(x,y,z) \tag{1}$$

The origin of the absolute Cartesian coordinate system is set at the lower-left corner of the sea surface corresponding to the surveyed area, and the direction of the projection of vector $\vec{p}$ onto the plane defined by vector xoy is taken as the positive direction of the x-axis in the absolute Cartesian coordinate system.

Let vector $\vec{p}$ be defined such that its angle with the horizontal direction is ε:

$$\varepsilon=\arccos\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) \tag{2}$$

Then, the angle between the plane perpendicular to vector A and the horizontal plane is the slope of the seabed incline.

$$\alpha=\frac{\pi}{2}-\varepsilon \tag{3}$$

In the scenario settings, it is required that the seabed incline must not exceed the sea surface level, as this would pose a risk of boats running aground or becoming stranded within the surveyed area. The angle γ, which is the angle of the projection of the survey line from the boat onto the seabed incline relative to the horizontal plane, can be represented as follows:

$$0\le\gamma\le\arctan\left(\frac{2H_0}{d}\right),\text{ at the same time, }\gamma\le\alpha \tag{4}$$

In this context, H_0 represents the depth at the center point of the surveyed area, d is the interval between the survey lines of the boat, and α is the slope of the seabed incline. The relative incline should also be less than or equal to the seabed incline given in the scenario. Only when these conditions are met can the exploration of the entire surveyed area be carried out. To ensure the accuracy of the measurements, the measurement frequency f and the speed $\vec{v}$ should be equal, that is $f = |\vec{v}|$. The goal is to carry out a measurement at every 1-meter interval, which facilitates easy positioning. During each survey, information is updated only for 1 meter in the direction of navigation, helping to save data space.

We have a sea area to be surveyed: both the length and the width are 4000 meters, with a slope of 1.5°, and the depth at the central point θ is 120 meters, with a measurement opening angle of 120°. By taking the projection of the boat on the slope as the origin, the direction of the projection of the seabed normal vector on the horizontal plane as the positive direction of the x-axis, and vertically upwards as the positive direction of the z-axis, a relative spatial Cartesian coordinate system is established. This allows us to obtain a schematic diagram of the boat's navigation during depth measurement in the sea area, as shown in Figure 1.

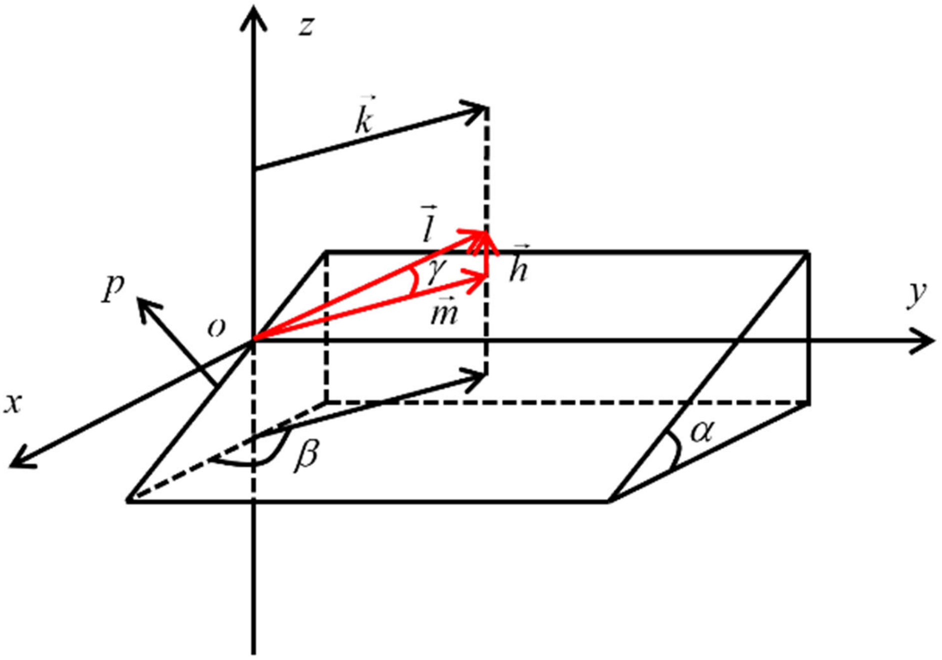

Figure 1. Three-dimensional diagram of the relative spatial Cartesian coordinate system

