The maximum drop height of high-altitude parachuting based on dynamic and thermodynamic analysis

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Abstract. This paper explores the maximum possible initial height for skydiving, taking into account the physical and biological constraints of the sport. It develops the dynamic and thermodynamic models of the skydiver before and after deploying the parachute, and uses numerical methods to obtain the optimal parameters for safe landing. It also examines the thermal properties of the protective suit and the maximum heat it can endure. The paper demonstrates that the initial height does not affect the landing speed, and that the heat generated by the fall is within the tolerable range of the skydiver. The paper shows that the key factor for determining the maximum skydiving height is the thermal resistance of the protective material, rather than the conventional dynamic considerations. The paper concludes that the highest height that can be skydived is 45400 meters, the opening height is 3000 meters, and the landing speed is 6.32 m/s. This paper provides a novel perspective on the physics and engineering of skydiving.

Keywords: Skydiving, Dynamic, Thermodynamic.

1. Introduction

Skydiving is an extreme sport that involves falling from a high altitude to the ground. The starting height of skydiving affects the safety and experience of the skydiver, which depends on many factors, such as air pressure, temperature, drag, and the characteristics of the parachute and the skydiver. Therefore, it is necessary to scientifically analyze and optimize the starting height of skydiving.

There are three aspects of research on the starting height of skydiving:

Dynamics analysis: This uses math formulas to calculate the skydiver’s motion and safety in the air. For example, Zhang Qiang[1] found that the skydiver’s speed depends on the air resistance, and the landing safety depends on the buffer time.

Experimental research: This uses instruments to measure the skydiver’s physical and mental effects at different heights. For example, Li Na[2] et al. found that the skydiver’s heart rate, blood pressure, muscle activity, and emotion change with the height.

Extreme research: This challenges the highest skydiving height and solves extreme problems like low pressure, low oxygen, low temperature, supersonic, space suit, etc. For example, Felix Baumgartner[3] set the world record of skydiving from about 39,000 meters in a balloon, breaking three other records.

The contribution of this paper is to re-optimize the dynamic model based on the atmospheric model, and introduce the thermodynamic model, and find out that the real determinant of the starting height of skydiving is not the dynamic challenge, but the maximum heat that the skydiver can withstand during the skydiving process.

The data not specified in this paper, such as some physical constants, are based on the national physics textbooks, and the parameters used for calculation, such as the human body’s windward area, are estimated according to common sense.

2. Research methods for high-altitude parachuting

2.1. Technical roadmap of the research process

The paper tackles the problem in four steps: identifying the factors influencing the safe landing of skydiving, constructing and computing mathematical models, refining the models, and validating the models and drawing conclusions.

First, the paper hypothesizes that thermodynamic and human factors affect the safe landing of skydiving, such as the low temperature at high altitude, the frictional heat during the descent, the dynamic behavior of the descent, and the physiological tolerance of humans. After reviewing the literature, the paper neglects some factors, such as the negligible damage to the body after breaking the sound barrier.

Second, the paper develops and calculates models based on the remaining factors. The paper first analyzes from the dynamic perspective, establishes an atmospheric model, and examines the velocity variation before and after opening the parachute. According to the calculation, the paper discovers that the initial height has little impact on the final speed. Therefore, the paper constructs a thermodynamic model and estimates the heat and temperature rise caused by falling from high altitude.

Third, the paper modifies the model according to the calculation results. The paper investigates the heat generation at different heights and ensures the final heat within the range that the human body can accommodate.

Finally, the paper verifies the model again and draws conclusions. After the previous steps, the paper identifies the risks and challenges of skydiving, and computes the maximum height for safe landing based on the model.

2.2. Physical model for fall before parachute deployment

2.2.1 Atmospheric model

The aerodynamic forces depend directly on the air density[4,5]. To solve the problem faced by this article, it is necessary to define a standard atmospheric model to describe the changes in the properties of the atmosphere. In fact, there are several different models to choose from - standard or average day, hot day, cold day and tropical day. These models are updated every few years to include the latest atmospheric data[6-8]. This model is based on the equations obtained by curve fitting of the average values of atmospheric measurements. This model assumes that the pressure and temperature only vary with altitude. The specific model shown here was developed in the early 1960s, and the curve fitting was given in SI units.

