Calculation of optical efficiency of the heliostat field based on Reverse Projection Box Counting Model

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Abstract. A more accurate and reasonable calculation model is proposed for the calculation of total optical efficiency, shadow blocking efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency and output thermal power of heliostat field in tower solar photovoltaic power plant. In this paper, under the condition that the relevant parameters of heliostat field are determined, the Monte Carlo Ray Tracing method is used as the basis for the calculation of each optical efficiency. At the same time, the innovative Reverse Projection Box Counting Model is established to accurately identify whether there is a shadow blocking problem or not, and the differential idea is used to effectively solve the problem of repeated calculation of the blocking shadow area and the irregularity of the blocking area difficult to be calculated caused by one heliostat being blocked by multiple heliostats, so as to estimate the shadow blocking efficiency of the heliostat field more accurately. Finally, the heliostat field in Gansu, China, is taken as an example for calculation, and the results are compared with those of other scholars and the real value of the heliostat field to prove the superiority of the model constructed in this paper and the accuracy of the solution results.

Keywords: Tower Solar Heliostats Field, Optic Efficiency, Reverse Projection Box Counting Model.

1. Introduction

Tower solar photovoltaic power generation is a new type of low-carbon and environmentally friendly clean energy technology [1], which is an important means to achieve the goals of "carbon peaking" and "carbon neutrality". The heliostat field, as the most important energy input link in the tower photovoltaic power generation system, the optical efficiency and thermal power of the heliostat field directly determine the highest performance of the power generation system and the rationality of its own layout [2]. In order to accurately calculate the integrated efficiency of heliostat field, some scholars have provided many specific calculation methods from different perspectives. For example, Wendelin et al. [3] discarded the parallel light assumption and calculated the optical efficiency of heliostat by using Monte Carlo ray-tracing method; Leonard et al. [4] designed a simulation program for calculating the optical efficiency of heliostat, and used the programming method to calculate the various types of optical efficiency of heliostat, and evaluated the overall performance of heliostat field. G-Sassi et al. [5] used the flat-plate projection method to calculate the shadow loss of heliostat fields, but the flat-plate projection method failed to take into account the case of shadow superposition or irregularly shaped shadows generated when shadow occlusion occurs between neighboring heliostats.

Based on the Monte Carlo Ray Tracing method [6], this paper constructs a corresponding algorithmic model for the composition of the optical efficiency, the distribution of the optical efficiency and the calculation of the thermal power of the heliostat field. At the same time, this paper innovatively establishes the Reverse Projection Box Counting Model, which effectively solves the problem of double-counting of the shadowing area caused by a heliostat being shaded by multiple heliostats, and accurately calculates the shadowing efficiency of the heliostat field. Finally, taking the heliostat field in Gansu, China, as an example, [3] the shadow blocking efficiency, cosine efficiency, collector truncation efficiency, total optical efficiency and thermal power of the heliostat field are calculated to verify the superiority of the model constructed in this paper and the accuracy of the solution results.

2. Optical efficiency and thermal power of the heliostat field

Tower solar photothermal power plant is through a large number of heliostats to gather solar energy, so that the low-density solar energy is reflected to the collector located on the top of the absorption tower to gather into high-density solar energy, and the collector will be converted from optical energy to thermal energy. Among them, the ratio of the energy received by the collector to the total energy of the incident light is called the optical efficiency of the heliostat field; the energy conversion rate of the collector that converts the light energy into thermal energy is called the output thermal power of the heliostat field. The optical efficiency and output thermal power of heliostat field are affected by many factors, such as the position of the sun, the atmospheric environment, the geographic location of the heliostat field, the layout of the heliostat field, and the parameters of the heliostat.

