

Research on multi-beam problems based on geometric modeling and underwater detection

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Abstract. This paper aims to establish and solve the mathematical model of coverage width under multi-beam bathymetry, rising from a one-dimensional sea area measurement problem to a two-dimensional sea area measurement problem. First, this paper takes the center of the sea area as the coordinate origin, establishes a one-dimensional coordinate system, and derives the functional relationship between the coordinate position of the measured ship axis and the angle of the seabed slope through the triangle edge relationship. At the same time, the mathematical relationship between the depth of the water body and the position of the measuring ship is constructed, and the relationship between the single coverage width of the detection system and the depth of the water body is found using the sine theorem. Subsequently, this paper established a two-dimensional sea area measurement model. Taking the center point of the sea area to be measured as the coordinate origin, a three-dimensional coordinate system was constructed. The normal vector projection direction was taken into account to obtain the new slope angle, and the two-dimensional sea area measurement was converted into a one-dimensional sea area measurement. Finally, this article uses MATLAB to conduct simulation experiments and gives the relationship between the coverage width and the position of the survey ship at different angles between survey line directions, providing an effective reference for the multi-beam sounding strategy.

Keywords: Geometric Modeling, Sea Area Measurement Model, Simulation Verification.

1. Introduction

In today's ocean science and engineering, a precise understanding of seafloor topography is critical for a variety of applications. Multi-beam bathymetry technology^[1-3] has become an important tool for obtaining deep-sea geographical information and is widely used in fields such as ocean survey, resource exploration, submarine pipeline layout^[4-6] and environmental research. However, there are some complex problems in the multi-beam sounding process: the coverage width of the multi-beam sounding strip changes with the change of the transducer opening angle and water depth. In order to prevent missed measurements from affecting the measurement accuracy and the measurement overlap rate being too high Affects measurement efficiency, it is necessary to study the quantitative relationship between coverage width and these parameters to assist decision-making. Current multi-beam bathymetry systems usually adopt different measurement schemes^[7-10], and there are complex trade-offs between these schemes. Therefore, an innovative approach is needed to achieve a balance of accuracy and efficiency in multi-beam bathymetry. Against this background, this study proposes an innovative mathematical model to provide an effective reference for multi-beam bathymetry.(Source of Data:<http://www.mcm.edu.cn/>)

2. Establishment of one-dimensional sea area measurement model

The seafloor slope forms an angle α with the horizontal plane, and the opening angle of the multi-beam transducer is θ . The survey ship measures the width of the slope, and there will be an overlapping area between the strips formed by adjacent survey lines.

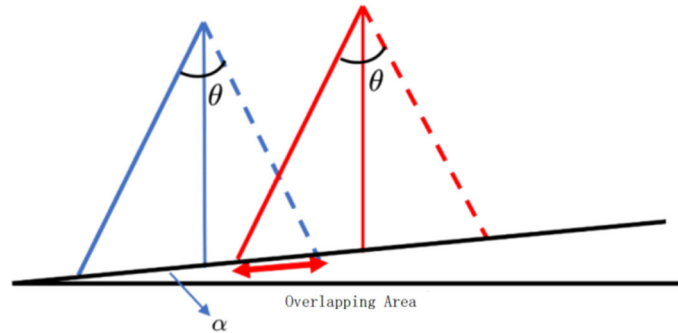


Fig. 1 Schematic diagram of the slope overlap area

It can be seen from Fig 1 that there is a certain geometric relationship between the seafloor slope areas measured by adjacent survey lines of the multi-beam bathymetry system.

This paper takes the horizontal right direction as the positive axis direction and the vertical upward direction as the positive axis direction, and establishes a rectangular coordinate system with the center point of the sea area as the origin of the coordinate system. This article believes that the seafloor slope on the right will be derived to the coast, as shown in Fig 2.

