Research on multibeam bathymetric strategy based on sine theorem and equivalent slope angle

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Abstract. Multibeam bathymetry is widely used in the fields of water body bathymetry and terrain survey, and the survey line laying scheme is of key significance to improve the measurement accuracy and enhance the measurement efficiency. Therefore, it is of practical significance to increase the research on the survey line laying scheme for different water bodies and different terrains. In this paper, we firstly establish a single-slope multibeam bathymetry coverage width and overlap rate model based on the sine theorem in the case of constant line direction, and solve a set of analytical solutions for seawater depth, coverage width and overlap rate; then we extend the study on the coverage width when the direction of the survey line is changed, and establish a multibeam coverage width model based on the equivalent slope; finally, the optimal line laying scheme is solved for a given rectangle area.

Keywords: Law Of Sines, Multibeam Bathymetry, Optimisation Models, Equivalent Slope.

1. Introduction

Single-beam bathymetry is a technique for measuring the depth of a body of water by utilizing the uniform linear propagation characteristics of sound waves in water. Sound waves propagate in a homogeneous medium at a uniform speed in a straight line and reflect at different interfaces. Using this principle, the depth of seawater can be calculated by transmitting acoustic signals vertically from the transducer of the survey vessel to the seafloor and recording the propagation time from acoustic emission to signal reception. Since single-beam bathymetry adopts a single-point continuous measurement method, the distribution of the bathymetric data is characterized by very dense data along the trajectory and no data between survey lines, i.e., there is a lack of data between survey lines[1].

In order to improve the accuracy and efficiency of bathymetry, multibeam bathymetric systems have been developed on the basis of single-beam bathymetry. The system can transmit dozens or even hundreds of beams at a time in the plane perpendicular to the track, and then the acoustic waves returned from the seabed are received by the receiving transducer [2,3]. In the sea floor flat sea area can be measured to the survey ship survey line as the axis and has a certain width of the full coverage of the bathymetry strip. Multibeam bathymetry system overcomes the shortcomings of single-beam bathymetry, so it is widely used in the field of water body bathymetry and terrain detection[4].

2. Multibeam bathymetric study of single slope based on sine theorem

2.1. Modeling

In this paper, the position of the measurement ship at the center of the sea is P, the intersection of the sound beam boundary and the seafloor slope on both sides of the transducer opening angle θ is M and N, respectively, and the intersection of the vertical straight line (i.e., the vertical line of the seafloor) and the seafloor slope is H. Let $\angle PMW = \beta$, $\angle PNM = \gamma$, $\angle PHM = \frac{\pi}{2} + \alpha$, $\angle PHN = \frac{\pi}{2} - \alpha$, as shown in Fig. 1

The water depth D at the center point of the sea area, the transducer opening angle θ and the slope angle α of the seafloor slope are all known quantities. As can be seen from Fig.1: In this scenario, the survey line direction of the survey ship is perpendicular to the paper surface outward, and the sound beam boundary lines on both sides of its transducer opening angle θ constitute an acoustic beam sounding PMN, because the water depth D remains unchanged during the sailing process of the survey ship. Therefore the mathematical model of single-slope multibeam bathymetry can be established in a two-dimensional planar PMN.

