Research on Multibeam Line Measurement Problem Based on Iterative Algorithms

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Abstract. With the continuous advancement of marine resource development, an efficient and reasonable multi beam acoustic survey line design scheme is crucial for accurate sea area measurement. Firstly, in order to make the model closer to the actual complex scene, a three-dimensional coverage model was derived that comprehensively considers the relative angle between the measurement line and the normal direction of the seabed terrain slope projected on the horizontal plane, referring to the three-dimensional sound beam propagation characteristics of multi beam sound waves. Then, based on the working principle and geometric relationship of multi beam recording, a formula for calculating the recording coverage width under multi-directional slope conditions considering measurement equipment parameters was derived, and an expression for the overlap rate of adjacent measurement lines was given. In addition, different measurement lines and slope normal direction angles α . Finally, based on the actual sea area situation of the rectangular area within the specific given range, iteratively traverse and set different combinations of survey lines, calculate the total length of the combination and the corresponding coverage area, and calculate the coverage rate. Select the survey line combination with the shortest total length and the highest coverage rate as the optimal solution. This article focuses on the design of multi beam acoustic survey lines and establishes a universal mathematical model to achieve optimization solutions in different practical scenarios.

Keywords: Multi beam survey line, Iteration, K-Means, Slope.

1. Introduction

Single beam bathymetry technology utilizes the principle of sound wave propagation in water to emit a single vertical sound beam to measure the depth of water bodies. However, the data obtained is only single point data from the seabed, which cannot effectively cover complex sea areas. Multi beam bathymetry technology can obtain strip-shaped depth data of the entire sea area by simultaneously emitting multiple acoustic beams in different directions and receiving seabed return signals, greatly improving measurement efficiency. However, multi beam measurement also faces challenges in line design. Due to the complex and ever-changing terrain of the actual sea area and the large fluctuations, using equidistant survey lines designed with average water depth or the shallowest water depth can lead to local missing measurements or data redundancy in local areas. The optimized survey line design should consider the specific terrain characteristics of the sea area and adopt unequal spacing survey line layout to achieve efficient coverage of the entire sea area.

2. Research on Overlap Rate of Adjacent Strips in Multi wave Velocity Sounding

2.1. Model establishment and derivation

BP This article assumes that the opening angle of the multi beam transducer is 120° , the water depth D is 70m, and the slope is α= 1.5°. The coverage width formula can be derived using the relevant knowledge of trigonometry [1]:

$$w = 2D\sin\left(\frac{\theta}{2}\right)\frac{\cos(\alpha)\cos\left(\frac{\theta}{2}\right)}{\cos\left(\alpha+\frac{\theta}{2}\right)\cos\left(\alpha-\frac{\theta}{2}\right)} \tag{1}$$

Then, the overlap rate formula can be used for calculation [2]:

$$\eta = 1 - \frac{d'}{w} \tag{2}$$

In the equation η Is the overlap rate; W is the coverage width; In the presence of a slope, the actual vertical distance between two measuring lines is no longer their horizontal distance. In this article, d 'is used to represent the actual distance between the two measuring lines, namely:

$$d' = \frac{d}{\cos\alpha} \tag{3}$$

Then, this article adjusts the water depth based on distance and calculates it using the following formula:

$$D = 70 - S\tan(\alpha) \tag{4}$$

2.2. Model solving

This article establishes mathematical models and theoretical calculations, and uses Matlab to write code to specifically calculate and solve problems. The solution process of this code mainly includes 5 parts. Firstly, define the known parameters in the problem, including the depth D of the sea center point_ Center, transducer opening angle theta(θ)And seabed slope(α), At the same time, define the distance sample distances from the center point, and initialize the result array based on the length of the distances, including seawater depth (D), coverage width (w), and overlap rate overlay(η),Used to save the calculated water depth, coverage width, and overlap rate at the corresponding distance point. Matching the iteration length in advance can avoid the problem of inconsistent size in the future. Secondly, calculate depth, width, and overlap point by point. The depth is adjusted based on the slope alpha and distance; Width is calculated based on water depth and opening angle; Overlap needs to determine whether it is the first line. The calculation process follows the mathematical relationship between the variables derived above, and finally, the four arrays of distance, water depth, coverage width, and overlap rate are directly combined into a table.