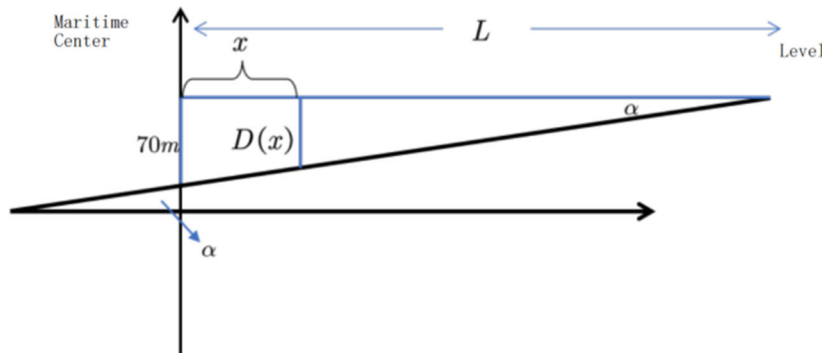


Fig. 2 Schematic diagram of sea area coordinate system

This article gives that the seawater depth at the origin of the sea area coordinate system is 70m. When the survey ship travels back and forth to measure the seabed slope area, it gradually moves away from the center of the sea area. In this paper, the distance between the measurement ship and the center of the sea area is x , and the distance between the measurement ship and the seabed slope, that is, the depth of the water body at different positions, is $D(x)$.

According to the relationship between the angles of the triangle, the distance L between the center of the sea area and the coast is obtained:

$$\frac{70}{L} = \tan \alpha \quad (1)$$

According to the properties of similar triangles - the ratio of any corresponding line segments of similar triangles is equal to the similarity ratio, this paper obtains the relationship between the distance x between the measurement ship and the center of the sea area and the depth of the water body $D(x)$ at different positions.

$$\frac{L-x}{L} = \frac{D(x)}{D} \quad (2)$$

Where D is the seawater depth at the origin of the sea coordinate system.

The width of the detection area of the survey ship at different positions in the sea coordinate system is divided into two parts by the measurement central axis of the multi-beam, as shown in Fig 3.

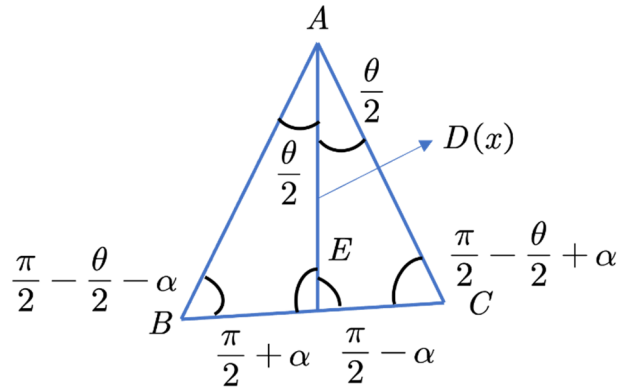


Fig. 3 Schematic diagram of measuring area width

This paper abstractly simplifies the problem of multi-beam measurement area width into the problem of calculating the side length of a triangle. BC is the width of the sea area measured by the survey ship, and AE is the distance between the survey ship and the seabed slope. Obtain the angles of a triangle based on the properties of triangles and related concepts of angles.

In the triangle ABE , the relationship between BE and AE is the relationship between the measured width $l_1(x)$ of the measuring ship in the area to the left of the central axis of measurement and the depth of the sea area.

$$l_1(x) = \frac{D(x) \sin \frac{\theta}{2}}{\sin \left(\frac{\pi}{2} - \frac{\theta}{2} - \alpha \right)} \quad (3)$$

In the triangle ACE , the relationship between CE and AE is the relationship between the measured width $l_2(x)$ of the measuring ship on the right side of the measurement axis and the depth of the sea area.

$$l_2(x) = \frac{D(x) \sin \frac{\theta}{2}}{\sin \left(\frac{\pi}{2} - \frac{\theta}{2} + \alpha \right)} \quad (4)$$

In triangle ABC , the relationship between the width of the measurement area and the depth of the sea can be obtained by measuring the width of the areas on both sides of the measurement central axis.

$$W(x) = l_1(x) + l_2(x) = \frac{D(x) \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} - \alpha \right)} + \frac{D(x) \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} + \alpha \right)} \quad (5)$$

There will be two situations in the overlapping area between the areas of adjacent survey lines. One is that the central axis of the measurement is passed, and the other is that the central axis of the measurement is not exceeded.