2.1. Calculation of optical efficiency and thermal power

The total optical efficiency of the heliostat is η:

$$\eta = \eta_{sb} \cdot \eta_{cos} \cdot \eta_{at} \cdot \eta_{trunc} \cdot \eta_{ref} \tag{1}$$

with:

$$\text{Shadow blocking efficiency: } \eta_{sb} = 1 - \text{Loss of shadow blocking} \tag{2}$$

$$\text{Cosine efficiency: } \eta_{cos} = 1 - \text{Cosine loss} \tag{3}$$

$$\text{Atmospheric transmission: } \eta_{at} = 0.99321 - 0.0001176 d_{HR} + 1.97 \times 10^{-8} d_{HR}^2 \quad (d_{HR} \leq 1000) \tag{4}$$

d_{HR} is distance from the center of the heliostat to the center of the collector (Unit:m)

$$\text{Collector truncation efficiency: } \eta_{trunc} = \frac{\text{Energy received by the collector}}{\text{Specular total reflection energy} - \text{Energy lost to shadow blocking}} \tag{5}$$

with: η_{ref} is specular reflectivity.

Output thermal power of the heliostat field is E_{field}:

$$E_{field} = DNI \cdot \sum_i^N A_i \eta_i \tag{6}$$

with: DNI is the normal direct radiant irradiance, N is the total number of heliostats, A_i is the light harvesting area of the ith heliostat, η_i is the optical efficiency of the ith heliostat

The normal direct radiant irradiance DNI (Unit: kw / m^2):

$$DNI = G_0 \left[a + b \cdot exp\left(-\frac{c}{sin\alpha_s} \right) \right] \tag{7}$$

$$a = 0.4237 - 0.00821(6 - H)^2 \tag{8}$$

$$b = 0.5055 + 0.00595(6.5 - H)^2 \tag{9}$$

$$c = 0.2711 + 0.01858(2.5 - H)^2 \tag{10}$$

with: the sun's constant is G_0=1.366kw / m^2, H is the altitude, $\alpha_s$is **solar elevation angle.**

2.2. Guidelines for the evaluation of heliostat field efficiency:

In order to comprehensively evaluate the performance of heliostat fields, this paper adopts the "monthly averaging method" as the evaluation index of heliostat fields. "The monthly averaging method has a large number of sample days and more accurate calculation results, which can better calculate the annual average optical efficiency and thermal power output of the heliostat field.

"Monthly averaging method" means: according to a certain way in a certain year in the 12 months are selected as a representative day of the month and in each day to select five typical time points, with these times to calculate the heliostat field of the annual average optical efficiency and thermal power output, the calculation formulas are as follows:

$$\tilde{\eta} = \sum_{i=1}^{12} \int_{t_1}^{t_2} \eta(t)\, dt \Big/ \left(\sum_{i=1}^{12} \int_{t_1}^{t_2} dt\right) \tag{11}$$

$$\widetilde{E_{field}} = DNI \cdot \sum_{m}^{N} A_m \widetilde{\eta_m} \tag{12}$$

with: i is the month serial number, t_1 and t_2 are the starting and ending time points of daily observations; m is the heliostat serial number.

3. Establishment of model

3.1. Selection of the coordinate system

In order to facilitate the modeling and calculation of the optical efficiency and output thermal power of the heliostat field, this paper firstly selects and inscribes the coordinate system of the heliostat field and the coordinate system of the heliostat surface.

Heliostat field coordinate system: take the center of the circular area in which the heliostat field is located as the origin, and select the east direction as the x-axis direction, the north direction as the y-axis direction, and the center of the absorption tower perpendicular to the ground upward as the z-axis direction for the establishment of the coordinate system, as shown in Fig. 1:

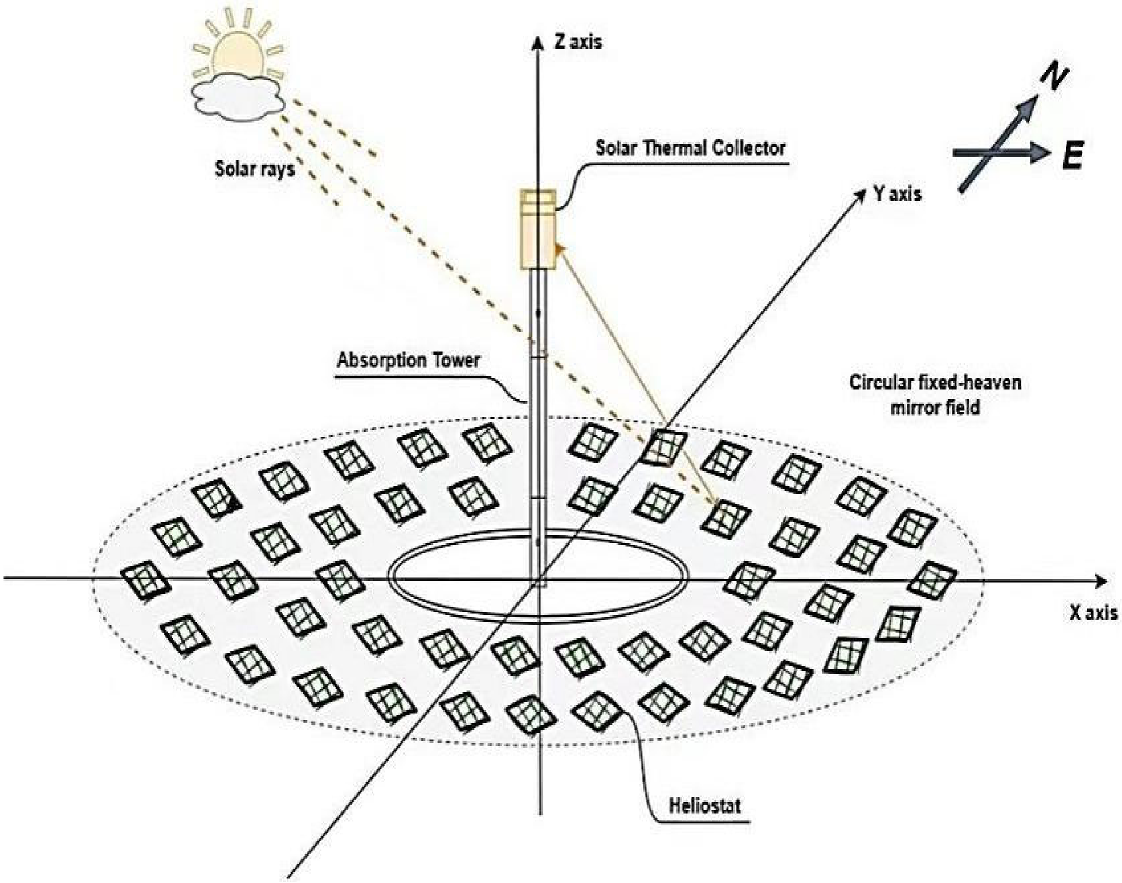

Figure 1. Coordinate system of the heliostat field

Heliostat coordinate system [7]: take the center of heliostat as the origin, the direction parallel to the ground is the x-axis direction, the direction parallel to the left and right sides of the mirror is the y-axis direction, and the direction normal to the mirror is the z-axis direction to establish a coordinate system, as shown in Fig. 2:

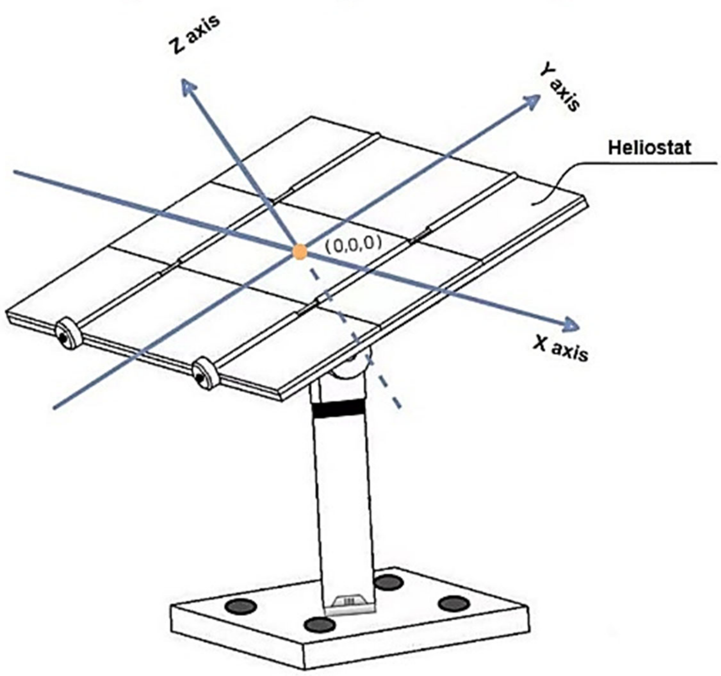

Figure 2. Coordinate system of the heliostat

3.2. Solar ray tracing model

The rotation and revolution of the Earth itself will cause the position of the Sun to change all the time, and in the research, the position of the Sun is usually measured by the earth plane coordinate system, which can be expressed by the solar altitude angle α_s and solar azimuth angle γ_S, and their calculation formulas are as follows [8]:

$$sin\alpha_s = cos\delta cos\varphi cos\omega + sin\delta sin\varphi \tag{13}$$

$$cos\gamma_s = \frac{sin\delta - sin\alpha_s sin\varphi}{cos\alpha_s cos\varphi} \tag{14}$$

with: φ is the local latitude, with north latitude being positive; ω is the solar time angle.

$$\omega = \frac{\pi}{12}(ST - 12) \tag{15}$$

with: ST is the local time, and δ is the solar declination angle.

$$sin\delta = sin\frac{2\pi D}{365} sin(\frac{\pi}{360} 23.45) \tag{16}$$

with: D is the number of days counted from March 21 as day 0.

According to the change of the sun position, this paper constructs a ray tracing model to analyze the trajectory of the sun rays into the heliostat and reflected to the collector of the absorption tower. The ray tracing model can be simulated to track the whole process of the ray from the sun to the reflection to the surface of the collector to perform the relevant analysis and calculation.

With the ray tracing model and the position of the sun, the solar incident light can be quantitatively calculated and its direction vector can be expressed as [9]:

$$\boldsymbol{i} = (\cos\alpha_s \cos\gamma_S, \cos\alpha_s \sin\gamma_S, \sin\alpha_s) \tag{17}$$

According to the law of reflection, it can be obtained:

$$\boldsymbol{i} + \boldsymbol{r} = 2 \cdot \boldsymbol{n} \cdot \cos\theta_n \tag{18}$$

with: $\boldsymbol{i}$ is the direction vector of the incident light from the sun, $\boldsymbol{r}$ is the direction vector of the reflected light from the sun, and $\boldsymbol{n}$ is the normal vector of the mirror surface of the heliostat, θ_n is the angle of incidence or reflection.

3.3. Calculation of shadow blocking efficiency - Reverse Projection Box Counting Model

3.3.1 Simplification of the calculation of shaded shelter area

In fact, the shadow blocking loss consists of two parts:

(1) The sunlight received by the rear heliostat is blocked by the front heliostat is known as shadow loss.

(2) The sunlight reflected by the rear-row heliostat is blocked by the front heliostat and does not reach the collector, which is called the light-blocking loss.

Since the light-blocking loss is relatively small, in order to simplify the calculation, this loss is ignored in the calculation of this paper, and only the shadow loss is considered. As shown in Fig.3:

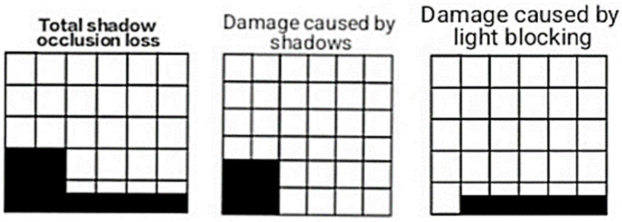

Figure 3. Classification of light reflection loss

3.3.2 Reverse projection model

As shown in Fig.4, the solar rays arrive at the covered heliostat D_2, and after mirror retro-reflection, forming a reflected beam l. According to the nature of solar retro-reflection and the vertical normal vector of the covered heliostat D_2, the geometric relationship can be constructed to solve the spatial linear equation of the reflected beam:

$$A_2x + B_2y + C_2z + E_2 = 0 \tag{19}$$

At the same time, the vertical normal vector of the front heliostat D_1 and the coordinates of its center are already known, so the equation of the space plane of the front heliostat D_1 can be solved:

$$A_1x + B_1y + C_1z + E_1 = 0 \tag{20}$$

Solve the two equations to obtain the intersection point $M(x, y, z)$ of the reflected beam l and the spatial extension plane of the front heliostat D_1. Project the intersection M and the front heliostat D_1 onto the $x0y$ plane to get the ground projection D_1' of the front heliostat D_1 and the ground projection M' of the intersection M.

