Optimal modeling of line paths based on multibeam bathymetric system

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Abstract. Multibeam ocean bathymetry is one of the major issues in exploring the ocean. For problems such as multibeam line ocean bathymetry, this paper starts from the related theory, analyzes the inherent geometric features of the problem, comprehensively establishes a multibeam line model with generalized significance within a certain error range, and according to the method of least squares, gives a model of the line with a fixed opening angle of the multibeam transducer in a certain range of the sea area. Firstly, from the geometrical point of view, the interpreted diagram and the schematic diagram given in the title are combined and simplified into a problem of solving a triangle. The trigonometric equations are listed through the sine theorem and solved algebraically. A mathematical model is developed about the coverage width and adjacent strips. Then, from the question, we know that we need to solve the modeling differences caused by the different angles of the measuring lines in different directions, i.e., to elevate the two-dimensional problem of solving triangles to a three-dimensional problem of three-dimensional geometry. By finding the angle between the coverage width and the horizontal plane γ , we know the relationship between it and the angle between the direction of the survey line β and the slope angle α , so as to establish the model of the coverage width of multibeam bathymetry. Summarized by the verification of the previous conclusions, this paper obtains: when controlling the width of two adjacent survey lines exactly meets the overlap rate of 10% with the previous survey line, and the survey line direction is perpendicular to the seafloor slope in the projection, the measurement length is the shortest at this time. Using the above summary as a constraint, the shortest path is calculated to be 66 nautical miles using the trigonometric method.

Keywords: Solving triangles, Cubic geometry, Trigonometry, Least-squares method.

1. Introduction

The ocean is an important resource for mankind, and exploring the ocean is an important topic for mankind. Under the background of the rapid development of science and technology, many advanced measurement systems are now applied to ocean surveying. In terms of ocean bathymetry, surveying science has developed from single-beam bathymetry to multibeam bathymetry. [1]Both single-beam and multibeam bathymetry utilize the propagation characteristics of acoustic waves in water to complete the measurement of water depth. Compared with single-beam measurement, multibeam measurement has many advantages, which greatly improves the efficiency of seabed topography measurement, and in the process of measurement, it can be carried out for a number of measurement points on the seabed, and obtain the corresponding measurement depth value. [2]Among them, the extended bathymetry, solving coverage and other issues is an important aspect. In the process of multibeam detection, in order to prevent leakage between the strips and detect targets of a certain scale, there is generally a certain degree of overlap between the strips. [3]The application of this technology in the process of marine measurement realizes the effect of surface measurement, which makes the efficiency and quality of marine measurement greatly improved and promotes the steady development of China's marine industry. At the same time, due to the complexity of the seabed terrain, the undulation changes, how to efficiently adjust the survey line spacing and get a good overlap rate so that the multibeam efficiently in any sea bathymetry has become an urgent problem to be solved.[4]

2. Determine the coverage width of multibeam bathymetry and the overlap rate between adjacent strips

First of all, it is necessary to establish a mathematical model of the coverage width of multibeam bathymetry and the overlap rate between adjacent strips in order to solve the problem, and this paper assumes that the direction of acoustic propagation will not be affected by seawater and marine organisms, suspended matter, and can accurately obtain the seabed topographic data. [5]In practice, the coverage of multibeam bathymetry is very large, with a very small possibility of being affected by seawater and marine organisms, suspended matter. [6]In order to idealize the model, it is assumed that the acoustic wave propagation direction will not be affected by seawater and marine organisms and suspended matter, and in addition, it is simplified into a problem of solving the equation of a triangle. As shown in Fig 1. (with brown triangles as small boats).