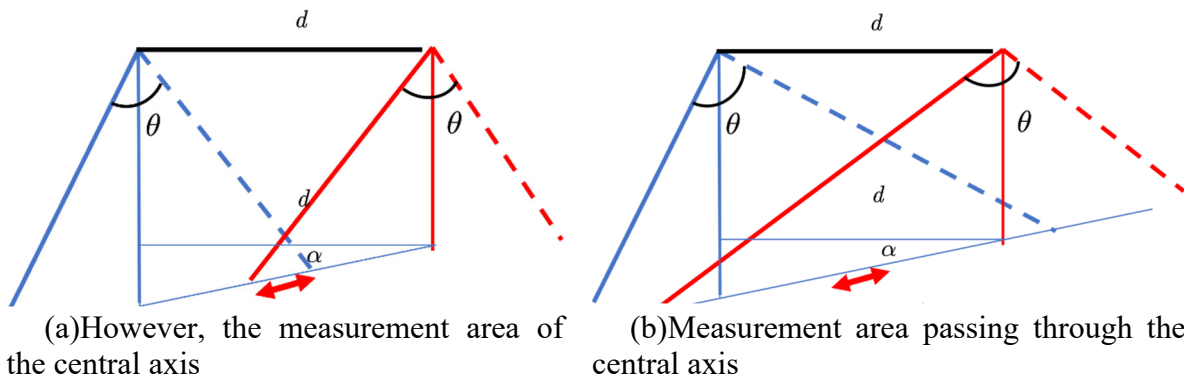


Fig. 4 Overlap of measurement areas of adjacent survey lines

It can be seen from Fig 4 that the relationship between adjacent survey lines is that there will be an overlapping area width h between the width of the measurement area on the right side of the central axis of the i -1th measurement and the width of the measurement area on the left side of the central axis of the i -th measurement.

This paper obtains the relationship between the two based on the definition of geometric relationship and overlapping area.

$$h = l_1(x_i) + l_2(x_i - 1) - \frac{d}{\cos \alpha} \quad (6)$$

Equation (6) can satisfy the two different overlap situations in Fig 4. The relationship between the width of the overlap area h and the width of the left and right sides of the measurement axis established in this article can satisfy a variety of overlap situations.

According to the definition of overlap rate, the coverage overlap rate of adjacent survey lines is obtained:

$$\eta = \frac{h}{W(x_i)} \quad (7)$$

3. Establishment and solution of two-dimensional sea area measurement model

3.1. Establishment of two-dimensional sea area measurement model

In this paper, the projection direction of the normal vector of the area to be measured on the horizontal plane is the positive direction of the y -axis and the vertical direction is the positive direction of the z -axis. The direction perpendicular to the x and z axes is the positive direction of the x -axis. The center point of the sea area is used as the coordinate. Establish a three-dimensional coordinate system with the origin. As shown in Fig 5:

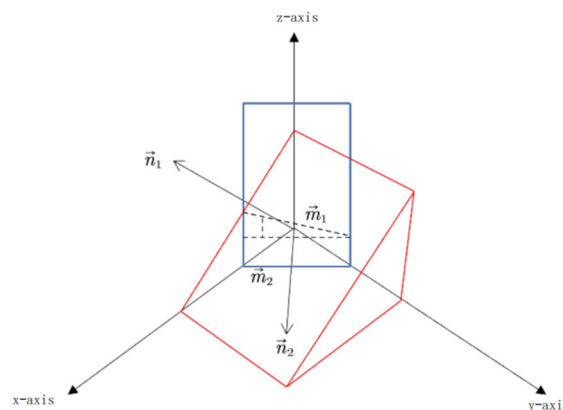


Fig. 5 Three-dimensional coordinate system for sea area measurement

Among them, \vec{n}_1 is the normal vector of the area to be measured, and \vec{n}_2 is the course direction vector of the survey ship (that is, the survey line direction vector). It is known that the multi-beam measurement and control system will emit measurement beams in a plane perpendicular to the course direction of the survey ship, and the normal vector of the vertical plane of the course is the survey line \vec{n}_2 . Where \vec{m}_2 is the intersection line of the heading vertical plane on the xoy plane, and \vec{m}_1 is the intersection line of the heading vertical plane and the surface of the area to be measured. Fig 6 is a simple diagram of each vector.