2.3. Calculation results

Table 1. Calculation Results of Multiwave Velocity Sounding

Distance from the measuring line to the center point/m	-800	-600	-400	-200	0	200	400	600	800
Sea water depth/m	90.9487	85.7116	80.4744	75.2372	70	64.7628	59.5256	54.28884	49.0513
Coverage width/m	315.8133	297.6276	279.4418	261.2560	243.0703	224.8845	206.6987	188.5130	170.3272
Overlap rate with the previous line/%	—	32.7789	28.4042	23.4205	17.6911	11.035	3.2076	-6.1299	-17.4613

From Table 1, it can be concluded that as the distance between the measuring line and the center point increases, the depth of seawater shows a gradually decreasing trend. This is because the terrain is a slope that is deep in the west and shallow in the east, with the deepest seawater at the center point. The farther away from the center point, the shallower the seawater depth. The coverage width also shows a gradually decreasing trend, as it is proportional to the water depth. The overlap rate of adjacent lines shows a trend of first increasing and then decreasing. As the distance increases, the overlap first increases, reaches its maximum value, and then begins to decrease. The reason for this phenomenon is that the distance between adjacent lines is fixed, while the coverage width decreases, which first leads to an increase in overlap rate. Later, as the coverage width continues to decrease, the overlap rate also decreases. Through the calculation of this set of data, the relationship between water depth, coverage width, and overlap rate can be intuitively seen, providing examples for subsequent measurement design. In addition, a negative coverage rate indicates the occurrence of missed measurements.

3. Research on the coverage width of multi beam bathymetry

This article assumes that the depth of the seawater at the center point of the sea area is D0=120m, and the slope angle is α= 1.5°, open angle=120°.

3.1. Model research and analysis

This article extends some of the two-dimensional models derived above to three-dimensional models. The theoretical derivation of three-dimensional models is as follows Fig 1[3]:

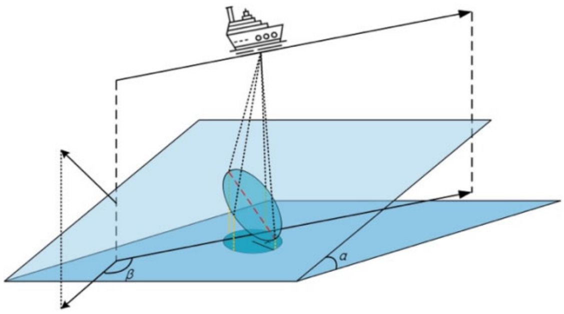

Fig. 1 Schematic diagram of slope and horizontal projection

(1) Sea water depth formula:

According to the triangular relationship, when the horizontal distance between the measuring vessel and the center point is s, the corresponding seabed elevation difference is:

$$\Delta H = S \tan\alpha \tag{5}$$

The depth of seawater at the measurement point is:

$$D = D_0 - \Delta H = D_0 - S\tan\alpha \tag{6}$$

(2) Elliptic equation of coverage area:

Let the opening angle of the multi beam transducer be θ, The angle at which it covers the sector is θ/ 2. In the absence of slope, the coverage area can be regarded as a circle with a radius of:

$$R = D\tan\left(\frac{\theta}{2}\right) \tag{7}$$

Considering the presence of slope α, Then, in the direction perpendicular to the slope direction, the coverage area still does not have a circle with a radius of R; In the slope direction, due to changes in seawater depth, the coverage area can be seen as an ellipse.

If the coordinate system is consistent with the slope direction, then in this direction, the coverage area is one half major axis a of the ellipse; In the direction perpendicular to the slope, the coverage area is a semi minor axis b of the ellipse.

From the triangular relationship, it can be seen that:

$$a = \frac{R}{\cos\alpha} = D\frac{\tan(\frac{\theta}{2})}{\cos\theta} \tag{8}$$

$$b = R = \tan\frac{\theta}{2} \tag{9}$$

According to the relevant research results in reference [4], the equation for the ellipse covering the region is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{10}$$

Substitute the expressions of a and b into the elliptic equation to obtain the covering region elliptic equation:

$$\frac{x^2}{[\frac{\mathrm{Dtan}\left(\frac{\theta}{2}\right)}{\cos\alpha}]^2} + \frac{y^2}{\left[\mathrm{Dtan}\left(\frac{\theta}{2}\right)\right]} = 1 \tag{11}$$