For $h > 25000$ (Upper Stratosphere)

$$\begin{cases} T = 141.94 + 0.00299h \\ P = 2.488 \cdot \left(\dfrac{T}{216.6}\right)^{-11.388} \end{cases} \tag{1}$$

For $11000 < h < 25000$ (Lower Stratosphere)

$$\begin{cases} T = 216.67 \\ P = 22.65 \cdot e^{1.73-0.000157h} \end{cases} \tag{2}$$

For $h < 11000$ (Troposphere)

$$\begin{cases} T = 288.19 - 0.00649h \\ P = 101.29 \cdot \left(\dfrac{T}{288.08}\right)^{5.256} \end{cases} \tag{3}$$

Now that we know the relationship between T, P and h, we need to solve the relationship between air density ρ and h, and the method is as follows.

$$\rho = \frac{m}{V} = \frac{nM}{V} \tag{4}$$

Then according to the ideal gas Clausius-Clapeyron equation, we know that

$$PV = nRT \tag{5}$$

where $R = 8.314J/mol$, the gas constant. So, we can have:

$$\frac{n}{V} = \frac{P}{RT} \Rightarrow \rho = \frac{MP}{RT} \tag{6}$$

The molar mass of the gas is $28.96\ g/mol$, and we turn the data back to the formula, finally get the relationship between ρ and h. And the relationship between T, P, ρ and h are shown in figure 1.

$$\rho = \frac{P}{0.2869 \cdot T} \tag{7}$$

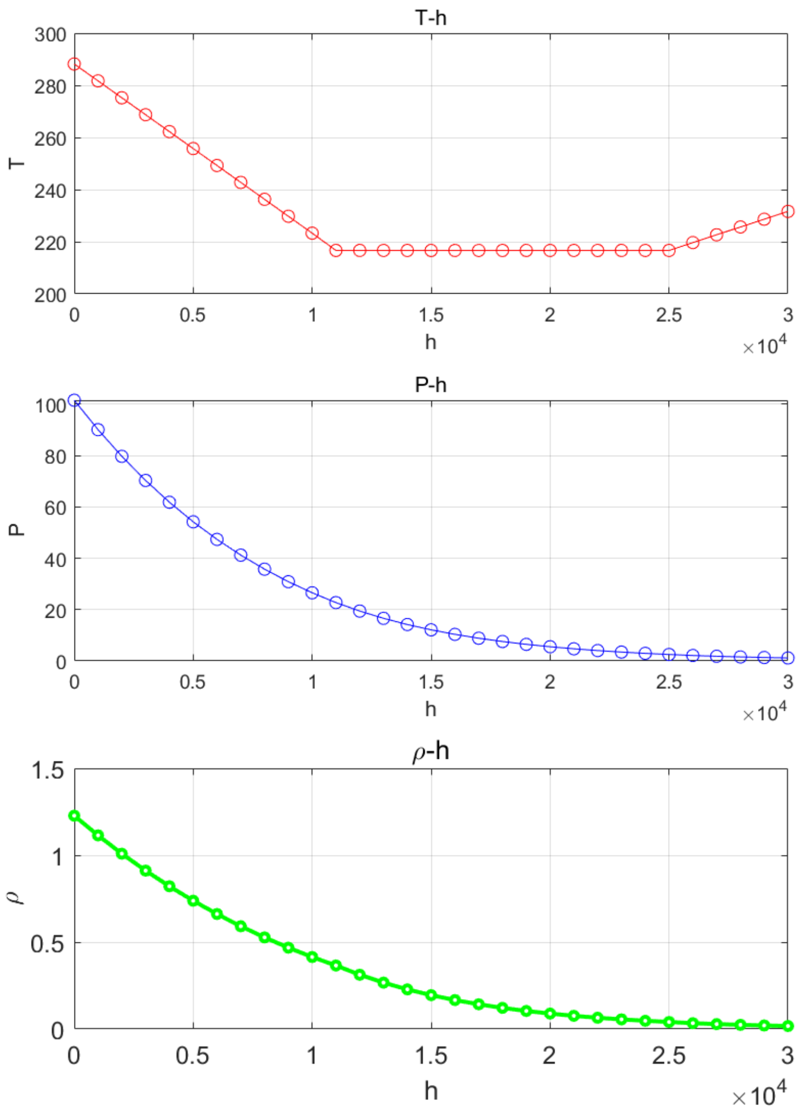

Figure 1. The relationship between $\boldsymbol{P, T, \rho}$ and $\boldsymbol{h}$