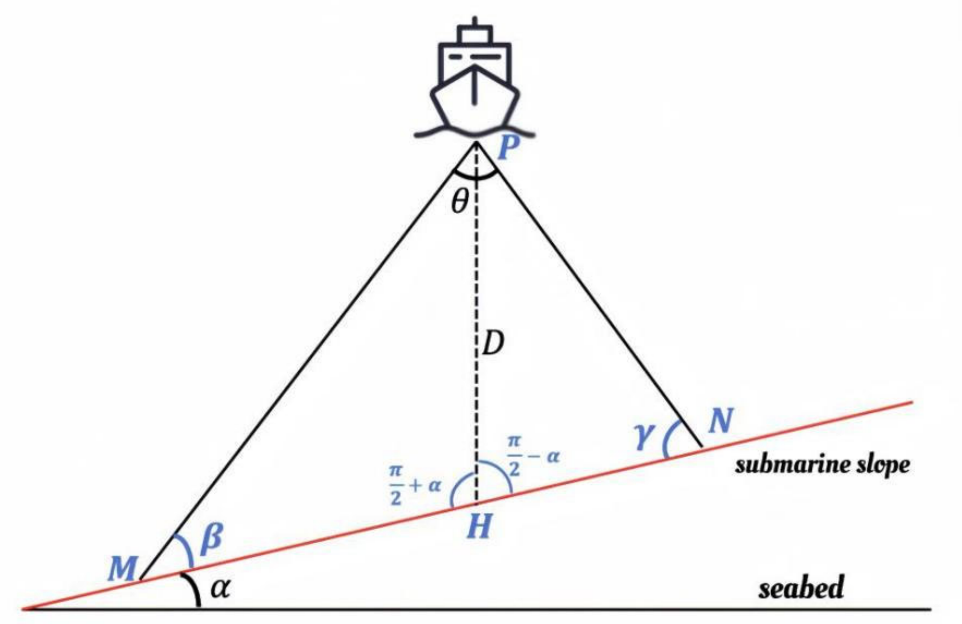

Fig. 1 Schematic diagram of the setting of the corners of the centerline of the sea area

First, consider one-way line measurement:

In triangle ΔPMH , $\beta = \frac{\pi}{2} - \alpha - \frac{\theta}{2}$ according to the triangle interior angle sum theorem, and by the sine theorem, we get

$$\frac{MH}{\sin\frac{\theta}{2}} = \frac{D}{\sin\beta} \tag{1}$$

In triangle ΔPNH , $\gamma = \frac{\pi}{2} + \alpha - \frac{\theta}{2}$ according to the triangle interior angle sum theorem, and by the sine theorem, we get

$$\frac{NH}{\sin\frac{\theta}{2}} = \frac{D}{\sin\gamma} \tag{2}$$

Next, the case of bi-lateral (with overlapping coverage widths) line measurement is discussed:

For the sake of the narrative, we make the following assumptions.

The initial position of the adjacent survey vessel is P'. The intersections of the beam boundaries with the seabed slope on both sides of the transducer opening angle θ are M' and N'. The intersection of a vertical line (i.e., the vertical line of the seabed) with the seabed slope is K. A parallel line of PP is drawn through M to intersect PM at O. The line is then drawn through M to intersect PM. The intersection of the vertical line drawn through P'(i.e., the vertical line of the seafloor) and the slope of the seafloor is H'. Make a parallel to PP' through point M' and intersect PM at point O. Assume that the survey line interval d is known. From the problem statement, $PP' = d$, and from the geometric relationship, $OM' = d$, as shown in Fig. 2

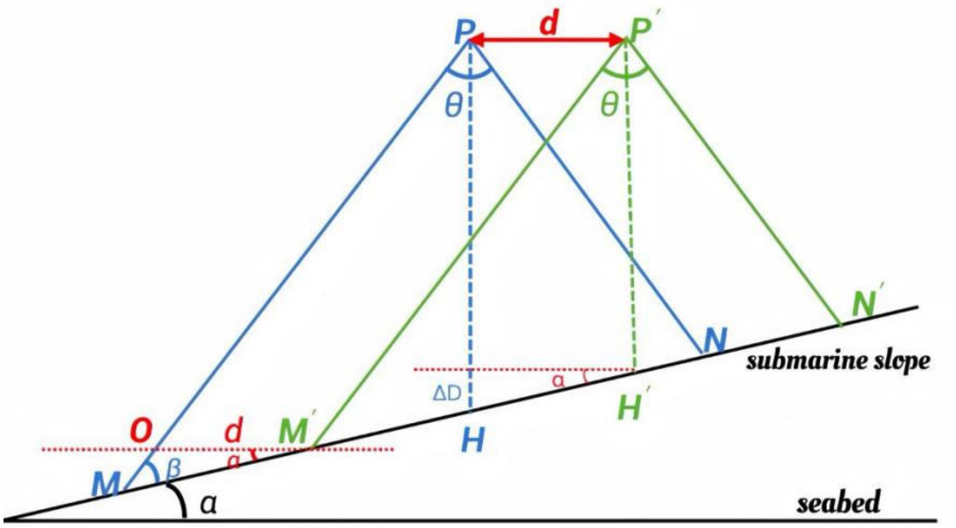

Fig. 2 Schematic diagram of edge setting for a two-way route

Firstly, this paper derives an expression for the depth of seawater D_i at a distance x from the center of the sea, for which the distance x is defined to be negative when the survey line is located in the area of water depths to the left of the center of the more sea, and positive otherwise, then

$$\Delta D = x \tan \alpha \tag{3}$$

$$D_i = D - x \tan \alpha \tag{4}$$

Therefore, the water depth D_i at any distance x from the center of the sea can be determined from this and can be considered as a known quantity in the modeling and calculation process.