3.2. Comparison of Grid Method Measurement and Equidistant Uniform Survey Line Method

3.2.1 Basic Principle of the Grid Measurement Method

The grid measurement method refers to the process in which the ship travels to a point and releases a detection signal. This method uses the point as the center to spread the detection waves around beneath the ship for depth measurement [2]. If the surveyed seabed area is a flat plane, the projection of the spread detection waves on the seabed is a circle. However, in the scenario described in this paper, the seabed of the surveyed area is a slope; thus, the projection of the spreading detection waves on the slope is an ellipse, with the center's horizontal coordinate of the ellipse consistent with the horizontal coordinate of the ship's measurement point. The geometric analysis model can be used to calculate the actual detection area on the slope, allowing for simple mathematical calculations.

Assuming a fixed interval along the y-axis to satisfy the formation of grid measurement, and to ensure that there are no missed areas in the survey, a vertical cross-section is taken along the shallowest part of the sea area in the direction of the ship's travel. The transverse interval should be the largest factor of 4000, excluding itself. The schematic diagram of the navigation cross-section is shown in Figure 2.

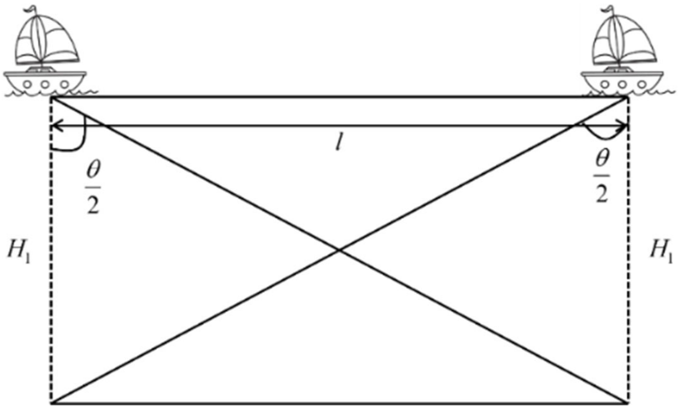

Figure 2. Schematic Diagram of the Navigation Cross-Section

Analysis shows that with $l \le 2H_1 \tan\frac{\theta}{2}$, representing a certain value derived from known conditions, $l \le 234.270807$ can be calculated accordingly. Since 4000 must be divisible by l without a remainder, the optimal scenario occurs when the largest possible value, $l = 200m$, is chosen.However, the distance in the ***x***-axis direction needs to change with the variation in seabed depth. The schematic diagram of the survey line cross-section is as shown in Figure 3.

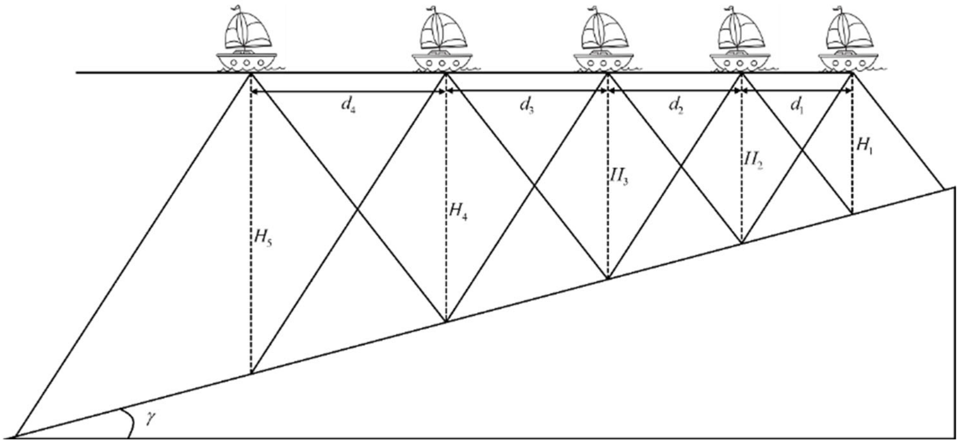

Figure 3. Schematic Diagram of the Survey Line Cross-Section

Each measurement's range must intersect with the projection of the ship on the slope. The analysis starts from the survey line where the depth is the shallowest. The schematic diagram showing the intersection of the projections is illustrated in Figure 4.