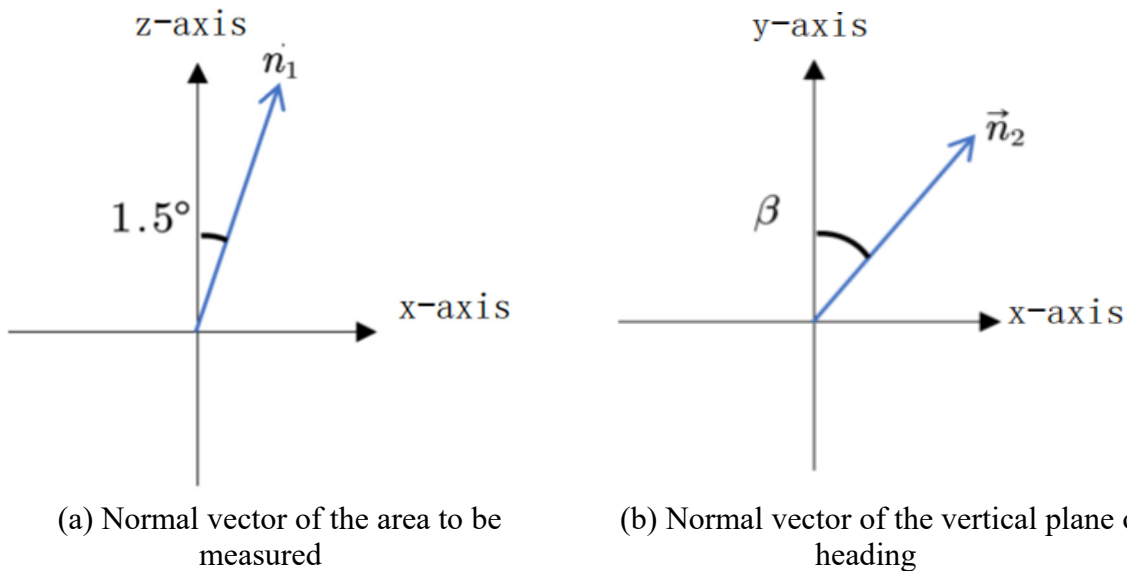


Fig. 6 Vector simple diagram

Calculate the coordinates of each vector based on the geometric relationship of the space vector and the problem conditions.

$$\begin{cases} \vec{n}_1 = (\sin 1.5^\circ, 0, \cos 1.5^\circ) \\ \vec{n}_2 = (\cos \beta, \sin \beta, 0) \end{cases} \quad (8)$$

When the angle β between the direction of the survey line and the normal vector of the seafloor slope projected on the horizontal plane (xoy plane) is 90 degrees, the intersection of the slope angle of the area to be measured, the normal vector of the area to be measured, and the vertical plane of the heading on the xoy plane can be found. The relationship between line vectors \vec{m}_2 . Fig 7 shows the situation when β is 90 degrees.

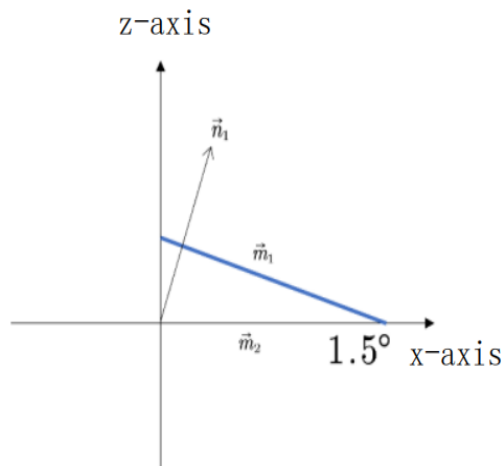


Fig. 7 Time vector diagram

It can be seen from Fig 7 that when β is a constant value, the two-dimensional sea area measurement problem can be converted into a one-dimensional sea area measurement problem. According to the three perpendicular theorem, it can be proved that the seabed slope angle is the angle between the intersection vector \vec{m}_1 of the vertical plane of the course and the surface of the area to be measured and the intersection vector \vec{m}_2 of the vertical plane of the course on the xoy plane.

(1) Select vector cross product direction

According to the three perpendicular theorem and the analysis of Fig 7, it can be found that the intersection vector \vec{m}_1 of the vertical plane of the heading and the plane of the area to be measured is obtained by the cross product of \vec{n}_1 and \vec{n}_2 . In this paper, for the intersection line between the area surface to be measured and the vertical surface of the heading, two non-collinear vectors on the two surfaces are selected for cross-multiplication to obtain the vector \vec{m}_1 .

$$\begin{aligned} \vec{m}_1 &= \vec{n}_1 \times \vec{n}_2 \\ &= \begin{vmatrix} i & j & k \\ \sin 1.5^\circ & 0 & \cos 1.5^\circ \\ \cos \beta & \sin \beta & 0 \end{vmatrix} \\ &= -\cos 1.5^\circ \sin \beta i + \cos 1.5^\circ \cos \beta j + \sin 1.5^\circ \sin \beta k \end{aligned} \tag{9}$$

Therefore, we get the vector $\vec{m}_1 = (-\cos 1.5^\circ \sin \beta, \cos 1.5^\circ \cos \beta, \sin 1.5^\circ \sin \beta)$.