(3) Coverage width formula:

The angle between the projection of the measuring line and the normal of the slope on the horizontal plane is β, The cross-section of the coverage area ellipse in the direction of the measuring line is the elliptical chord, and its length is the coverage width W. The specific derivation process is as follows:

In the elliptic equation derived from (2), if y=0, there is:

$$\tan\theta = \frac{x}{b}) \tag{12}$$

$$x = a\sin\theta$$

The coverage width W can be obtained according to the triangular relationship:

$$W = 2x = 2b\tan\alpha \tag{13}$$

Combining x and tan θ By substituting the expression and simplifying it, the expression for the coverage length can be obtained:

$$W = \frac{2\sqrt{\tan^2(\beta)+1}}{b^2+a^2\tan^2(\beta)}\sqrt{a^2b^2[b^2+a^2\tan^2(\beta)]} \tag{14}$$

Wherein$a = \frac{\mathrm{Dtan}\left(\frac{\theta}{2}\right)}{\cos\alpha}$; $b = \mathrm{Dtan}\left(\frac{\theta}{2}\right)$

This formula describes the coverage length of multi beam bathymetry, taking into account the influence of slope, opening angle, and line direction on the projection angle of the normal phase of the seabed slope on the horizontal plane.

3.2. Actual operation of the model

Based on the above derivation process, this article converts the actual problem into a mathematical model and uses Matlab software to write code for the final solution. This code aims to calculate the value of coverage width under different ship heading and distance. The following is a detailed explanation and solution steps for this code.

Firstly, in this article, θ represents the radar beam width in radians, and α represents the radar elevation in radians.

Conversion_ Factor: used to convert nautical miles into meters.

Beta_ Degrees: The heading angle of a series of ships, expressed in degrees.

Distances_ Nm: The distance corresponding to the heading angle of each ship, in nautical miles.

Next, the code converts the angle from degrees to radians for mathematical calculations. Next, through a nested loop structure, the code traverses different combinations of heading angles () and distances () to calculate the radar coverage width.

The calculation and research process of coverage width is as follows:

a) Calculate parameter D: Based on the given formula, parameter D was calculated using 'distance', which plays a crucial role in the calculation of coverage width.

b) Calculate parameters a and b: Based on the value of D, the code calculates parameters a and b related to coverage width calculation.

c) Calculate coverage width: The most critical step is to use the values of a, b,, and to calculate the coverage width. This calculation involves complex mathematical formulas that consider the interrelationships between measuring beam width, elevation, ship heading angle, and distance.

All the above calculation results are stored in a matrix called 'coverage'_ Widths'. In this matrix, rows correspond to different heading angles and columns correspond to different distances. This makes it easy to find the radar coverage width values under different heading and distance combinations.

Finally, the code presents the value of radar coverage width by displaying the calculation results in the MATLAB command window.

3.3. Model calculation results

Table 2. Calculation results of depth measurement coverage width

Coverage width/m		Measure the distance/nautical mile between the ship and the center point of the sea area							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Angle betweenmeasuring line directions/°	0°	415.8347	466.2508	516.6670	567.0831	617.499	667.9154	718.3315	768.7477
	45°	415.7634	451.4069	487.0504	522.6939	558.337	593.9808	629.6243	665.2678
	90°	415.6922	415.6922	415.6922	415.6922	415.692	415.6922	415.6922	415.6922
	135°	415.7634	380.1199	344.4765	308.8330	273.189	237.5460	201.9025	166.2590
	180°	415.8347	365.4186	315.0024	264.5863	214.170	163.7540	113.3379	62.9217
	225°	415.7634	380.1199	344.4765	308.8330	273.189	237.5460	201.9025	166.2590
	270°	415.6922	415.6922	415.6922	415.6922	415.692	415.6922	415.6922	415.6922
	315°	415.7634	451.4069	487.0504	522.6939	558.337	593.9808	629.6243	665.2678

From Table 2, it can be concluded that:

3.3.1. In terms of seawater depth,

As the distance between the survey line and the center point increases, the seawater depth data shows a gradually decreasing trend. In this rectangular sea area, the seabed terrain is a uniform slope that is deep in the west and shallow in the east, with the deepest seawater at the center of the sea area. As the distance from the center point increases, the depth of the seawater gradually becomes shallower, consistent with the terrain characteristics. The regularity of data changes confirms the rationality of the results calculated by the team.