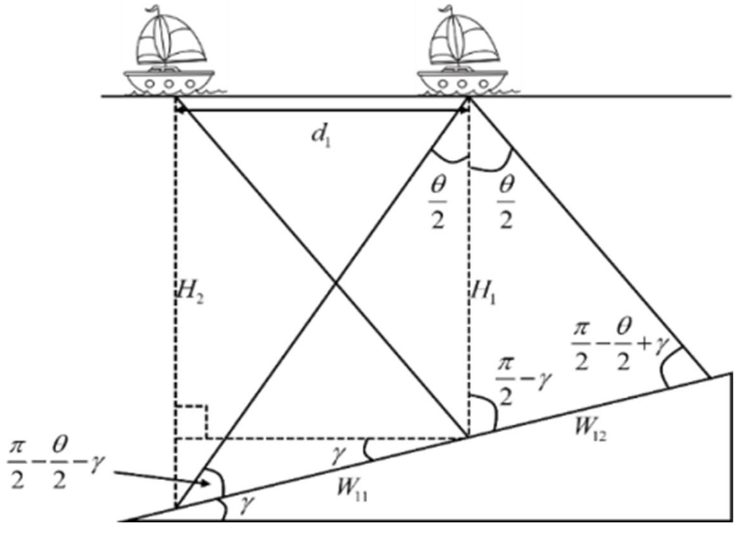

Figure 4. Projection Intersection Diagram

According to the law of sines, the maximum length of the seabed slope projection on both sides of the first survey line, $W_{i,m}(i=1,2,\cdots 21;m=1,2)$, can be obtained as follows:

$$W_{i,m}=\frac{H_1\sin\frac{\theta}{2}}{\sin\left[\frac{\pi}{2}-\frac{\theta}{2}+(-1)^{m-1}\gamma\right]} \tag{5}$$

Among them, H_1 represents the depth of the seabed corresponding to the initial survey line of the boat, θ is the opening angle of the detector when the boat is measuring depth, and γ is the angle between the projection of the boat's survey line on the seabed and the horizontal plane. By $W_{1,1}$, it can be calculated that the distance between the initial survey line of the boat and the second survey line is d_1, and the depth corresponding to the second survey line is H_2:

$$d_1=W_{1,1}\cos\gamma \tag{6}$$

$$H_2=H_1+d_1\times\tan\gamma \tag{7}$$

By extension, two formulas can be derived to calculate the depths corresponding to all survey lines:

$$d_i=W_{i,1}\cos\gamma \tag{8}$$

$$H_{i+1}=H_i+d_i\times\tan\gamma\,(\,i\geq 1,i\in Z\,) \tag{9}$$

From the analysis of the geometric model of the point-grid measurement method, it is known that the horizontal axis a_i of the ellipse corresponding to the i -th survey line and the x -axis coordinate corresponding to the center of the ellipse are respectively:

$$a_i=\frac{W_{i,1}+W_{i,2}}{2} \tag{10}$$

$$x_i=\sum_{n=1}^{i-1}d_i+\frac{W_{i,1}-W_{i,2}}{2}\cos\gamma \tag{11}$$

$W_{i,1}$ and $W_{i,2}$ are respectively the maximum lengths of the seabed slope projections on both sides of the i -th survey line, then the depth h_i corresponding to the center of the ellipse of the i -th survey line and the vertical axis b_i of the ellipse are respectively:

$$h_i=H_i+(x_i-x_{i-1})\tan\gamma \tag{12}$$

$$b_i=h_i\tan\frac{\theta}{2} \tag{13}$$

By calculating the projections of all the ellipses on the slope and using the method of calculus to subtract the area outside the region, the sum of the areas of all ellipses within the region can be determined. Using trigonometric function relationships, the area of the seabed slope in the surveyed sea area can be calculated. The partial results of the ellipse projection areas from the 1st to the 5th survey lines are shown in Table 1.

Table 1. Partial Display of the Elliptical Projection Area Results Using the Grid Survey Line Method

The i-th survey line	1	2	3	4	5
Depth	67.76756	70.98721	74.35983	77.89268	81.59337
Horizontal Axis (a)	117.3769	122.9535	128.795	134.9141	141.3239
Vertical Axis (b)	117.4171	122.9956	128.8391	134.9603	141.3723
S=Pi (π)*a*b	43297.6	47509.49	52131.1	57202.29	62766.79

The sum of the elliptical areas minus the difference with the sloping surface area, divided by the sloping surface area, is the redundancy rate of the grid method within the surveyed sea area. The calculated redundancy rate is 295.61%, which places high demands on data