(2) Select the direction of the intersection vector of the heading vertical plane on the xoy plane

This article selects the intersection vector direction of the vertical plane of the course on the xoy plane based on the vector direction:

$$\vec{m}_2 = (-\sin \beta, \cos \beta, 0) \tag{10}$$

The intersection vector \vec{m}_1 of the course vertical plane and the surface of the area to be measured is scalarly multiplied by the intersection vector \vec{m}_2 of the course vertical plane on the xoy plane to obtain the new slope angle γ of the seafloor:

$$\begin{aligned} \cos \gamma &= \frac{\vec{m}_1 \cdot \vec{m}_2}{|\vec{m}_1| \cdot |\vec{m}_2|} \\ &= \frac{\cos 1.5^\circ \sin^2 \beta + \cos 1.5^\circ \cos^2 \beta}{\sqrt{\cos^2 1.5^\circ \sin^2 \beta + \cos^2 1.5^\circ \cos^2 \beta + \sin^2 1.5^\circ \sin^2 \beta}} \end{aligned} \tag{11}$$

This article uses the center point of the sea area as the origin of the coordinate system to establish a rectangular coordinate system. It is believed that the seafloor slope on the right will be derived from the coast, as shown in Fig 8.

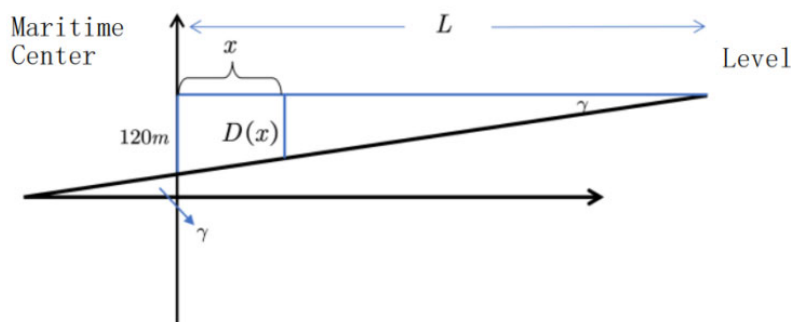


Fig. 8 Schematic diagram of the two-dimensional sea area coordinate system

This article gives that the seawater depth at the origin of the sea area coordinate system is 120m. When the survey ship travels back and forth to measure the area of the seabed slope, it gradually moves away from the center of the sea area. This article assumes that the distance between the measurement ship and the center of the sea area is x , and the distance between the measurement ship and the seabed slope, that is, the depth of the water body at different positions, is $D(x)$.

According to the relationship between the angles of the triangle, the distance L between the center of the sea area and the coast is obtained.

$$\frac{120}{L} = \tan \gamma \tag{12}$$

According to the properties of similar triangles - the ratio of any corresponding line segments of similar triangles is equal to the similarity ratio, this paper obtains the relationship between the distance x between the measurement ship and the center of the sea area and the depth $D(x)$ of the water body at different positions.

$$\frac{L-x}{L} = \frac{D(x)}{D} \tag{13}$$

Where D is the seawater depth at the origin of the sea coordinate system.

Since the new slope angle γ of the seafloor changes with the angle β projected by the normal vector of the seafloor slope on the horizontal plane (xoy plane), so $D(x)$:

$$\begin{cases} D(x) = x \tan \gamma + 120 (\gamma \in (0,90) \cup (270,360)) \\ D(x) = 120 - x \tan \gamma (\gamma \in [90,270]) \end{cases} \tag{14}$$

By analyzing the movement of the ship and calculating the change of the new slope angle with the β angle, it can be found that when the measurement ship moves on the two-dimensional sea area to be measured, three situations will occur in the measurement area of the detection system, as shown in Fig 9. The vertical plane of the measurement ship's track "cuts" the area to be measured on the xoy plane, yoz plane and different quadrants.