Finally, determine whether M' belongs to the definition domain where the ground projection D_1' is located; if so, there is an occlusion; if not, there is no occlusion.

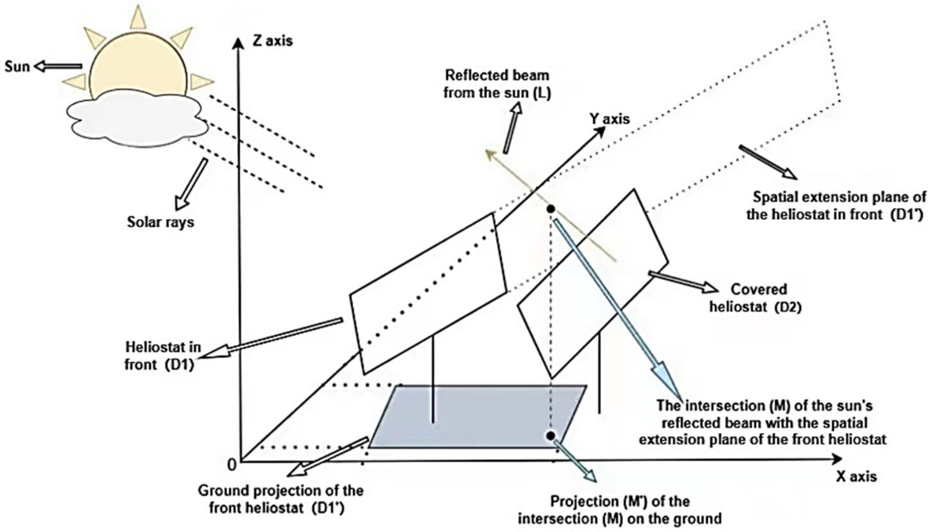

Figure 4. Schematic of the inverse projection model

3.3.3 Box counting model

The Box counting model is an extension of the differential calculus idea, which can effectively avoid the problems of repeated calculation of the occluded area and the irregularity of the occluded area that is difficult to calculate. The basic idea is to split the target plane into as many dense and uniformly continuous small squares as possible, select a number of representative small squares as the representative area, and consider the characteristics expressed in this representative area as the characteristics of the whole, and then solve the problem.

The steps for approximating the shaded occlusion area by the Box count method are as follows:

Step1: Divide the occluded mirror (assumed to be mirror K) into as many uniformly dense small squares (assumed to be number Z) as possible, which constitute the set of squares $S_k = (p_i \in K, i = 1,2, \dots, Z)$, where p_i is the number of each small square, and its coordinate is $(x_i, y_i, z_i), i = 1,2, \dots, Z$.

Step2: Select the value of Z. In this paper, we use the elbow rule in the clustering algorithm to calculate the cost function by changing the number of clustering categories, and select the key point in the cost function where the curve reduction becomes slower as the optimal Z value.

Step3: Shoot the p_i along the reverse direction of the light ray to the plane where the other mirrors are located, and form a projection. If the projection is on the mirror being projected, then mark the p_i, repeat the process to get the squares that have been marked several times, and equate these squares to the squares that have been marked only one time. This effectively solves the problem of the occluded shaded area being double-counted.

Step4: Ratio the number of marked squares to the total number of squares Z. Then multiply the ratio with the area of the mirror to get the shadow blocking area A_{sb}.