In the triangle $\Delta OMM'$, the sine theorem gives

$$\frac{MM'}{\sin(\pi - \beta - \alpha)} = \frac{d}{\sin \beta} \tag{5}$$

It is assumed that the stylus spacing d is measurable and therefore MM' can be represented.

2.2. Calculation of coverage width and overlap ratio for multibeam bathymetry on single slopes

Discussions based on model preparation, set the coverage width to W.According to equations (1) and (2).

$$MN = MH + NH = D \sin \frac{\theta}{2} \left(\frac{1}{\sin \beta} + \frac{1}{\sin \gamma} \right) \tag{6}$$

Coupled with $\beta = \frac{\pi}{2} - \alpha - \frac{\theta}{2}, \gamma = \frac{\pi}{2} + \alpha - \frac{\theta}{2}$, therefore the coverage width

$$W = MN = D \sin \frac{\theta}{2} \left(\frac{1}{\sin\left(\frac{\pi}{2} - \alpha - \frac{\theta}{2}\right)} + \frac{1}{\sin\left(\frac{\pi}{2} + \alpha - \frac{\theta}{2}\right)} \right) \tag{7}$$

Let there be a total of k (k>0) lines in a certain sea area, neighboring lines sailing in opposite directions. Let the route number be i $(0 < \mathrm{i} < \mathrm{k})$,and i increases in the direction of shallower seawater depth. According to the definition of overlap rate: The overlap ratio between the i-th line of measurement and the neighboring (i—1)-th line of measurement is equal to the ratio of the length of the overlap between the i-th line of measurement and the (i—1)-th line of measurement to the width of the coverage of the (i－1)-th line of measurement, thus the overlap rate η_i for the i-th line of measurement is

$$\eta_i = \frac{MN - MM'}{MN} = 1 - \frac{MM'}{MN} \tag{8}$$

According to equations (5)

$$MM' = \frac{d}{\sin\beta}\sin\left(\pi - \beta - \alpha\right) \tag{9}$$

wherein $\beta = \frac{\pi}{2} - \alpha - \frac{\theta}{2}, \gamma = \frac{\pi}{2} + \alpha - \frac{\theta}{2}$.Therefore, the overlap rate η_i between the survey vessel with route number $i(1 < i < k)$ and the survey vessel with route number $i - 1$ is

$$\eta_i = -\frac{\dfrac{d}{\sin\left(\dfrac{\pi}{2} - \alpha - \dfrac{\theta}{2}\right)}}{\tan\dfrac{\theta}{2}\left[\dfrac{D}{\sin\left(\dfrac{\pi}{2} - \alpha - \dfrac{\theta}{2}\right)} + \dfrac{D}{\sin\left(\dfrac{\pi}{2} + \alpha - \dfrac{\theta}{2}\right)}\right]} \tag{10}$$

2.3. Calculus Analysis

Under condition $d = 200m$、$\alpha = 1.5$, depth of seawater、coverage width and overlap are calculated respectively according to equations (4), (7), and (10) in the model. The results are given in Table 1.

Table 1. Calculation of the given indicator

Distance of the Survey Line from the Center Point/m	Depth of Seawater/m	Coverage Width/m	Overlap with Previous Line /%
-800	90.9487	315.813	/
-600	85.7116	297.628	33.64
-400	80.4744	279.442	29.59
-200	75.2372	261.256	25
0	70	243.070	19.78
200	64.7628	224.885	13.78
400	59.5256	206.699	6.81
600	54.2884	188.513	-1.39
800	49.0513	170.327	-11.17

According to Table 1, the overlap rate is less than zero at a distance of 800 m from the center of the survey line. Thus there's been a leakage. The rationale for this phenomenon is analyzed below.