2.2.2 Air resistance analysis

The equation of air resistance can also be expressed by the Reynolds number, which is a dimensionless number that reflects the ratio of the inertial force and the viscous force of the fluid, defined as:

$$Re = \frac{\rho VL}{\mu} \tag{8}$$

where ρ is the fluid density, V is the relative velocity of the fluid, L is the characteristic length of the object, and μ is the dynamic viscosity of the fluid[9]. The greater the Reynolds number, the greater the inertial force of the fluid and the smaller the viscous force, making the flow more likely to produce turbulence and separation. The smaller the Reynolds number, the greater the viscous force of the fluid, the smaller the inertial force, and the more likely the flow will remain laminar and adherent[10-11].

When the Reynolds number is large ($Re > 1000$)[12], the air resistance is proportional to the square of the velocity. In the problem dealed with in our paper, because the mass of the athlete is large enough, the Reynolds number is much greater than 1000, and the formula for drag is as follows:

$$F_f = \frac{1}{2} \cdot \rho C_d A v^2 \tag{9}$$

2.2.3 Kinetic equations

The paper analyzed the force on the object, considering the gravity, air resistance and Coriolis force it received. According to Newton's second law, we can get that:

$$mh' = -\frac{GMm}{(R+h)^2} + \frac{1}{2}\rho v^2 C_d A + 2m\omega v sin(\phi) \tag{10}$$

After analyzation, the paper find that the Coriolis is too small so that we can ignore, therefore, we finally get:

$$h' = -GM/(R+h)^2 + \frac{1}{2m}\rho v^2 C_d A \tag{11}$$

2.3. Physical model for fall containing parachute deployment

2.3.1 Principle and process for opening the parachute

The principle of a parachute is to use air resistance to slow down the falling object and ensure a safe landing. The magnitude of air resistance depends on the shape, area, velocity, and air density of the falling object. The design of a parachute aims to maximize the air resistance and minimize the falling velocity.

The process of opening a parachute is a complex fluid-structure interaction process, involving multiple stages such as deployment, inflation, and stabilization. Generally speaking, the process of opening a parachute can be divided into releasing, deployment, inflation and stabilization.

2.3.2 Adjustment of drag parameters

First return to equation (9), which is the drag force without a parachute.

During the opening process of the parachute, since it takes time for the parachute to open and stabilize, the frontal area A and the drag coefficient C_d should be replaced by functions of t.

$$F_f(t) = \frac{1}{2} \cdot \rho C_d(t) A(t) v^2 \tag{12}$$

where $t = 0$ when the parachute starts to open and $t = t_1$ when the parachute becomes stable.

Therefore, it is easy to see that the drag force increases monotonically, and when the parachute is fully opened, the drag force becomes proportional to v^2 again.

3. Results and analysis

3.1. Preliminary solutions to the physical models

3.1.1 Solutions to fall before parachute deployment

By looking into formula (11), we can transform $h'' = \frac{dh'}{dh} \cdot \frac{dh}{dt} = \frac{1}{2} \cdot \frac{d(h)^2}{dh}$ and $GM = gR^2$. Therefore, we can get:

$$\frac{1}{2}\frac{d(h)^2}{dh} = -\frac{gR^2}{(R+h)^2} + \frac{1}{2m}\rho v^2 C_d A \tag{13}$$

Let $(h')^2 = y, p(h) = \frac{\rho}{2m}C_d A, q(h) = \frac{-gR^2}{(h+R)^2}$, so we can get $\frac{1}{2}\frac{dy}{dh} - \frac{1}{2}p(h)y = -q(h)$. Then multiply $e^{\int_{h_0}^{h} -p(x)dx}$ on both sides, we can get:

$$y = 2e^{\int_{h_0}^{h} p(x)dx}\left[c + \int_{h_0}^{h} -q(x)e^{\int_{h_0}^{x} -p(s)ds}\,dx\right] \tag{14}$$

When $h = h_0, c = 0$. So, we get:

$$v = \sqrt{2e^{\int_{h_0}^{h} p(x)dx}\left[\int_{h_0}^{h} -q(x)e^{\int_{h_0}^{x} -p(s)ds}\,dx\right]} \tag{15}$$

3.1.2 Solutions to fall containing parachute deployment

The parachute deployment process can be divided into three stages: inflation, stabilization, and descent. The inflation stage refers to the process of releasing the parachute from the pack, filling it with air, and forming the canopy. This stage takes about 2.5 seconds .