Assuming that the area of the mirror is S, the number of markers is I, then [10]:

$$A_{sb} = {}^{I}/_{Z} \cdot S \tag{21}$$

the total shadow blocking efficiency is:

$$\eta_{sb} = \frac{A_i}{A_t} = 1 - \frac{\sum_i^N A_{sb_i}}{A_t} \tag{22}$$

with: A_i is the total effective light gathering area of the mirror, and A_{sb_i} is the shadow blocking area of the ith mirror, and A_t is the total mirror surface area.

3.4. Calculation of cosine efficiency

The cosine loss arises from the reduction in the area of the reflective surface of the heliostat relative to the visible area of the sun's rays due to the fact that the surface of the heliostat cannot always be kept perpendicular to the incident sunlight and presenting an angle of inclination. Under local climatic conditions, the cosine efficiency is related to the magnitude of the angle of incidence of the sun's rays and the position of the heliostat. Using the solar ray tracing model developed above, the cosine efficiency formula can be obtained by geometrical analysis:

$$\eta_{cos} = cos\theta_n \tag{23}$$

The geometric analysis diagram is shown in Fig.5:

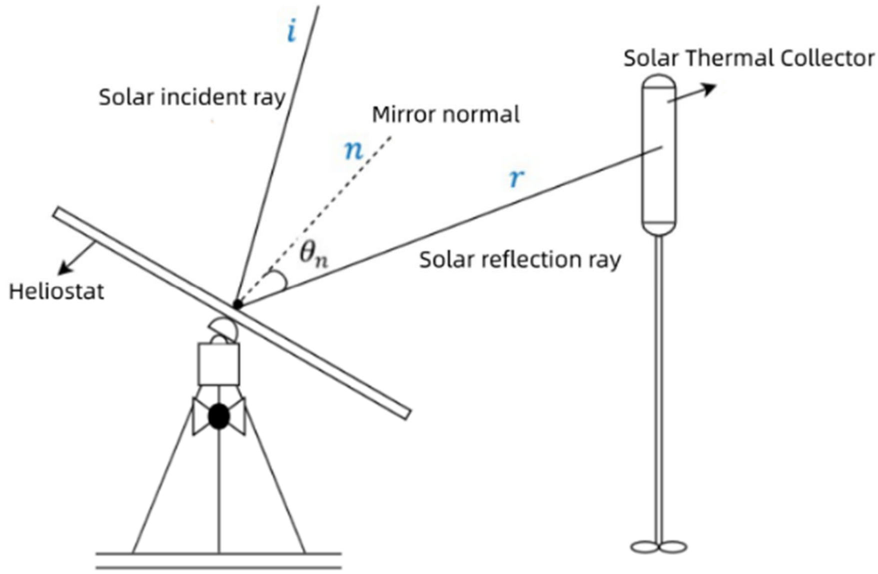

Figure 5. Geometric analysis chart

3.5. Calculation of collector truncation efficiency

When the reflected sunlight from the heliostats on the collector, due to tracking errors, light scattering and other factors, not all of the sunlight reflected from the heliostats can be received by the collector, resulting in part of the light irradiated outside the collector, thus generating part of the energy loss.

In this paper, based on the HFLCAL model proposed by DLR (German Aerospace Center), it is found that the energy flow density of the heliostat is close to a two-dimensional Gaussian distribution in the collector plane [11], therefore, the collector truncation efficiency is calculated by integrating the boundary range of the energy flow density in the collector plane, and the formula is as follows:

$$\eta_{trunc} = \frac{1}{2\pi\sigma_{tot}^2}\int_x \int_y \exp\left(-\frac{x^2+y^2}{\sigma_{tot}^2}\right) dydx \qquad (24)$$

with: σ_{tot} is the overall error, which is used to describe the degree of divergence of the reflected light.