Since this paper defines the direction in which the depth of the sea water changes from deep to shallow as the positive direction in which the distance of the survey line from the center of the sea area increases, As is shown in Fig.3, the arrow points in the direction of increasing x. As x increases, the depth of the sea PH at the position of the corresponding measuring vessel is constantly decreasing, due to $\Delta PHM \cong \Delta P'H'M'$, the result is that the coverage width is getting smaller and smaller, in other words, within the range (-800,800), both the coverage width and the repetition rate should decrease as x increases, and leakage occurs when the acoustic beam boundaries of the two do not intersect.

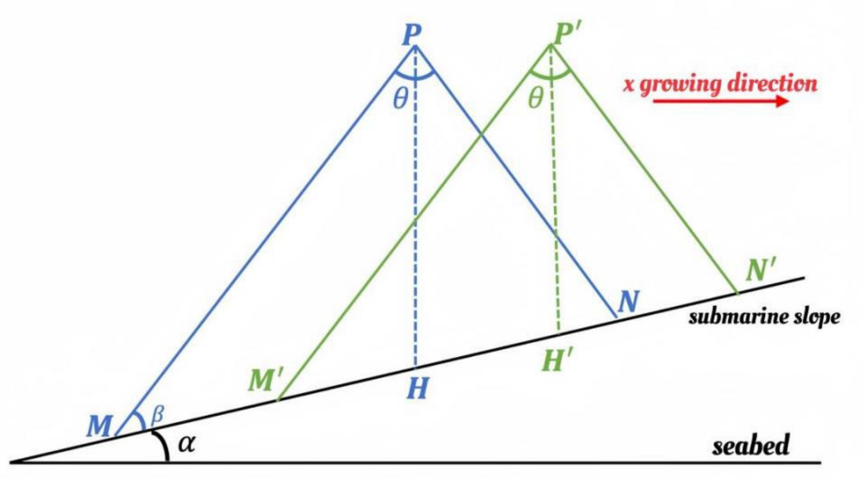

Fig. 3 Schematic of the positive direction of increase in the distance of the survey line from the center of the sea area

3. Results

3.1. The building of the model

Select the intersection line between the seabed slope and the horizontal plane to the right as the positive direction of the x-axis, set the line that passes through the center of the sea area and is parallel to the x-axis as the reference line, and set the angle between a certain line and the positive direction of the x-axis as the offset angle ω of the line. Points M and N are the intersections of the sound beam boundary with the seafloor slope. MN is the coverage width, the projections of points M and N on the horizontal plane are H and F, the intersection of the extension lines of MN and HF is Q; Extend HQ to intersect the x-axis at point T; Make ML perpendicular to the x-axis on the submarine slope through the point M, connect HL. As is shown in Fig.4.

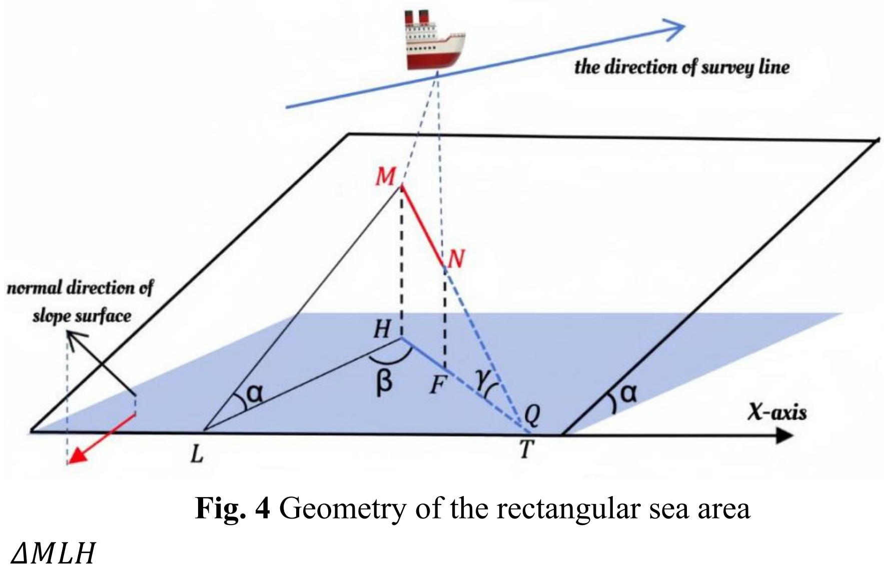

Fig. 4 Geometry of the rectangular sea area

In triangle ΔMLH

$$\tan\alpha = \frac{MH}{LH} \tag{11}$$

In triangle ΔMHQ

$$\tan\gamma = \frac{MH}{QH} \tag{12}$$

In triangle ΔHLQ

$$HQ = \frac{LH}{\cos\beta} \tag{13}$$

Associative formulas (11), (12), (13)