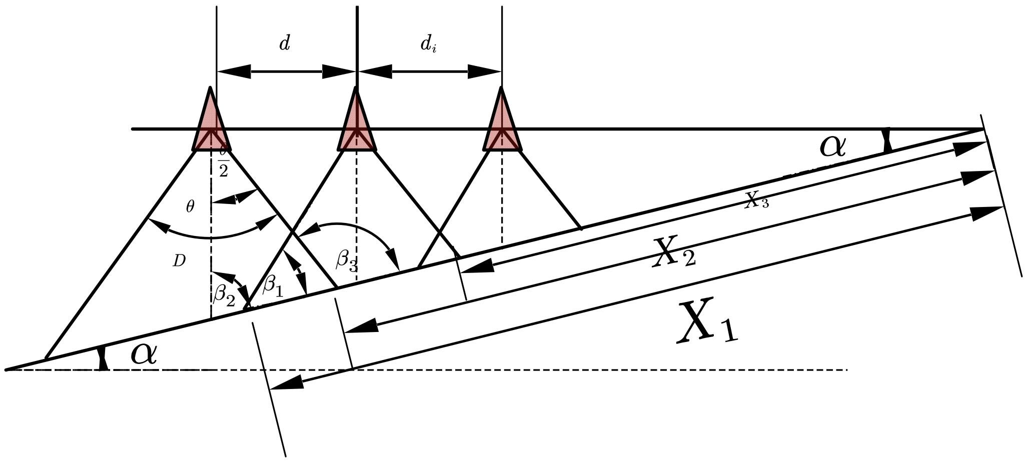

Fig. 1 Survey ship scan slope cross-section

It is obtained from the geometric relationship:

$$\beta_1 = 90° - \alpha - \frac{\theta}{2} \tag{1}$$

$$\beta_2 = 90° - \alpha \tag{2}$$

$$\beta_3 = 90° - \alpha + \frac{\theta}{2} \tag{3}$$

$$\mathrm{d}_i = \frac{D}{\tan\alpha} \tag{4}$$

$$d_i^{'} = d_i + d \tag{5}$$

It also follows from the sine theorem:

$$\frac{X_1}{\sin(\frac{\theta}{2}+90°)} = \frac{d_i}{\sin\beta_1} \tag{6}$$

$$\frac{X_2}{\sin(90°-\frac{\theta}{2})} = \frac{d_i^{'}}{\sin\beta_3} \tag{7}$$

$$\frac{X_3}{\sin(90°-\frac{\theta}{2})}=\frac{d_i}{\sin\beta_3} \tag{8}$$

In this paper, the overlap ratio is equal to the coverage width of the overlapping portion of two neighboring survey lines over the length of the coverage width of either line. Thus generalizing the coverage of flat terrain to that of terrain with slopes can be obtained:

$$\eta=1-\frac{X_2-X_3}{X_1-X_3} \tag{9}$$

The joint solution is obtained:

$$\eta=1-\frac{d(\frac{1}{\tan\frac{\theta}{2}}-\tan\alpha)}{2D} \tag{10}$$

Finally, according to the conditions given in the question:

$$\theta=120°,\alpha=1.5°,\text{The initial } D=70m \tag{11}$$

3. Establishment of a mathematical model of multibeam bathymetric coverage width in any line direction

Assume that the depth from the initial position of the ship to the vertical of the seabed slope is known here; assume that the depth of the sea water at the initial position D_0 is already known.

First derive the change in sea depth for a ship along the angle β from a sea depth of D_0 to D_1 at Δh .

Let Δy be the displacement in the direction parallel to the normal to the bottom of the slope; let Δh be the magnitude of the depth change; and the displacement of the survey ship sailing under the above conditions is $S_{D_0D_1}$

Fig. 2 shows a three-dimensional schematic diagram, Fig. 3 shows a top view of Fig. 2, and Fig. 4 shows a side view (left view) of Fig. 2.