3.3.2. In terms of coverage width

(1) When the direction of the survey line is parallel to the normal of the slope (i.e. 0 ° and 180 °), the coverage width shows a linear decreasing trend[5], and the decrease is the largest. Because in this case, the measuring line extends along the slope direction, with significant changes in depth, resulting in a decrease in coverage width.

(2) When the direction of the survey line is perpendicular to the normal of the slope (i.e. 90 ° and 270 °), the coverage width remains constant, independent of the distance from the center point. Because the measuring line is perpendicular to the direction of depth variation, the influence of depth is relatively small.

(3) For other angles between the above two situations, the change in coverage width shows a trend of decreasing first and then increasing, with a minimum value. This is the result of the combined influence of distance from the center point, depth, and line direction angle on the team.

3.3.3. Curve characteristics of coverage width at different azimuth angles

(1) Azimuth angles that differ by 180° (such as 0 and 180), and the coverage width curves are mirror symmetric to each other[6]. This is consistent with their symmetry in the direction of the slope normal.

(2) Overall, the coverage width of the parallel azimuth angle varies the most, while the coverage width of the vertical azimuth angle remains constant. This fully reflects the influence of the direction of the survey line and the angle of the slope normal.

4. Design and determination of survey line scheme

4.1. Determine the direction of the survey line

The requirement for the survey line in this sea area is that the survey line needs to fully cover the entire sea area to be surveyed, and the overlap rate between adjacent strips needs to be controlled between 10% and 20%.

Regarding the overlap rate, the definition given in the question is: $\eta = 1\text{-}d/W$. Where d is the distance between two adjacent measuring lines, and W is the coverage width of the strip. According to this definition, if the distance d between adjacent lines is fixed and the coverage width W increases, the overlap rate η It will decrease.

In the case of uneven seabed terrain, if the ship navigates the survey line in an oblique direction, the coverage width W between adjacent survey lines will change due to changes in water depth, making it difficult to control the overlap rate within the required range of 10% to 20%,If the ship navigates the survey line in a constant direction[7], the coverage width W between adjacent survey lines can remain stable, and the overlap rate can be adjusted by controlling the distance d between survey lines to ensure that it is within the required range of 10% to 20%[8].

So from the perspective of ensuring the overlap rate of adjacent strips, ships need to navigate the survey line in a fixed direction rather than an oblique direction in order to meet the constraints of the coverage requirements of the problem. The following figure shows a situation where the overlap rate may be 100% if the measuring line is driven diagonally,as shown in Fig 2[9].