$$\gamma = \arctan\left(\tan\alpha\cos\beta\right) \tag{14}$$

At this point, the variable sailing equivalent slope angle of the survey line is known to convert the solution model to the coverage width model in 2.1, Replacing α in Eq. (7) with γ

$$W = MN = D\sin\frac{\theta}{2}\left(\frac{1}{\sin\left(\frac{\pi}{2}-\gamma-\frac{\theta}{2}\right)}+\frac{1}{\sin\left(\frac{\pi}{2}+\gamma-\frac{\theta}{2}\right)}\right) \tag{15}$$

Through the above discussion this paper also found that, under condition $\omega = \frac{\pi}{2}\ or\ \frac{3\pi}{2}(\beta = 0°or\ \beta = 180°)$, $\gamma = \alpha$, measuring vessel moving on a fixed slope, As shown in Fig.5. In this process the coverage width is constant

$$W = 2D\tan\frac{\theta}{2} \tag{16}$$

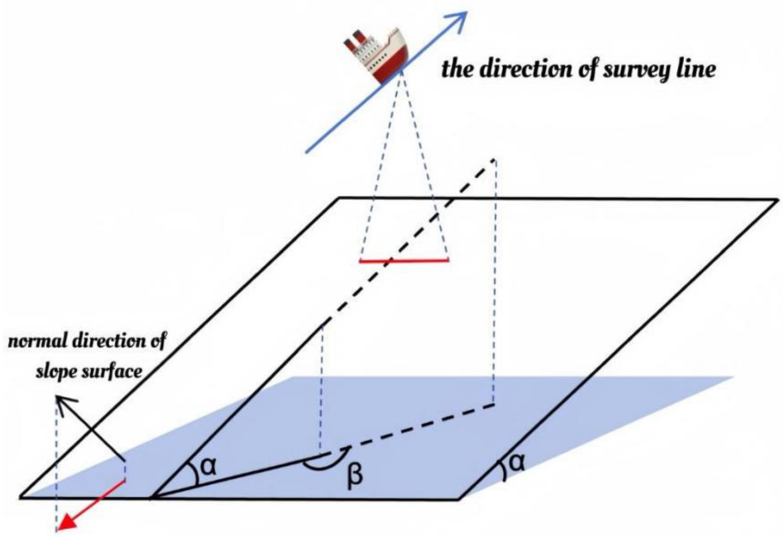

Fig. 5 Schematic diagram of special cases

In summary, the expression for the coverage width is

$$W = \begin{cases} 2D\tan\frac{\theta}{2}, \beta = 0^{\circ}\text{or}180^{\circ} \\ D\sin\frac{\theta}{2}\left(\frac{1}{\sin\left(\frac{\pi}{2}-\gamma-\frac{\theta}{2}\right)}+\frac{1}{\sin\left(\frac{\pi}{2}+\gamma-\frac{\theta}{2}\right)}\right), \beta = \text{else} \end{cases} \tag{17}$$

3.2. Calculus Analysis

Under condition $d = 0.3n\ mile$、$\alpha = 1.5°$, The left side of the direction of progress of the survey vessel is defined as the direction of increasing distance from the center point, the coverage width data is calculated from equation (17) in the model and the image is plotted as shown in Fig. 6.