After the parachute fully opens, C_d, A becomes $C_d(t_1), A(t_1)$, therefore, the formula of the acceleration of the parachutist becomes:

$$h'' = -\frac{R^2 g}{(R+h)^2} + \frac{1}{2}\frac{\rho C_d(t_1)A(t_1)}{h}v^2 \tag{16}$$

As we assume that the parachutist opens the parachute at a moderate altitude, moreover, since the air density at low altitudes can be approximated as a constant, the formula can be simplified as:

$$h'' = -g + \mathrm{Cst} \cdot \mathrm{v}^2 \tag{17}$$

where Cst is a constant. Now the formula can be turned to:

$$\frac{1}{2}\frac{d(h')^2}{dh} = -g + \frac{1}{2}Cst \cdot v^2 \tag{18}$$

Take $v^2 = y$

$$\Rightarrow \frac{1}{2}\frac{dy}{dh} = -g + \frac{1}{2}Cst \cdot y \tag{19}$$

$$\Rightarrow \frac{1}{2}\int \frac{dy}{-g + \frac{1}{2}Cst \cdot y} = \int dh \tag{20}$$

$$\Rightarrow y = \frac{2g}{Cst} + \frac{2e^{Cst\cdot(h-h_0)}}{Cst}\left(-g + \frac{1}{2}Cst \cdot y_0\right) \tag{21}$$

where y_0 is the square of velocity when the parachute has been fully opened, and h_0 is the altitude when the parachute has been fully opened.

According to the formula, the relationship between v^2 and h can be drawn, which is shown in figure 2, and we find that the curve drops rapidly and then stabilizes. This shows that the parachutist can reach the terminal velocity in a very short time after opening the parachute. After the calculation, the landing velocity is $6.32m/s$.

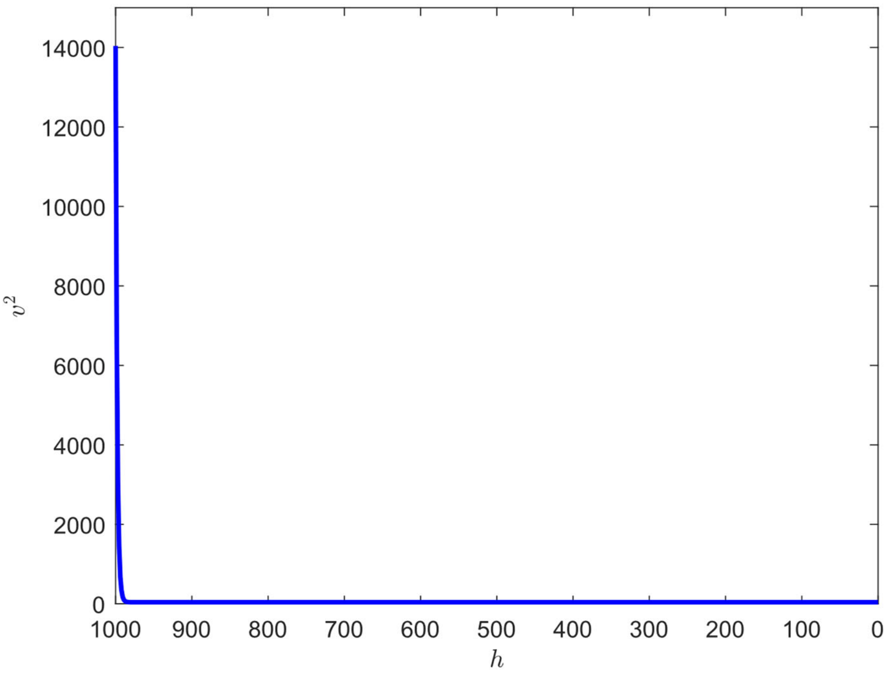

Figure 2. Relationship between $\boldsymbol{v^2}$ and $\boldsymbol{h}$ after parachute deployment

3.2. Analysis of the solutions

3.2.1 Skydiving altitude and parachute deployment altitude

The paper estimates that the human body's surface area is about $1.72m^2$, so we choose the frontal area $A = \frac{1}{2} \cdot Bodysurface$ which is about 0.8 and C_d to be around 0.4.