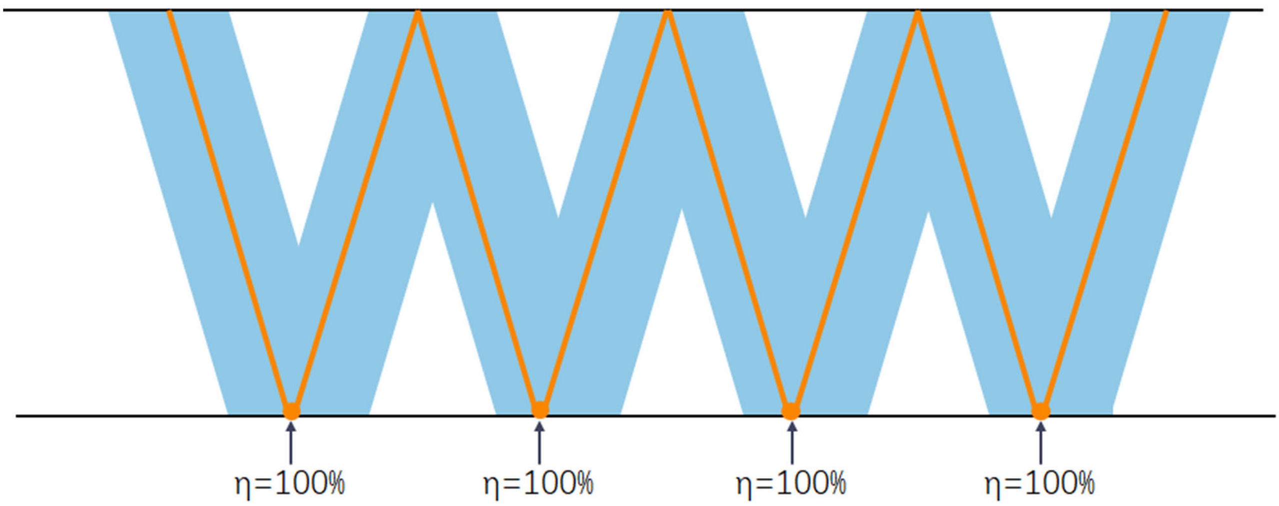

Fig. 2 Special case with 100% coverage

4.2. Visualization of Line Design

The terrain of a given sea area is deep in the west and shallow in the east, and the water depth shows a single directional variation trend. If the ship navigates along the east-west or north-south survey lines, the coverage width W between adjacent survey lines can remain relatively stable due to small changes in water depth.

According to existing research results [10], when the measuring line is parallel or perpendicular to the normal of the slope, the overlap rate between adjacent strips can be easily controlled within the required range of 10% to 20%. This article refers to the research idea and determines the layout plan of the survey line,as shown in Fig 3

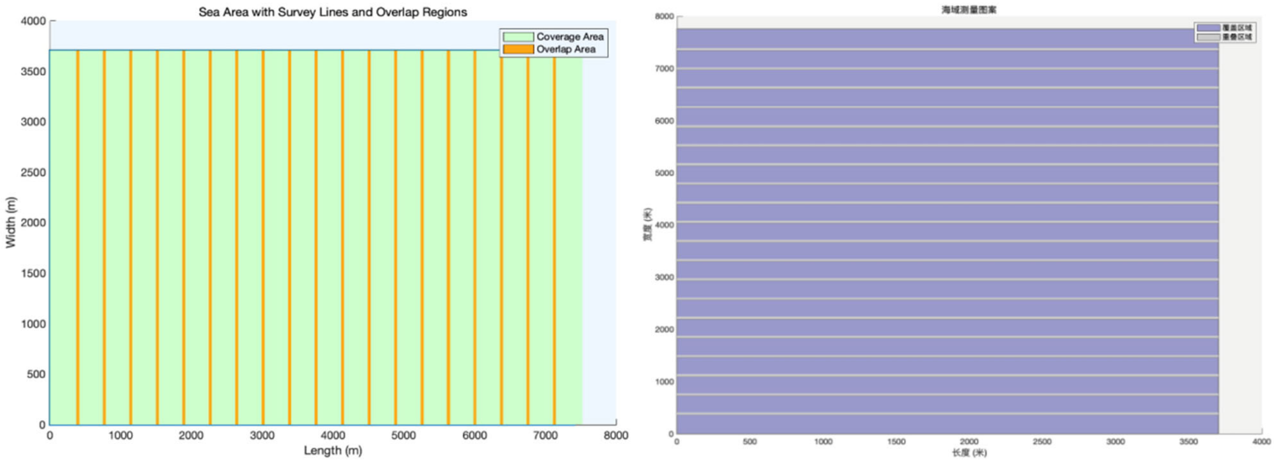

Fig. 3 Schematic diagram of the north-south direction measuring line (left); Schematic diagram of east-west measuring line (right)

5. Conclusions

This article establishes a calculation model for the coverage width W of multi beam bathymetry and the overlap rate between adjacent bands. According to the principle of multi beam measurement, there is a relationship between W and the opening angle of the transducer and the water depth D. Considering the seabed terrain as an inclined plane with an angle to the horizontal plane, the relationship between W and D can be derived based on the trigonometric function relationship. On the other hand, the definition of can obtain the relationship between W and the distance d between adjacent measuring lines. By substituting the calculation formula for W, the calculation formula can be derived. At the same time, this article establishes a calculation model for the coverage width W of multi beam bathymetry. It can be inferred from the triangular relationship that W also has a definite relationship with. Therefore, it is possible to establish a calculation formula for W that comprehensively considers, D, and multiple factors, in order to fully utilize the given information.The design of the survey line in this article fully considers the undulating changes of the seabed terrain, avoiding the problems caused by only using the average water depth design. Then, the average coverage width of different regions was calculated to achieve reasonable control of overlap rate based on terrain. Finally, different schemes were evaluated and optimized iteratively to make the survey line design more reasonable.

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