According to the schematic diagram:

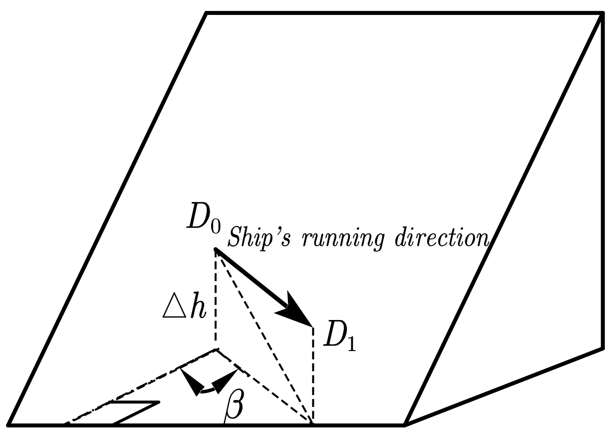

Fig. 2 Three-dimensional diagram of Angle relation

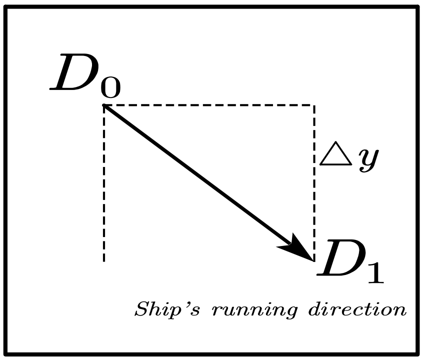

Fig. 3 Angle relationship Top view

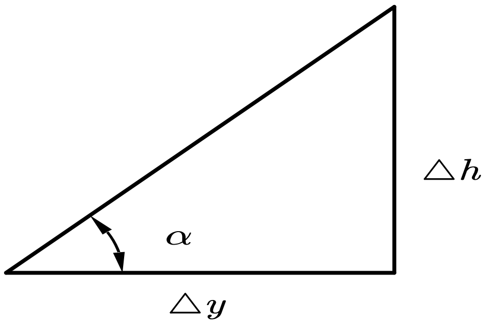

Fig. 4 Angle relationship left view

$$\Delta y = (S_{D_0D_1})\cos\beta \tag{12}$$

$$\Delta h = \Delta y \tan\alpha \tag{13}$$

$$D_1 = D_0 + \Delta h \tag{14}$$

assume (office)

$$D_1 = D_0 + (S_{D_0D_1})\cos\beta\tan\alpha \tag{15}$$

Secondly, a mathematical model is established for the measurement line angle at (0,90]. Then based on this model is extended to the measurement line direction angle located in the (90,180], (180,270], (270,360] mathematical models

As shown in Fig 5, BD is the projection of the width of the cover on the horizontal plane, and BC is a line parallel to the line of intersection of the submarine slope and the horizontal plane, which can be obtained from the geometric relationship, $\angle DBC = \beta$.

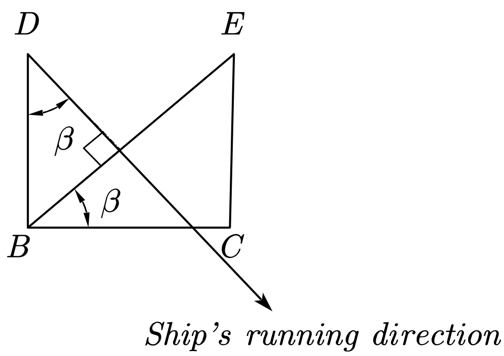

Fig. 5 Angle relation planar graph

Let the angle between the width of the cover and the horizontal plane be $\angle ABD = \gamma$ from the geometric relations in Fig. 6:

$$\tan\gamma = \sin\beta\tan\alpha \tag{16}$$

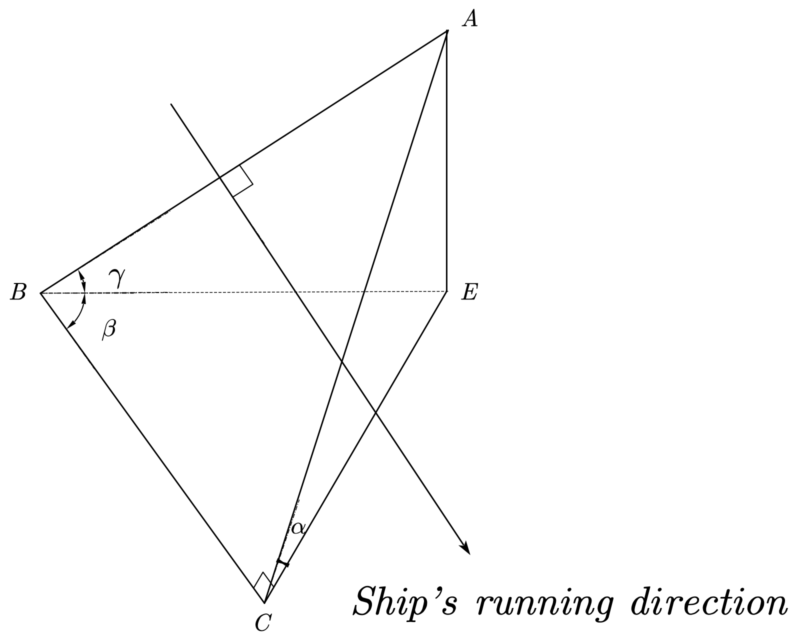

Fig. 6 Stereogram of angular relationship

Also as in Fig. 7 can be obtained: let the intersection of the angular bisector of the open angle of the multibeam transducer and the coverage width AB be O, $AO = X_{AO}, BO = X_{BO}$

This is obtained by the sine theorem:

$$\frac{X_{AO}}{\sin\frac{\theta}{2}} = \frac{D_1}{\sin(90° - \frac{\theta}{2} + \gamma)} \tag{17}$$

$$\frac{X_{BO}}{\sin\frac{\theta}{2}} = \frac{D_1}{\sin(90° - \frac{\theta}{2} - \gamma)} \tag{18}$$

Based on:

$$W = X_{AO} + X_{BO} \tag{19}$$

The simplification yields:

$$W = X_{AO} + X_{BO} = \frac{D_1\sin\theta\cos\gamma}{\cos(\frac{\theta}{2} - \gamma)\cos(\frac{\theta}{2} + \gamma)} \tag{20}$$

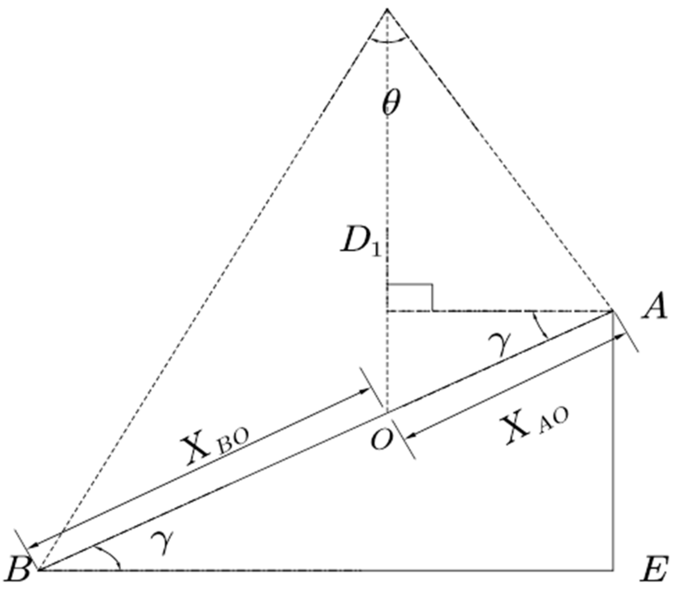

Fig. 7 Scanning slope range geometric relationship diagram

Where the triangle A'B'C' of Fig. 9 is the interval of the angle of the direction of the line of measurement $(90°,180°]$ with which the corresponding fig

According to Fig 8 and 9:

$$\tan\gamma = \sin(180° - \beta)\tan\alpha \tag{21}$$

assume (office)

$$\tan\gamma = \sin\beta\tan\alpha \quad \alpha \in (90°,180°] \tag{22}$$

The same conclusion can be drawn for the remaining two intervals.Substituting equation (21) into equation (18) gives:

$$W = [D_0 + (S_{D_0D_1})\cos\beta\tan\alpha]\bullet\frac{\sin\theta\sqrt{\sin^2\beta\tan^2\alpha+1}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}(\sin^2\beta\tan^2\alpha)} \tag{23}$$