First consider the speed when falling from a high altitude to 3000 meters, and we use the computer to substitute the formula for different initial heights. The paper chooses 10000 meters, 50000 meters, and 1000000 meters as the initial heights and find that the speed at 3000 meters is about 114.9693 m/s for all of them. This surprising result shows that as the height increases, the speed at 3000 meters is not sensitive to the initial height.

Finally, the paper also considers the emergency situation, which may require about 20 seconds of reaction time. The terminal velocity in a few seconds, so and it is reasonable to choose 3000 meters as the place to open the parachute with 20 seconds for preparation.

However, because the above analysis is only from the perspective of dynamics, it doesn't mean that as long as the parachute is opened at 3000 meters. Next, start from the perspective of thermodynamics and roughly understand the heat generated by the fall.

According to the principle of conservation of energy:

$$\Delta U - \Delta E_k = E_H \tag{22}$$

$$\Rightarrow \frac{GMm}{R + h_1} - \frac{GMm}{R + h_2} - \frac{1}{2}mv^2 = cm\Delta T \tag{23}$$

where h_1 is the deployment altitude and h_2 is the initial altitude. We take $h_1 = 3000, h2 = 1000000$ and c as the heat capacity of water which equals to $4200J/kg \cdot K$. Then we can find that:

$$\Delta T = 2008K \tag{24}$$

The temperature will increase $2008K$ even for $190kg$ water. Such results are definitely unacceptable, so the material of the flight suit becomes an important factor to consider in this paper. For detailed analysis, please refer to the model revision part later.

3.2.2 Force and acceleration analysis of the parachute opening process

The paper can analyze the forces acting on the person and the parachute during the process of opening the parachute, to study how many times the gravitational acceleration they will experience as a whole. The paper writes the following equation:

$$\frac{1}{2}\rho v^2 C_d A - mg = ma \tag{25}$$

After calculation, the air density ρ at 3000 meters is about 0.9 to 1, we take 0.9 here. Since it takes time for the wind to fully open the umbrella when opening the parachute, we take the middle stage of opening the parachute, the windward area is about $5m^2$, C_d is about 0.8. Since the speed will quickly drop to the terminal velocity, we also take half of the speed at the moment before opening the parachute, which is $50m/s$. Substitute these original data, we can get the result:

$$a = 13.88 \cdot g \tag{26}$$

The human body's tolerance to extreme acceleration depends on multiple factors, in such a short time, it only needs to let the parachutist's protective suit pressurize the astronaut's lower limbs, so that the blood can flow back, which can ensure the astronaut's life safety.

3.3. Optimization and final solutions of the model

3.3.1 Heat evaluation

In the whole process, we consider that the total heat energy generated is equal to the heat absorbed from the surrounding environment before opening the parachute E_{H1}, the heat exchange between the parachutist and the surrounding environment before opening the parachute E_D, and the heat generated after opening the parachute E_O. The formula can be:

$$\Sigma E = E_{H1} - E_D + E_O \tag{27}$$

First, consider the heat conditions after opening the parachute, namely, under $3000m$. We know that the parachutist can reach the terminal velocity, which is about 6m/s, in a short time. Using 3000m divided by this speed, we can roughly estimate that the parachutist will spend 8 minutes in the last 3000 meters. Such a long time will not make the parachutist's temperature continue to rise, which means that E_O can be ignored and approximated as 0.

Therefore, focus on the heat absorbed in the process before opening the parachute. The paper first study the heat exchange between the parachutist and the surrounding environment. The state of the whole motion, at the beginning, due to the small air resistance, it can be approximately seen as free fall. Then, as the speed approaches the speed of sound, a sonic boom is formed, and the resistance is large, so we consider that the speed remains at the speed of sound in this stage. As it continues to fall, the air resistance increases, and the parachutist's falling speed decreases continuously. We think that the resistance and gravity are balanced in this section, so the formula of speed can be got as a function of height and the paper draws the figure of speed and height in figure 3:

$$v = \begin{cases} \sqrt{2(h_0 - h)g}, < h_1 < h < h_0 \\ 340, h_2 < h < h_1 \\ \sqrt{\dfrac{2mg}{\rho C_d A}}, h < h_2 \end{cases} \tag{28}$$