4. Model solving and validation

4.1. Parameters of the heliostat field

In order to verify the accuracy and reasonableness of the above model, this paper takes the heliostat field in Gansu, China, as an example, and chooses the 21st day of each month in 2021 as the simulation time and selects five typical time points (9:00, 10:30, 12:00, 13:30, and 15:00) within each date. Each optical efficiency and output thermal power of the heliostat field was calculated using the constructed model with each parameter given. The relevant parameters are shown in Table 1:

Table 1. Parameters of target heliostat field

Parameters	Numerical value	Parameters	Numerical value
Latitude/(°)	39.4	Heliostat width/(m)	6
Longitude/(°)	98.5	Heliostat length/(m)	6
Altitude/(m)	3000	Number of heliostats	1745
Absorption tower height/(m)	80	Heliostat field radius/(m)	350
Collector height/(m)	8	Heliostat height/(m)	4
Collector diameter/(m)	7	Specular reflectance	0.92

4.2. Calculation results of the model

The calculation results are shown in Table 2:

Table 2. The result of the model calculation

$\eta/year$	$\eta_{cos}/year$	$\eta_{sb}/year$	$\eta_{trunc}/year$	E_{field} $/year\ (MW)$	$E_{field}/mirror/year$ $(kW\ /\ m^2)$
0.4453	0.7598	0.9082	0.7401	27.4967	0.4358

4.3. Validation of the results

Compared with the calculation methods of other scholars, the Reverse Projection Box Counting Model proposed in this paper makes the calculation results of shadow blocking efficiency closer to the real value, and then the total optical efficiency and output power are obtained more accurately. At the same time, due to the errors in the calculation and the simplification of modeling, the calculated value of collector truncation efficiency in this paper deviates from the true value.

The comparison of the calculation results and real values for different methods is shown in Fig. 6:

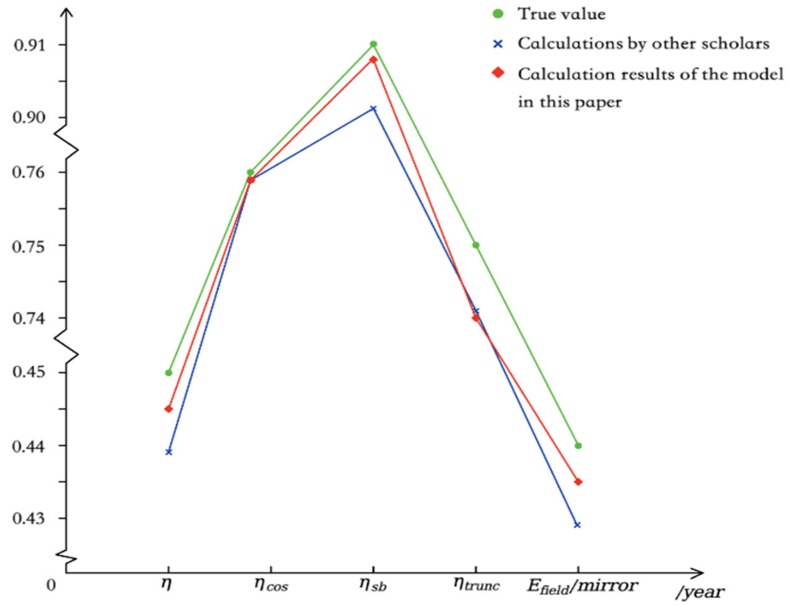

Figure 6. Comparison of the calculated results of different methods and the true values

5. Conclusion

In this paper, a more accurate and reasonable calculation model is proposed for the calculation of each optical efficiency and output thermal power of heliostat field in tower photovoltaic power plant. After solving the calculation and analyzing, the following conclusions are obtained:

1) The calculation results obtained from the model established based on Monte Carlo Ray Tracing method are close to the real value although there are some errors, which proves the accuracy and rationality of the model established in the article.

2) The Reverse Projection Box Counting Model proposed in this paper can effectively judge the blocking conditions between heliostats, and at the same time, it solves the problem of double counting of the blocking shadow areas caused by the blocking of one heliostat by multiple heliostats, which reduces the amount of calculations, and the calculations are highly efficient and accurate.

3) Based on the model established in this paper, the output thermal power of heliostat field is calculated with higher accuracy compared with other models. Taking the heliostat field in Gansu, China as an example, the accuracy of the calculation of the output thermal power per unit of heliostat is improved by about 12%. This provides a reference for the comprehensive efficiency of heliostat field and the rationality of its layout.

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