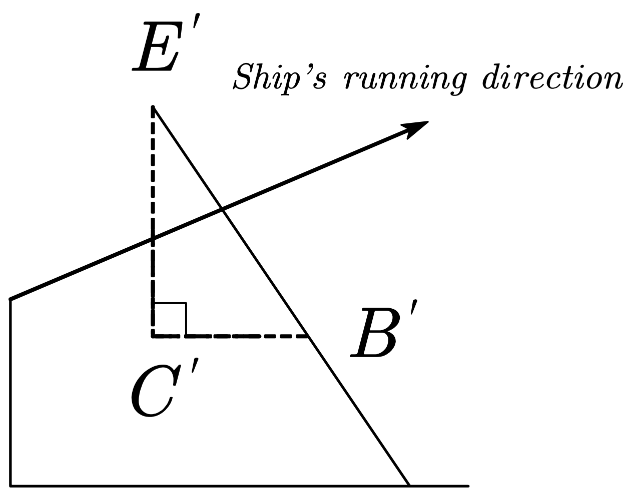

Fig. 8 Top view when the Angle of the line is obtuse

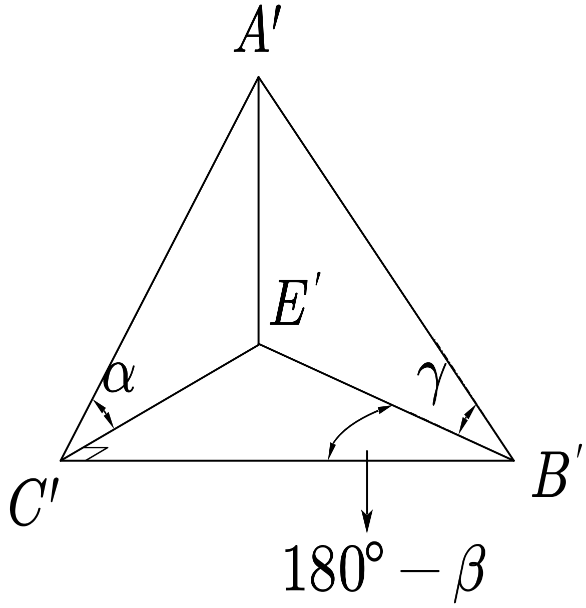

Fig. 9 A plan of the top view with obtuse angles

4. Minimum line lengths required to cover the known sea area with a line overlap of 10-20 per cent

In this paper, it is assumed that the influence of sea surface fluctuation on the ship is not taken into account, i.e., the heading is always stable and the center of the measurement is always perpendicular to the seabed; and it is assumed that the sea area given in the title is a flat and large area, i.e., there are no islands, reefs, rocky pebbles, and other terrain. In practice, the real sea conditions such as waves will cause errors in the measurement, and more work needs to be put in to minimize the errors.[7]Therefore, in this question, it is assumed that the influence of sea surface fluctuation on the ship is not taken into account, i.e., the heading is always stable and the measurement center is always perpendicular to the seabed. At the same time, the topography of the sea, such as islands, rocks and pebbles, will have a certain impact on the planning of the survey line. Therefore, this question assumes that the sea area given in the question is a large flat sea area, i.e., there are no islands, rocks and other terrain.

As can be seen from the previous section, due to the depth in the west and the shallowness in the east, the route starts from the westernmost part and covers the greatest width. When the overlap rate with the previous measurement line is more than 10%, the overlap rate of the latter measurement line is bound to be greater than 10%; in the case of the same conditions of the multibeam transducer opening angle and water depth, the coverage width is the largest when the direction of the measurement line and the seafloor slope are perpendicular to the projection, i.e., when $\beta = 90^\circ$ is used. Therefore, when the direction of the measurement line is perpendicular to the seabed slope in projection, the measured area is the largest and the efficiency is the highest; according to the slope and three-dimensional geometry characteristics, the depths of the east and west sides can be derived from the depths of the middle sea area.