3.2.2 Basic Principle of Equidistant Uniform Survey Line Method

γ is the angle between the projection of the survey line on the seabed slope and the horizontal plane, which is the angle between the bathymetric path vector $\vec{l}$ and the horizontal plane; Vector $\vec{k}$ is the actual navigation vector of the boat on the sea surface, which is generally set to one measurement interval, and in the set scenario, the measurement interval is set to 200m; h is the gradually increasing height of the slope inside the seabed as the boat travels over a measurement interval; $\vec{m}$ is a vector perpendicular to axis y, pointing in the direction of the navigation vector from axis y; the angle between the two vectors should be in the range of (0~π). When β is greater than π, it can be symmetrically processed, and a brute force search algorithm is used on MATLAB to discuss β.

Regarding the discussion of angle γ between the survey line and the horizontal direction, there are two cases: obtuse and acute angles. Taking the obtuse angle as an example:

In the perspective view of the slope, in the relative spatial orthogonal coordinate system, β is an obtuse angle, as shown in the top view in Figure 5 and the side view on the right in Figure 6.

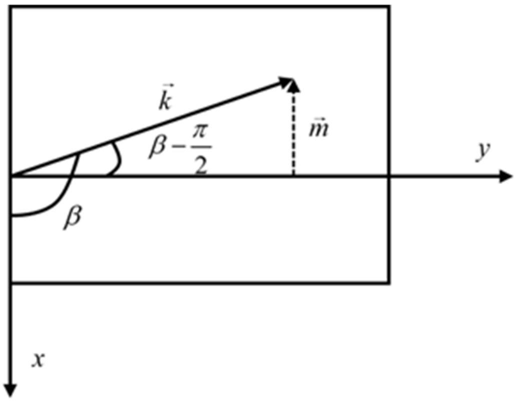

Figure 5. Top view of the slope when angle ***β*** is obtuse.

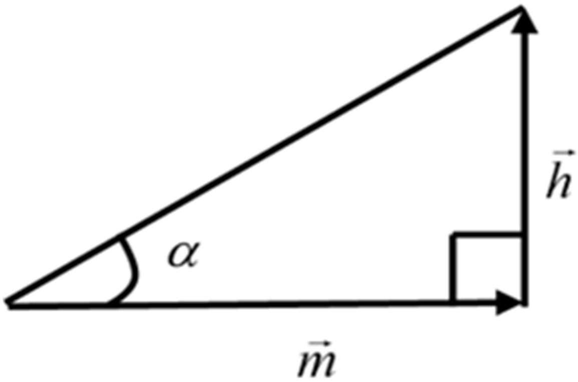

Figure 6. Side view (right) of the slope when angle ***β*** is obtuse.

Based on the trigonometric function relationships, we can obtain:

$$|\vec{m}| = |\vec{k}| \times \sin(\beta - \frac{\pi}{2}) \tag{14}$$

Where, $\vec{k}$ is the displacement vector of the boat; $\vec{m}$ is the horizontal component vector of the boat's displacement; β is the angle between the projection vector of the boat's displacement vector on the horizontal plane and the positive direction of the x axis in the relative spatial rectangular coordinate system. When β is an obtuse angle, the cross-sectional view of the survey line is shown in Figure 7:

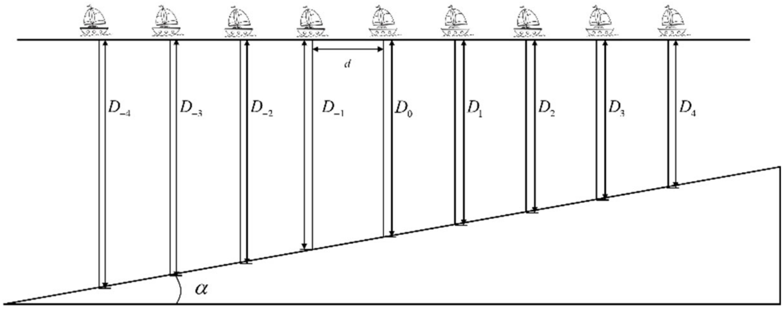

Figure 7. Cross-sectional view of the survey line when β is an obtuse angle.