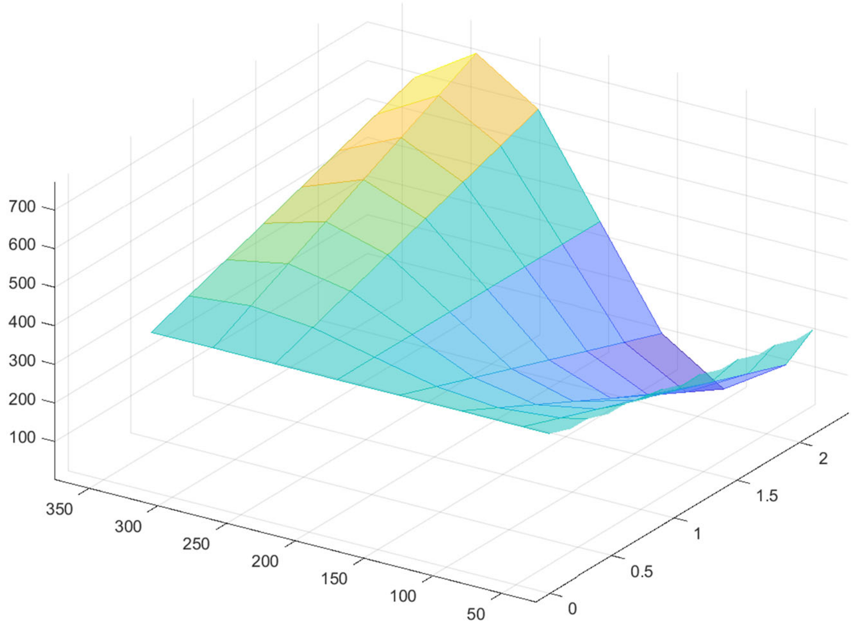

Fig. 6 Coverage width schematic

When the angle $\beta = 0°\ or\ 180°$, Coverage width is a constant 415.6922 meters. When $0 < \beta < 180°$, the width of the line coverage decreases as the distance of the survey vessel from the center of the sea increases. When $180° < \beta < 360°$, As the distance of the survey vessel from the center of the sea increases, the width of the survey line coverage increases, as shown in Fig. 7 d

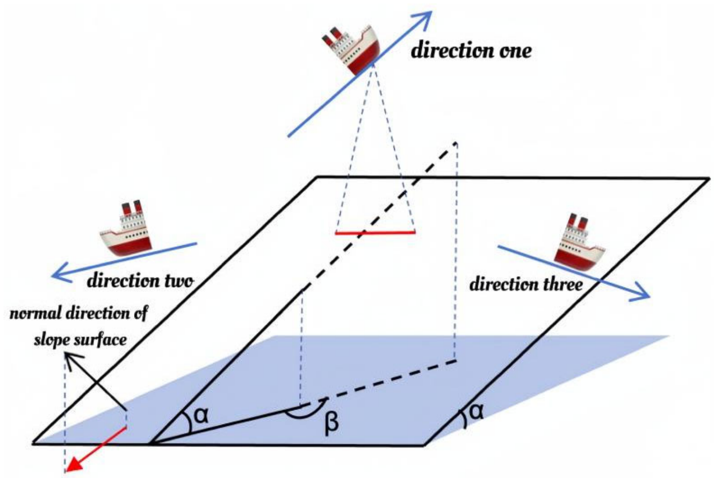

Fig. 7 Coverage width schematic

4. Research on optimal survey line placement in rectangular sea area based on iterative method

Based on the regulations on the layout of survey lines in the sea area, most of the sea area currently adopts the method of parallel isobaths or perpendicular isobaths to lay survey lines [5,6]. For a set of determined survey line distribution, the indicators to evaluate the advantages and disadvantages of its deployment include the coverage of the sea area, the total length of survey lines and the overlap rate between survey lines [7].

4.1. Rectangular Seas Line Laying Study

This paper takes a rectangular sea area as an example. When the direction of the survey line is parallel to the isobath, the water depth D is kept constant during one measurement, set the width of the water in the east-west direction of the rectangular sea area as d_{we}, and set the survey lines numbered from 1 to n from west to east, respectively, and set the corresponding coverage width of the nth survey line as W_n, the spacing with the nth-1st line as d_n, the overlap rate as η_n, and the corresponding x-axis coordinates as x_n, where $d_n = x_n - x_{n-1} (n \geq 2)$, and the line placement scheme is required to cover the entire sea area with an overlap rate between 10 and 20 per cent.