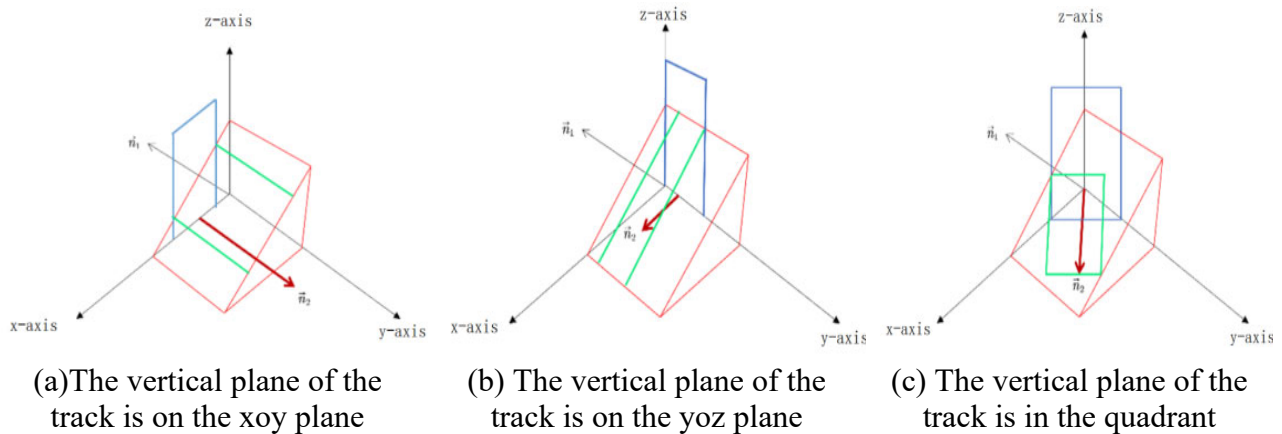


Fig. 9 Three situations of the vertical plane of the track

It can be seen from Fig 9 that when the vertical plane of the track "cuts" the area to be measured on the xoy plane, the slope angle of the slope does not change and remains the α angle. The width of the measurement area of the measurement ship has nothing to do with the distance of the measurement ship from the center point of the sea area, that is, the width of the measurement area is a constant value.

$$W(x_i) = \frac{120 \cdot \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} - \alpha \right)} + \frac{120 \cdot \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} + \alpha \right)} \quad (15)$$

When the vertical plane of the track "cuts" the area to be measured on the yoz plane, the slope angle of the slope does not change and remains the α angle, but the width of the measurement area of the measuring ship is related to the distance between the measuring ship and the center point of the sea area.

$$W(x_i) = \frac{120 \cdot \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} - \alpha \right)} + \frac{D(x_i) \cdot \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} + \alpha \right)} \quad (16)$$

According to the above formula 3-5, the relationship between the width of the measurement area and the depth of the sea area can be obtained by obtaining the measurement width of the two-dimensional area in the same way. When the vertical plane of the track "cuts" the area to be measured on the yoz plane, the slope angle of the seabed slope will change. , and as the measurement ship gradually moves away from the center point of the sea area, the width of the measurement area will also change.

$$W(x_i) = l(x_i)_1 + l(x_i)_2 = \frac{D(x_i) \cdot \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} - \gamma \right)} + \frac{D(x_i) \cdot \sin \frac{\theta}{2}}{\cos \left(\frac{\theta}{2} + \gamma \right)} \quad (17)$$

3.2. Solution to the two-dimensional sea area measurement model

This article gives the opening angle of the multi-beam transducer as 120 degrees, the slope as α angle as 1.5 degrees, and the seawater depth D at the origin of the sea coordinate system as 120m, and also defines the distance between the ship and the center position.

This article uses matlab to write a program to solve the problem of converting the two-dimensional sea area measurement model into a one-dimensional sea area measurement problem, the sea water depth and the measurement width of the measurement system, and visualizes the results to obtain Fig 10.