In summary, The shortest measurement length is achieved when the width of two adjacent survey lines is controlled to meet exactly the 10% overlap with the previous line, and the direction of the line is perpendicular to the seabed slope in projection.(a_1 is the length of the waist at the second station in the direction of the survey line):

Fig.10 shows the schematic diagram of the relevant mathematical model: Figure 8 is the schematic diagram of the ideal case (a1 is the direction of the measurement line Waist length of the second stand) :

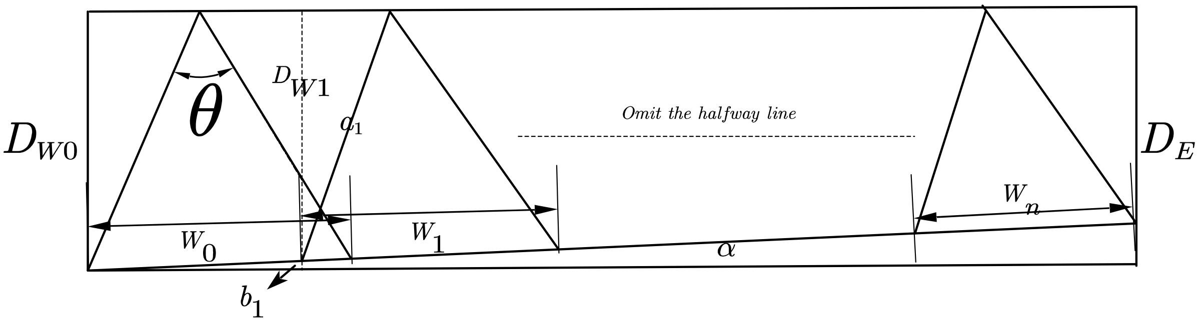

Fig. 10 Diagram of lateral overlap rate relationship in ideal case

The overlap rate of any two neighboring survey lines can be obtained by combining the previous ideas.

According to Fig.8 can be obtained:

$$b_1 = 0.9W_0 \sin 1.5° \tag{24}$$

$$D_{W1} = D_W - b_1 \tag{25}$$

$$a_1 = \frac{D_1}{\sin 30°} \tag{26}$$

by the sine theorem:

$$W_1 = a_1 \frac{\sin 120°}{\sin 31.5°} \tag{27}$$

Let the initial line length be W_0 and the final line length be W_T

there are

$$W_T = 0.9E + W_1 \tag{28}$$

Using a for loop in MATLAB, if $W_T < W_H$, then proceed to $E = E + W_1$, where E is the parameter of the loop in the loop statement[8-10],The shortest survey line length is then obtained by trigonometry as 66 nautical miles.

5. Conclusion

The multibeam bathymetric system is one of the widely used means of national ocean exploration at present, and has good bathymetric effects. By sweeping known obstructions, it can present a more vivid three-dimensional image of the seafloor topography.The working principle of the multibeam system is that the transmitting base array of the transducer transmits an irradiated acoustic wave to the bottom of the water in the direction of narrow parallel to the track line and wide perpendicular to the track line, and there exists a fan-shaped receiving area in the direction of wide parallel to the track line and narrow perpendicular to the track line in the receiving base array of the transducer as well. When the acoustic wave is emitted, through proper processing of the received signals, multiple bathymetry values can be measured in a single detection, and then necessary corrections such as positioning, attitude, sound speed, tide level and so on, can be obtained and plotted accurately and reliably underwater three-dimensional topography.This paper establishes a model of the measurement line with fixed multibeam transducer opening angle in a certain sea area and a mathematical model of the relationship between the coverage width and the adjacent strips. In the solution of the whole problem, this paper focuses on its intrinsic mathematical relationship, using mathematical knowledge to turn abstraction into simplification, and finally into mathematical formulas, which has good generalization. At the same time, a large number of schematic diagrams are used to simplify the problem, and the mathematical logic behind the problem is studied more intuitively. In the mathematical model of multibeam bathymetry coverage width in any line direction, the direction of the line should be north-south, which coincides with the relevant literature, which confirms the correctness of the relevant derivation and lays the foundation for the promotion of the relevant research.

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