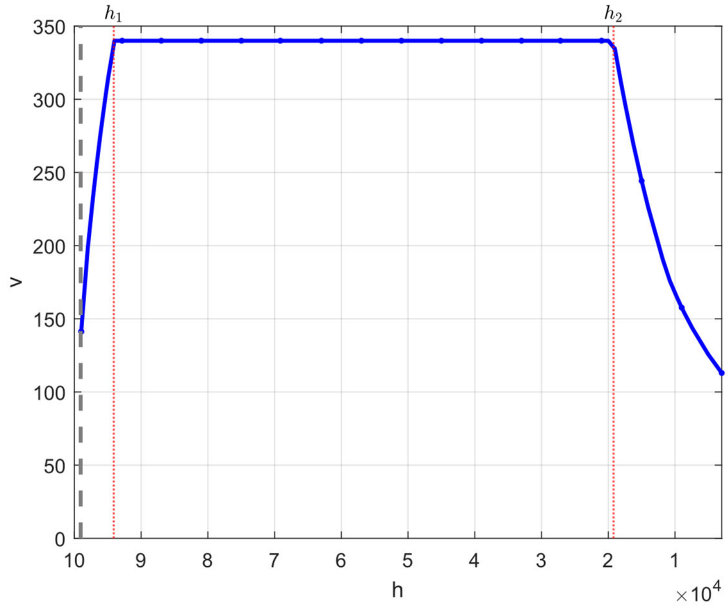

Figure 3. Relationship between $\boldsymbol{v}$ and $\boldsymbol{h}$ before deployment

It can be seen from the above figure that this process takes about 3 minutes, so we can use this time to calculate the heat transfer formula:

$$E_D = \alpha \cdot S\Delta Tt \tag{29}$$

where α is heat dissipation constant, S is the whole surface, ΔT is the temperature difference between the parachutist and the environment. We can take $\alpha = 30, S = 1.7, \Delta T = 150, t = 180$, therefore the heat energy caused by heat transfer is about a million joules of data.

Finally, consider the heat absorption process. The paper has roughly understood that the value of E_{H1} is in the order of hundreds of millions, so we can ignore E_D and focus on calculating the heat generated by the heat absorption process, and use this to calculate the maximum height from which it can fall.

3.3.2 Maximum height under the maximum tolerated heat

The paper can use formula (22) to figure out the maximum initial altitude:

$$Q = \frac{GMm}{R + h_1} - \frac{GMm}{R + h_2} - \frac{1}{2}mv^2 \tag{30}$$

where h_1 is the deployment height, which is 3000.

Let's first consider $\frac{1}{2}mv^2$, as $m = 190, v = 114$, the kinetic energy is calculated to be about a million level, which can be ignored compared to the billion level of heat energy. Therefore, the paper can ignore it in the subsequent calculations.

The paper learns that:

$$Q = c_1 m_1 \Delta T_1 + c_2 m_2 \Delta T_2 \tag{31}$$

where the model treats the protective suit as 80 kg, with 50 kg of water and 30 kg of residual material inside. c_1 is the specific heat capacity of water, m_1 is the mass of water, c_2 is the specific heat capacity of other materials, and m_2 is the mass of the remaining material.

What the paper needs to pay special attention to here is ΔT_1 and ΔT_2. ΔT_2 is the temperature difference between the protective suit and the outside, which we assume to be 150K. ΔT_1 is the temperature rise of the water, but we consider the process of water starting from ice, liquefying, then heating up, and then vaporizing. So, the reaction is roughly $100 + 120 + 80 = 300K$. By substituting this data into equation (27), we can calculate that the maximum height to start falling is about 45400 meters.

4. Conclusions

The paper presents a model of skydiving from 45400m height, considering the dynamics and thermodynamics of the process. The model accounts for the possibility of parachute failure, the effects of friction and drag, and the parameters of the spacesuit and the landing. The paper also discusses the challenges and risks of such a high-altitude jump. This article analyzes the theoretical basis for the maximum height that skydiving can reach, and provides the theoretical foundation and further development direction for the future skydiving sport.

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