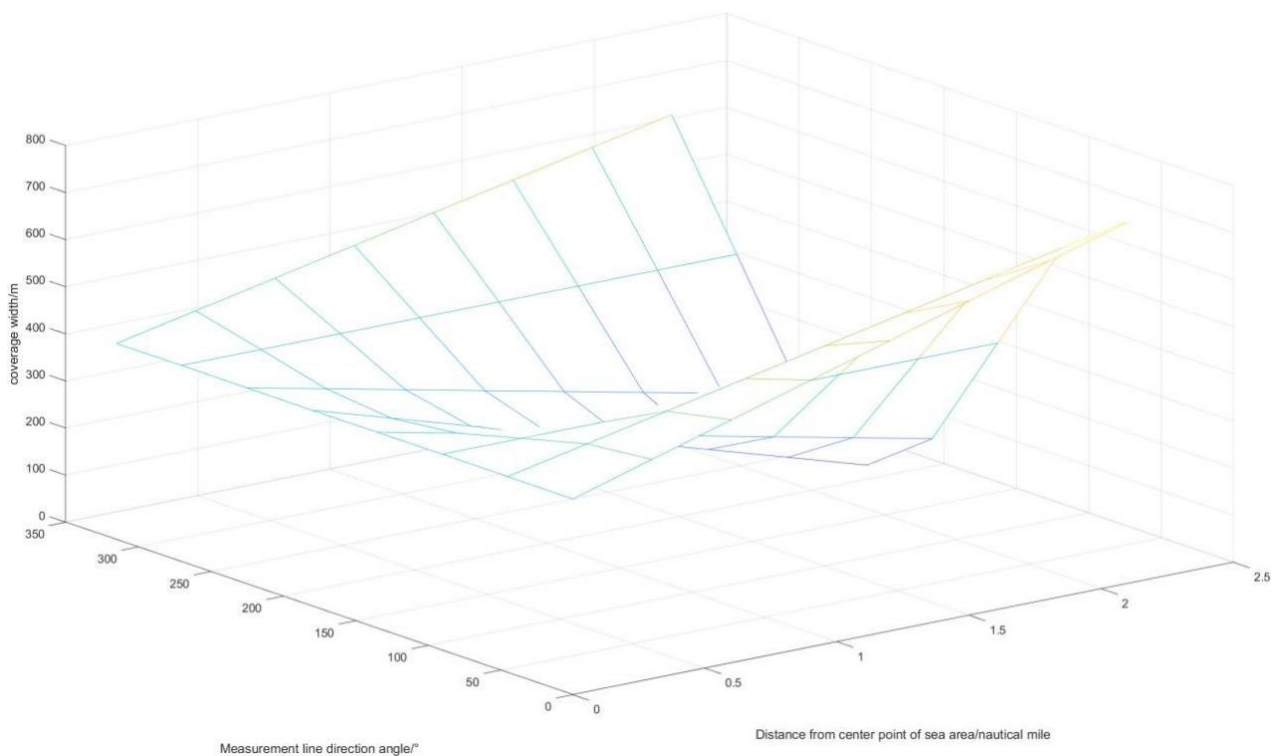


Fig. 10 Relationship between coverage width and survey line direction and center distance

It can be seen from Fig 10 that when the angle between the survey line direction is 0 degrees and 315 degrees, the coverage width increases as the distance between the survey ship and the center point of the sea area increases; when the angle between the survey line direction is 90 degrees and 270 degrees, The coverage width has nothing to do with the position of the survey ship from the center point of the sea area. When the vertical plane of the track is in the remaining quadrants, the coverage width and the position of the survey ship from the center point of the sea area show a decreasing trend.

When the survey ship is located at the center point of the sea area, the coverage width does not change with the direction angle of the survey line. In addition, when the distance between the surveying ship and the center point of the sea area remains unchanged, the coverage width decreases as the angle in the direction of the survey line increases.

4. Conclusions

This paper introduces a new approach to address the challenges of multibeam bathymetry, the transition from one- to two-dimensional ocean survey models. These models provide a mathematical foundation for understanding the parameters that influence coverage width during multi-beam detection. The one-dimensional model defines the width of the measurement area based on the measurement vessel position, seabed depth and sensor opening angle. On this basis, the two-dimensional model takes into account the normal vector projection, taking into account changes in the seafloor tilt angle and its impact on the coverage width. Simulations and visualizations highlight the effectiveness of the model. This paper improves the accuracy and efficiency of multi-beam bathymetry, which is beneficial to oceanography, resource exploration and environmental research. This model provides valuable guidance for multi-beam detection decisions, promoting the balance between measurement accuracy and efficiency as technology develops.

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