Optimization is performed with the objective of minimizing the total length of the survey line:

$$\max\left(\min d_n\right) \tag{18}$$

where $n \geq 2$ and d_n is the interval between the nth measurement line and the n-1st measurement line.

$$s.t.\begin{cases} 10\% \leq \eta_n = 1 - \dfrac{d_n}{W_{n-1}} \leq 20\% \\ \displaystyle\sum_{i=2}^{n} W_i = \dfrac{d_{we}}{\cos\alpha} \end{cases} \tag{19}$$

where d_{we} is the width of the watershed in the east-west direction of the rectangular sea area, W_n is the coverage width corresponding to the nth survey line, d_n is its spacing from the n-1st survey line, and η_n is the overlap rate.

The total length of the line (m) is:

$$l_{\text{total}} = n \times 2 \times 1852 \tag{20}$$

When the direction of the survey line is perpendicular to the isobath, the water depth D increases or decreases during a voyage, and the coverage area of a voyage is a trapezoidal shape that is narrow at the top and wide at the bottom: the side lengths are wide in the deeper part of the water, and the side lengths are narrow in the shallower part, i.e., a trapezoidal shape that is wide in the west and narrow in the east, and the deeper part of the water is certainly not missed when there is no leakage of the shallow area. Therefore, when the short edges meet, i.e., when there is a critical overlap in the shallow areas, it is a line design programme that meets the requirements for the shortest total length of the line and the range of overlap rates and full coverage of the sea area. This is shown in figures 8 and 9.

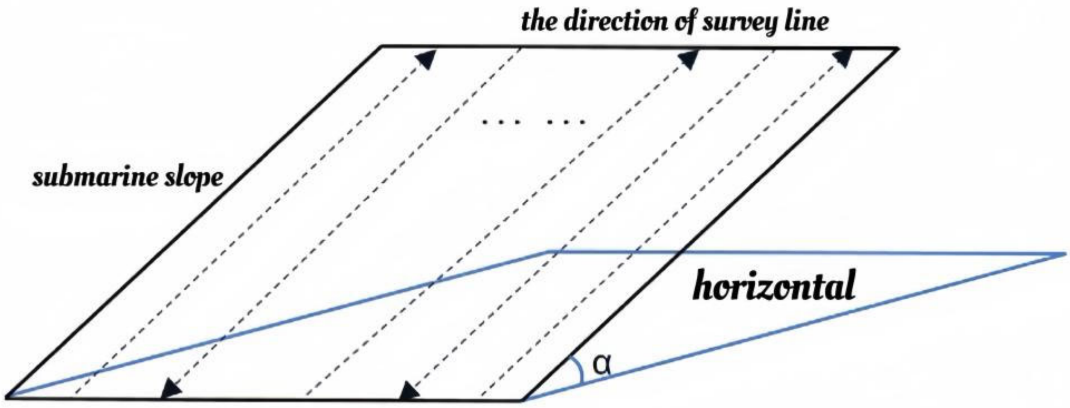

Fig. 8 Schematic of the slope projection of the survey line

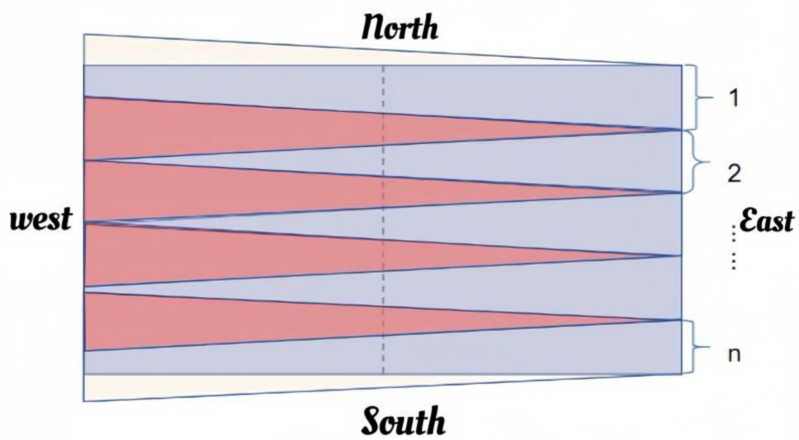

Fig. 9 Schematic illustration of the overlap of horizontal sea coverage widths

In Fig.9, any short edges are equal and summable, and the minimum number of survey lines required under this scheme is when the short edges are in close proximity to each other. Let the nth short edge be Δl_n (in metres), then the number of survey lines required for full coverage of the sea area is at least:

$$n = \left\lceil \frac{2 \times 1852}{\Delta l_n} \right\rceil \tag{21}$$

Since the direction of the survey line is perpendicular to the isobath, the total length of the survey line, l_{total} is:

$$l_{\text{total}} = n \times 4 \times 1852 \tag{22}$$

By solving the model developed in this paper, the optimal survey line deployment scheme for this rectangular sea area can be obtained as shown in Fig.10.