Similarly, if ***β*** is an acute angle, then we have:

$$|\vec{m}| = |\vec{k}| \times \sin(\frac{\pi}{2} - \beta) \tag{15}$$

D_i represents the corresponding depth of the ***i***-th survey line:

$$D_i = D_{i-1} + id\tan\gamma \tag{16}$$

Where, $i = (-n, 1-n, \cdots, 0, \cdots, n-1, n)$; n is the result of the surveyed area's width being evenly divisible by the survey line spacing d, with a total of $2n+1$ survey lines; d is the uniform survey line spacing; when $\beta = \frac{\pi}{2} + k\pi$, $k \in Z$, that is when the direction of the boat's travel is perpendicular to the projection of the seafloor slope on the horizontal plane, the measured depth remains constant.

In summary, the calculation formula for depth allows us to determine:

$$|\vec{m}| = |\vec{k}| \times \sin(\left|\frac{\pi}{2} - \beta\right|) \tag{17}$$

$$|\vec{h}| = |\vec{m}| \times \tan\alpha \tag{18}$$

Where, $\vec{h}$ is the vertical component vector of the boat's displacement; ***β*** is the angle between the projection vector of the boat's displacement on the horizontal plane and the positive direction of the ***x***-axis in the relative spatial orthogonal coordinate system. The projection *γ* of the survey line on the seabed slope changes according to the variation of ***β***, resulting in the formula:

$$\gamma = \arctan\left(\frac{|\vec{h}|}{|\vec{k}|}\right) = \arctan\left(\frac{|\vec{m}| \times \tan\alpha}{|\vec{k}|}\right) = \arctan\left(\sin(\left|\beta - \frac{\pi}{2}\right|) \times \tan\alpha\right) \tag{19}$$

On the boat's heading, a cross-section is taken, which yields a profile of the survey route, as shown in Figure 8.

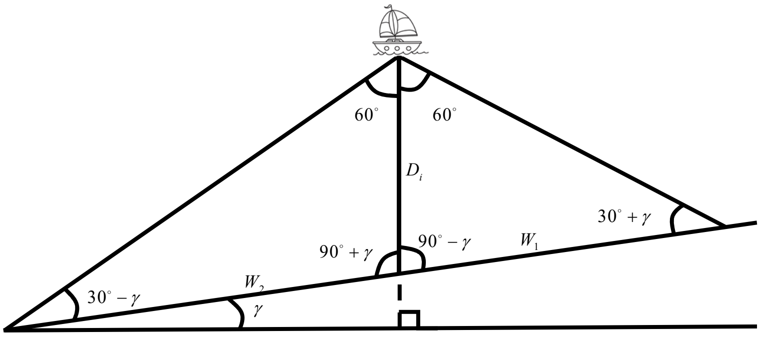

Figure 8. Cross-section of the navigation route.

In the relationship shown in the cross-section of the survey line, using a geometric analysis model similar to the grid survey line method and employing the Law of Sines, we can determine the maximum length of the seabed slope projection on both sides of the ***i***-th survey line, noted as $W_{i,1}$ and $W_{i,2}$, as well as the coverage width at the ***i***-th survey line, and the redundancy rate at the ***i***-th survey line. The redundancy rates from the 1st to the 5th survey lines are partially shown in Table 2:

Table 2. Partial Display of Redundancy Rate Results Using the Equidistant Uniform Transect Method.

Row i	1	2	3	4	5
d	200	200	200	200	200
W(i,1)+W(i,2)	598.5495	580.3637	562.1779	543.9921	525.8064
Coverage Width(W)	598.3444	580.1648	561.9853	543.8057	525.6262
Coverage Width(η)	0.665744	0.65527	0.644119	0.632222	0.619501

When surveying the depth by a small boat throughout the entire surveyed marine area, the redundancy rate should be replaced by the average of the redundancy rates at all transects. The calculated redundancy rate is 48.14%, which is much lower than the redundancy rate measured by the grid transect method, and there are non-negligible advantages in terms of storage space and storage performance.

4. Conclusions

In the marine area to be measured, which has a length and width of 4000 meters each, a slope of 1.5 degrees, and a central depth of 120m; under the condition of a survey opening angle (θ) of 120°, the result of the brute force search algorithm in the equidistant uniform transect method is best when A is considered, with the redundancy rate calculated to be 48.14%. In contrast, the grid transect method has a redundancy rate of 295.61%. In the above research, we have significantly reduced the redundancy rate of depth measurement by the small boat, saving the data volume to less than 1/6 of the original, and ensured that there are no omissions in the survey. However, while calculating the omission rate for the grid transect method, it is assumed that there are no missing parts; in reality, the results of the grid transect method could only be worse.

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