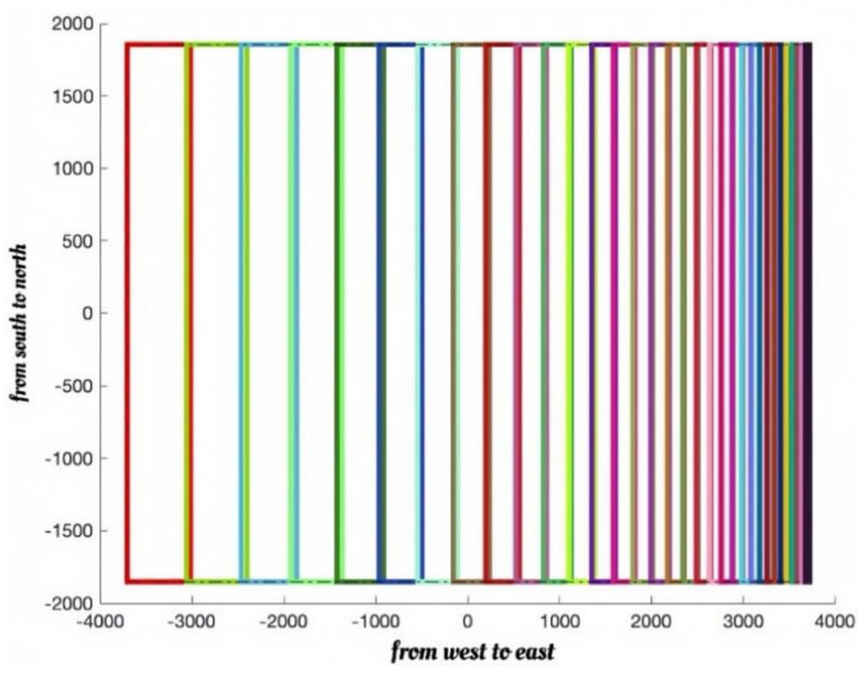

Fig. 10 Coverage of strips in the direction of parallel isobaths

At the same time, this paper also examines the case of laying lines perpendicular to the isobaths, and since at least 78 lines are required under this scenario when the trapezoidal strips on the eastern side of the sea are densely connected as shown in Fig. 9, which is much less than the number of lines required at least for the laying of lines parallel to the isobaths, it will not be repeated here.

4.2. Strategies for the placement of survey lines in general marine areas

In this paper, a more detailed study is made of the survey line laying programme in rectangular sea areas. For general sea areas, a least-squares fit can be made to the seabed topography of the sea area, resulting in a surface of the seabed topography. Since the surface can be replaced by a series of planes, the uneven seabed terrain can be gradually replaced by slopes, and each slope can be considered as a rectangular sea area.

5. Conclusion

Multibeam sounding has the advantages of high efficiency and high accuracy compared with the traditional single-beam measurement scheme[8,9], but the quality of its measurement is more dependent on the layout of the survey line, so it is crucial to lay the survey line in a reasonable way. In this paper, the sine theorem is used to study the survey line arranged along the isobath direction of a single inclined slope, and the mathematical model of the depth of seawater and the interval width of the survey line affects the precision parameters of the survey line; at the same time, this paper utilizes the equivalent slope angle to study the general case of variable direction of the survey line of the slope surface, and obtains the mathematical model of the coverage width of the survey line in the context of the situation; due to the actual problem of the need for bathymetric survey of the sea area of the terrain is variable, Because the topography of the sea area to be surveyed in the actual problem is variable and the area is not fixed, this paper gives an example analysis of the survey line layout for a single slope rectangular sea area, and at the same time, for the more general sea area to be surveyed[10], we use the least-squares fitting and topographic sectioning to discuss the survey line layout scheme for the general situation, which enhances the completeness and practicability of the research scheme described in this paper.

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